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SUGGESTIONS FOR THE DEFORMABLE SUBREFLECTOR

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Observations with the present experimental version (Engineering Div. Internal Report 109, July 1978) have shown that a deformable subreflector can indeed correct the gravitational astigmatic deformation of the main reflector to a good degree. At 20° elevation, south and east, the efficiency at λ = 1.345 cm was already increased by more than a factor of two, in spite of the limited deformation range of the present version, and enlarging this range would be desirable. However, the other three possible modes of deformation did not give significant changes of efficiency, but we do not know at present whether or not the reflector shell actually did deform by sufficient amounts in these other modes.

The present MEMO treats three items. First, we suggest measuring the shell stiffness for all four possible modes of deformation. Second, we define a desirable range for the astigmatic deformation. Third, regarding some planned new feed locations on the Cassegrain tower, we calculate the efficiency as a function of feed location, suggesting rather narrow limits for the latter. How much of all these suggestions is actually feasible will then depend on the available money, manpower and time.

I. Shell Stiffness for Different Modes

Figure 1 defines and describes the four modes of deformation which are possible with four actuators. It is immediately clear than an open segment of a curved shell is much softer against an astigmatic deformation than against any other kind. From the four modes of Figure 1, the shell will be stiffest against Mode 2, but this also seems to be the least important one. Regarding the strong EW deformations observed in Report 109 (Fig. 12 - 15), it could well be that Mode 4 is important enough for application, and maybe also Mode 3 for some smaller non-astigmatic NS deformation. It thus seems worthwhile to investigate these possibilities.

The proper procedure seems, first, to measure the stiffness, in terms of kilogram of force as applied to the shell at the actuating points, divided by the resulting amplitude of the rim deformation in millimeter. This should be done for each of the four modes separately. Second, we must define a desired range for these amplitudes. The astigmatic mode will be treated in the following section. For the other three modes, we need enough deformation to measure a significant change of the efficiency, for the better or worse; and I think the desired range should be at least 3 mm, say, for the rim amplitude, which would give about 1 mm rms over the whole surface with 15 db taper.

Third, we need some estimate or guess of how much force the shell surface can take without breaking or being pulled apart. This limiting force, divided by the stiffness of a mode, gives a safety range for the rim amplitude of this mode. Fourth, if this safety range is much smaller than the desired range, for some mode, then this mode cannot be used. Fifth, for each of the remaining usable modes, we find the needed force (desired or safety, whichever is smaller), and the next question is whether or not we find motors which deliver these forces with the required accuracy ($\leq$ 0.2 mm, say) for a reasonable price.

Previous measurements of the subreflector shape have been done along 24 radii with 6 points per radius (total of 145 points). If possible regarding manpower, it would be good to do it now the same way, six times: first with the undeformed surface, then once for each of the four modes, using just the maximum force (or maximum travel, whichever comes first) of our present motors, and finally an undeformed case again. In addition to the value for the stiffness, one would like to obtain, if possible, a complete contour map for each deformation mode. Furthermore, in each mode a small number of points should be measured with various different forces, as a check for linearity and accuracy.

As a measure of the force applied to the actuating points, it seems best to use the voltage supplied to the motors. This would need a calibration first, in kg/volt.

II. Desired Astigmatic Range

We write the astigmatic deformation as

$$\Delta z = A\, r^2 \cos(2\Omega) \tag{1}$$

where $r = 1$ at the rim, $\Omega = 0$ north, and where the rim amplitude A depends only on elevation β as

$$A(\beta) = -A_o + (1 - \sin\beta)\, B. \tag{2}$$

The two constants have been measured in 1977 with an elongated feed horn as $A_o = 1.72$ mm and $B = 8.70$ mm. With these values, $A(\beta)$ has been plotted in Figure 2, together with the air mass $M(\beta) = 1/\sin\beta$.

How close to horizon should we be able to correct the astigmatism? Because of the steep increase of the air mass, observations at short wavelengths will seldom go below 15° elevation (but occasionally down to 10° for spectrum lines of southern sources). As explained in Figure 2, we suggest to specify $\beta = 15°$, but to add a safety margin of $S = 1.3$ mm, yielding a desired range of $-3.0 \text{ mm} \leq A \leq +6.0 \text{ mm}$.

The subreflector could then be used for a feed at the design location and at a second location rotated about the vertex by 180°. It is better, however, to provide two more locations, at 90° and 270°, which needs equal range for both signs. The desired astigmatic range then is

$$-6.0 \text{ mm} \leq A \leq +6.0 \text{ mm}. \quad (3)$$

This is the desired range for the <u>rim</u> amplitude. The forced deformation at the actuated points is less because they are further in, but the travel range of the motors must be larger because the backup frame has some flexibility.

As compared to our present range, $-0.44 \text{ mm} \leq A \leq +1.81 \text{ mm}$, we have the following desired improvement factors for the astigmatic mode (the other modes may need more force):

$$\text{force factor} = 6.0/1.81 = 3.31, \quad (4)$$

$$\text{range factor} = (6.0+6.0)/(1.81+0.44) = 5.33. \quad (5)$$

III. Feed Location and Efficiency

The design location of the feed is $\phi_o = 22.5°$ north of west for our present subreflector, see Figure 3. If some other angle ϕ is chosen, the subreflector must be rotated by $\Delta\phi = \phi-\phi_o$ about the telescope axis. Its stiff diagonal and its actuators then are set off from their proper place by $\Delta\phi$, which degrades the astigmatic correction. This degradation has been calculated, and the following gives only the results without derivations.

If A is the astigmatic rim amplitude of the main reflector, and $\phi \neq \phi_o$ is the feed angle in Figure 3, then the best deformation amplitude for the subreflector, a, is not any more equal to A but smaller, and a least-squares treatment gives

$$a = A \cos 2(\phi-\phi_o). \quad (6)$$

Using this best amplitude, the remaining residual, averaged over the whole surface and with 15 db taper, is

$$\Delta z_a = \text{rms}\ (\Delta z_{astgm}) = 0.329\ A\ |\sin 2(\phi - \phi_o)| . \tag{7}$$

We add quadratically the rms surface error Δz_o without gravitational deformation, and obtain the total surface error

$$\Delta z = \sqrt{\Delta z_o^{\ 2} + \Delta z_a^{\ 2}} . \tag{8}$$

The aperture efficiency then is (with η_o for $\lambda \to \infty$)

$$\eta = \eta_o\ e^{-(4\pi\Delta z/\lambda)^2} . \tag{9}$$

Figure 3 was calculated with A = 4.00 mm for β = 20° elevation, assuming Δz_o = 1.20 mm and η_o = 60%. We see that the four maxima are more narrow than the rather deep and wide minima. The maximum efficiency in this calculated example is η_{max} = 17.1%; if we do not want to lose much, say $\eta \geq 15\%$, then the feed location must be in the range $\phi = \phi_o \pm \Delta\phi$, with

$$\begin{aligned} \Delta\phi &= 8.4° \quad \text{for} \quad \beta = 20° \text{ elevation} \\ \Delta\phi &= 7.1 \qquad\qquad \beta = 15 \\ \Delta\phi &= 6.1 \qquad\qquad \beta = 10 . \end{aligned} \tag{10}$$

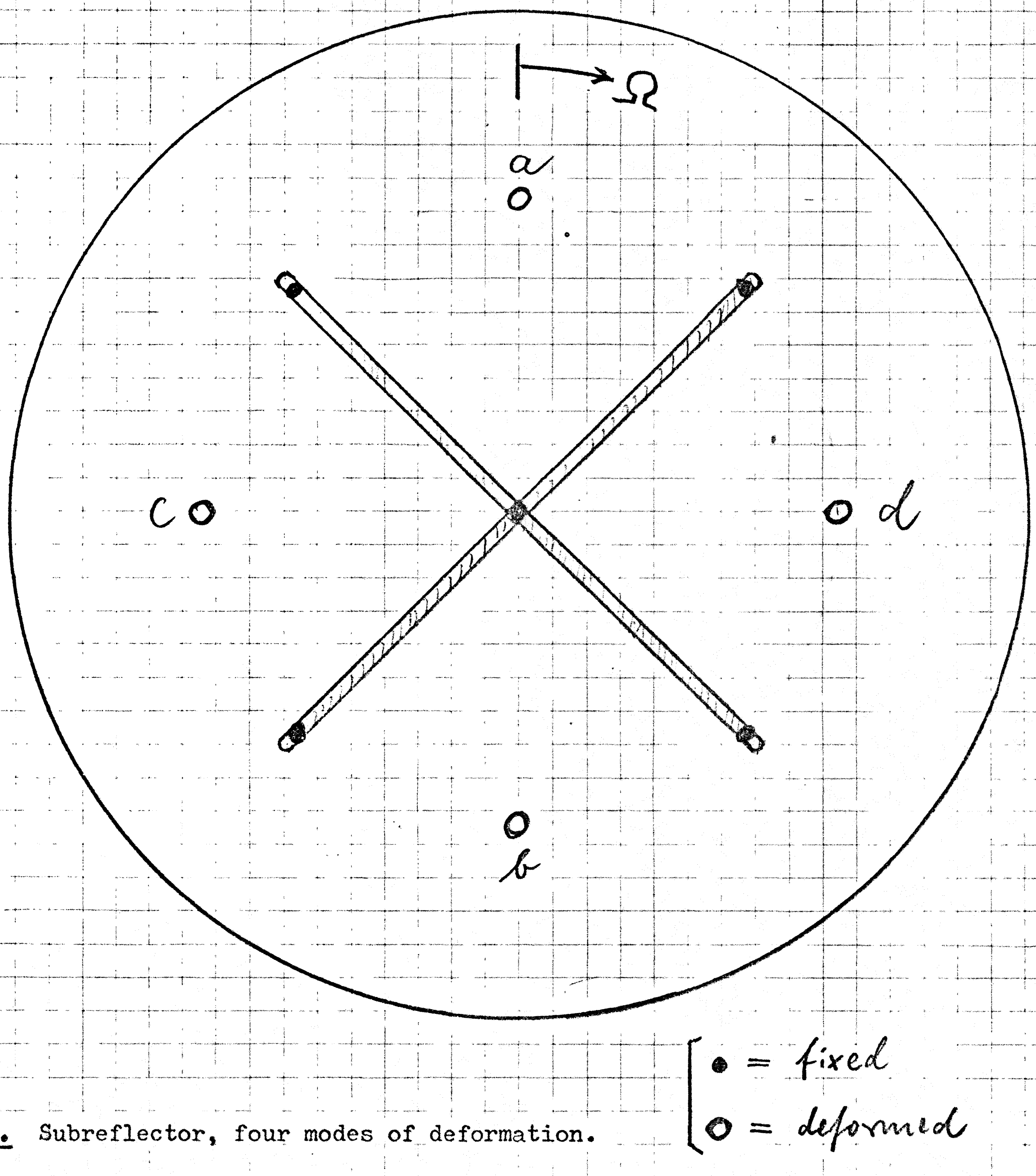
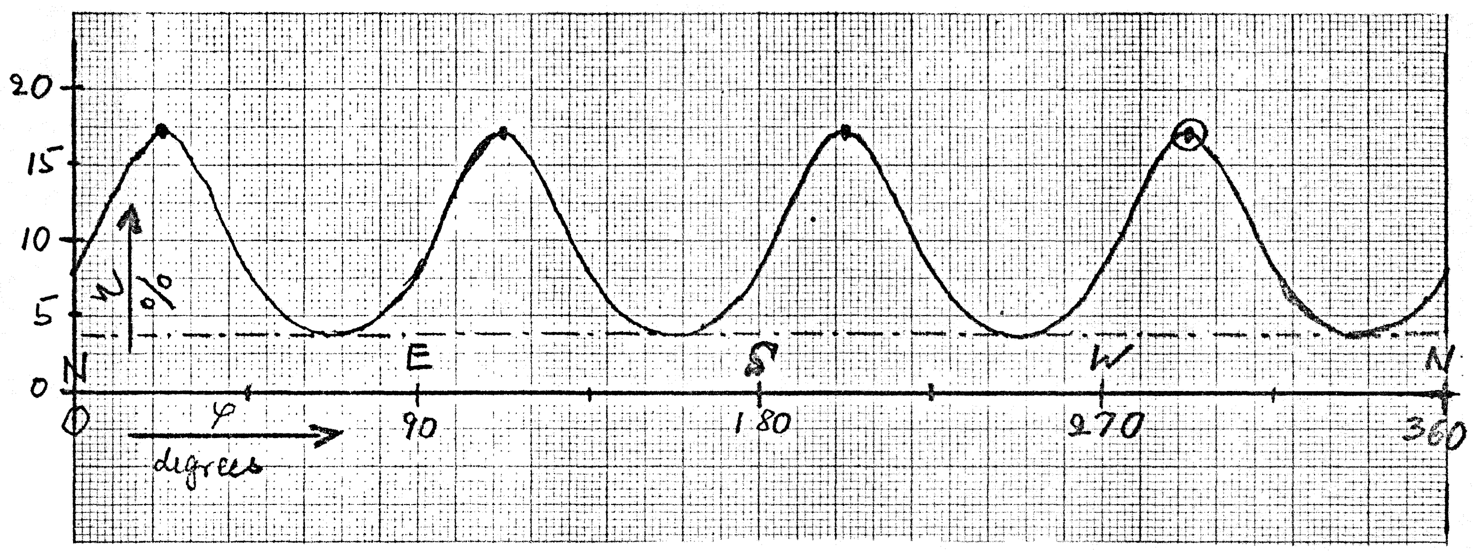
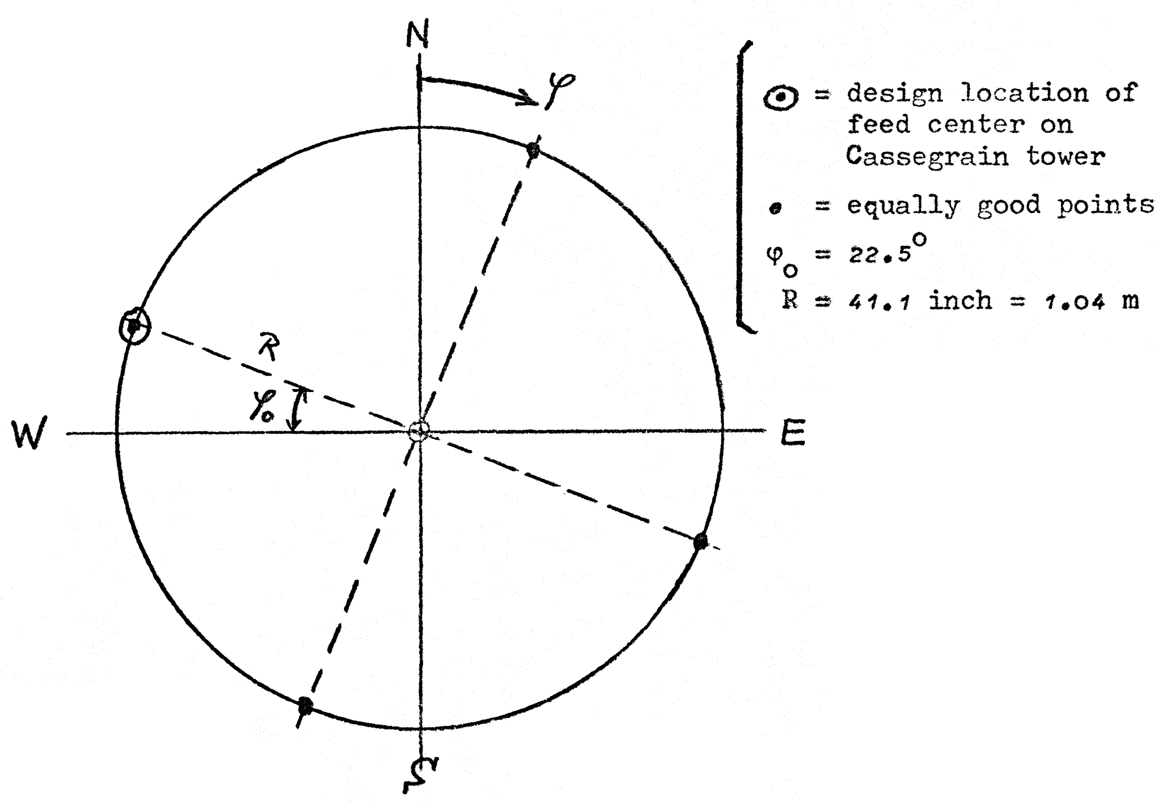


Fig. 1. Subreflector, four modes of deformation.

Mode No.	deformations a	b	c	d	description
1	+1	+1	−1	−1	astigmatism, $\Delta z \sim \cos(2\Omega)$
2	+1	+1	+1	+1	next order, $\Delta z \sim \cos(4\Omega)$
3	+1	−1	0	0	vertival shift of half-beam
4	0	0	+1	−1	horizontal shift of half-beam





<u>Fig. 3.</u> Aperture efficiency η , as a function of the feed location angle φ; with (————) and without (—·—·—·) subreflector deformation.

Calculated for elevation $\beta = 20^{\circ}$, and with 15 db illumination taper. If the rim deformation of the subreflector can be made equally large for both signs, $-6.0 \leq A \leq +6.0$ mm, then there are four equally good locations.