An Analysis of the Effects of Phase Dithering in a Lag-based Fringe-Stopping XF Correlator

NRC-EVLA Memo# 002

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ABSTRACT

In NRC-EVLA Memo# 001 [1], it was stated that it is not known precisely what the effect on visibility amplitudes of phase dithering in a lag-based, quantized-phase, fringe-stopping XF correlator chip might be. This correlator chip architecture is being proposed for the WIDAR-EVLA correlator since it may result in a complex correlator chip with lower power and fewer transistors than the alternative 'conventional' architecture. This is important since full 4-bit correlation is being contemplated for improved spectral dynamic range and sensitivity. This short memo will show that varying degrees of phase dithering do not incur any additional SNR loss over the case where there is no dithering. This is a key point that guarantees that visibility amplitudes will not, in any way, be modulated by varying dithering patterns. Varying patterns could result from a combination of different frequency shifts at different antennas, and Doppler phase compensation in the correlator. Additionally, an accurate value for the SNR loss from a proposed 5-level fringe rotation function is established to be 2.25%—roughly ½ the loss of a 3-level function.

Introduction

The proposed WIDAR correlator for the EVLA [1] requires a complex correlator to remove different frequency shifts imposed on the antenna Local Oscillators, to facilitate fully digital 1/16th sample delay tracking, and to simultaneously remove fringe/Doppler phase[1] due to the motion of the antennas. The best known way to perform this phase rotation in an XF correlator is with a 3-level complex digital mixer (or 5-level for reasons explained in [1]). There are two ways that this can be done:

1. Determine the phase difference ($\phi_x(t) - \phi_y(t)$) and rotate *one* data stream (Y) at one end of the lag chain to generate in-phase and quadrature Y data streams. At

[1] This is optional. Doppler phase could be removed by modifying the phase of the antennas' Local Oscillators, but since the correlator contains phase-rotators it would seem to be more efficient to do it in the correlator.

each lag, perform the complex correlation with the X data. This is the method normally used in 'conventional' XF VLBI correlators. This method requires two multipliers at each lag—requiring a fairly significant amount of additional logic (and power) at each lag for a 4-bit correlator. Also, now three data streams—all operating at the high bit rate—must be carried through the correlator chip delay line. This extra data stream will contribute to power dissipation in the chip.

2. Quantize the X and Y station phase to 4-bits and carry the phase along with the data through the lag chain. Form the lag-based quantized phase ($\hat{\phi}_x(t-\tau_x)-\hat{\phi}_y(t-\tau_y)$) and then perform the complex mixing and correlation at the lag. This method requires only one simple multiplication, followed by complex mixing. Although phase must be carried with the data through the lag chain, phase (for the wide bands proposed for the EVLA) is changing very slowly and contributes virtually nothing to power dissipation. This method is described in detail in [1] and [2].

With method **1.** there is no phase dithering since the baseline-based phase generator operates with a high number of bits. With method **2.**, phase dithering (oscillation between two adjacent phase states when changing to a new phase state) occurs if the X and Y phase rates are not harmonically related. This is because phase is coarsely quantized before the phase subtraction occurs. This memo performs a numerical analysis to show that the SNR loss of a 3 or 5-level digital mixer with 4-bit phase quantization is constant and independent of how much dithering there is or whether there is any dithering at all.

Analysis Methods and Results

The analysis closely follows the treatment for determination of 3-level fringe rotation efficiency in [3]. The efficiency is calculated as the square root of the ratio of the power of a pure sine wave optimally fit[2] to the digital function, to the power of the digital function. Using Parseval's Power Theorem this ratio can be written as:

$$\eta_{digital}=\sqrt{\frac{\sum_{n=-\infty}^{\infty}\left[A\sin(\phi_x(n)-\phi_y(n))\right]^2}{\sum_{n=-\infty}^{\infty}\left[\hat{f}_{digital}(\hat{\phi}_x(n)-\hat{\phi}_y(n))\right]^2}}$$

Where A is the amplitude of the pure sine wave optimally fit to the digital function, $\hat{f}_{digital}$ is the digital mixing function, and the ^ symbol indicates discrete or quantized values. For a digital 3-level fringe rotation function with 16 phase steps over 360°, A=1.18[3] and with or without[4] dithering $\eta_{digital}=0.960$—identical to the value stated in [3]. For a 5-

[2] i.e. least-squares fit.
[3] Further refinement as part of this investigation found that A=1.176.
[4] Resulting in the inevitable conclusion that the additional fringe rotation loss from dithering stated in [2] is incorrect.

level fringe rotation function with 16 phase steps, A=1.955 and with or without dithering $\eta_{digital}=0.9775$—corresponding to a 2.25% SNR loss. Note that the equation calculates $\eta_{digital}$ over an infinite number of points. Calculation over a finite number of points to achieve the same result requires an integer number of cycles of differential phase in the integration. In general, the correlator will not integrate over an integer number of phase cycles, nor will it be able to integrate over an infinite number of cycles. This will result in small changes in $\eta_{digital}$. For example, for an integration over ~9.5 cycles with 5-level fringe rotation, the calculated $\eta_{digital}$ is 0.9769—a difference from the ideal of a few parts in 10^4. In the real correlator, this example is equivalent to a 0.95 millisecond integration time with a Local Oscillator offset of 390 kHz in one antenna and 400 kHz in the other antenna. Clearly, longer incoherent or coherent integration times will reduce this difference until it is essentially negligible.

Some example 5-level dithered and non-dithered functions are shown in the following figures. Intuitively, it is clear how Parseval's Theorem calculates the same power on the digital function in dithered and non-dithered cases. In all cases, the frequency difference is 10 kHz.

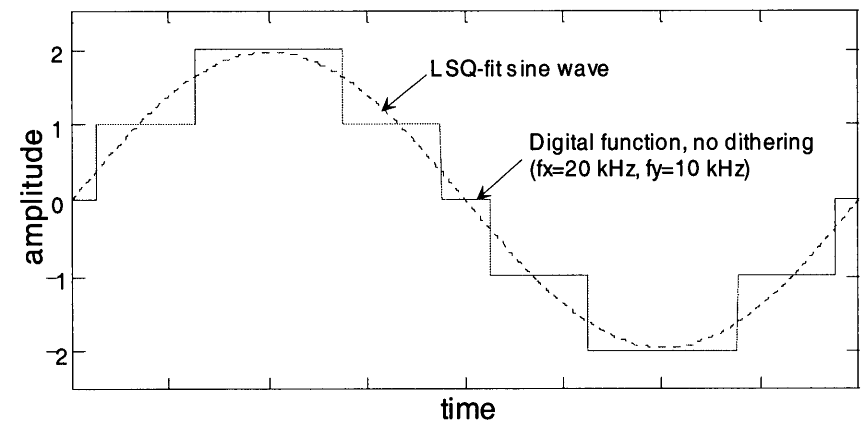

Figure 1 Digital mixing function and least-squares-fit sine wave with no dithering. There is no dithering because the X and Y frequency shifts are harmonically related.

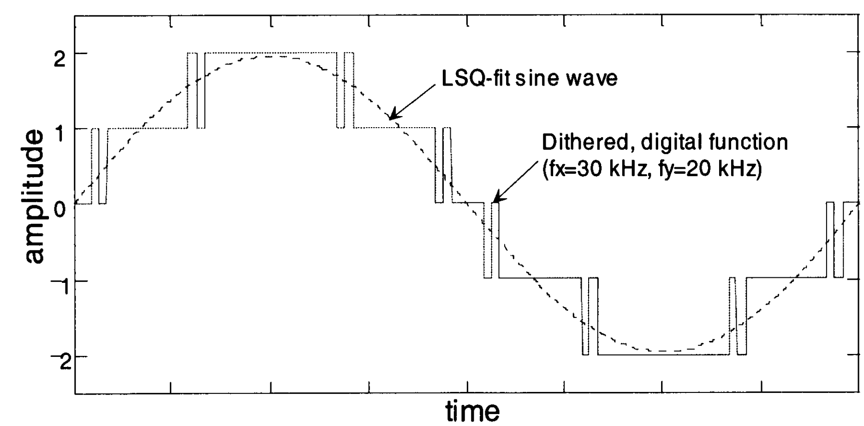

Figure 2 Digital mixing function and least-squares-fit sine wave with some dithering. Not much dithering occurs during the transitions because the frequency difference is close to the absolute frequency shifts in both antennas.

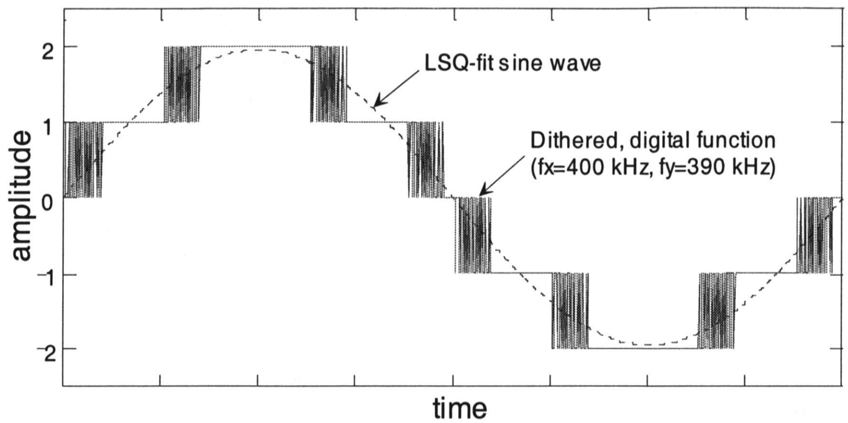

Figure 3 Digital mixer function and least-squares-fit sine wave with considerable dithering at each transition. Dithering occurs to this extent because the frequency difference is small compared to absolute frequency offsets.

Conclusions

This short memo has demonstrated that the 3-level or 5-level digital mixer function achieves the same efficiency whether the resulting differential-phase digital-mixer function has dithering or no dithering. This is a crucial conclusion to ensure that the proposed correlator chip architecture outlined in [1] is feasible for the WIDAR-EVLA correlator. A theoretical efficiency of 0.9775 for the proposed 5-level digital mixer function was also determined.

References

[1] Carlson, Brent, A Proposed WIDAR Correlator for the Expansion Very Large Array Project: Discussion of Capabilities, Implementation, and Signal Processing, NRC-EVLA Memo# 001, May 18, 2000

[2] Carlson, B.R., Dewdney, P.E., Burgess, T.A., Casorso, R.V., Petrachenko, W.T., Cannon, W.H., The S2 VLBI Correlator: A Correlator for Space VLBI and Geodetic Signal Processing, Publications of the Astronomical Society of the Pacific, 1999, **111**, 1025-1047.

[3] Thompson, A.R., Moran, J.M., & Swenson, G.W., 1986, Interferometry and Synthesis in Radio Astronomy, Wiley, New York.