EVLA Memo 163
Recirculation in WIDAR with phase serialization

R.J. Sault

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Introduction

A fundamental aspect of the WIDAR correlator is that the sampled astronomical datastream from each antenna is accompanied by a model phase stream. This model phase accounts for, amongst other things, natural fringe rate and the $f_{\rm shift}$ frequencies that are used to frequency shift the data stream from each antenna. The phase models of the two antennas of a baseline are used for fringe stopping and $f_{\rm shift}$ removal in the lag correlator cells that make up WIDAR.

This memo investigates some characteristics of WIDAR response when the phase model is sampled at a lower rate than the astronomical data. This will introduce a phase noise which can lead to significant decorrelation if the phase rates are sufficiently high. This issue is of interest with some modes (but not all modes) of recirculation in WIDAR. WIDAR can use baseboard stacking (sometimes called 'static recirculation') as well as classical recirculation to achieve large numbers of spectral channels with high spectral resolution. Hardware restrictions of one of the modes of driving these forms of recirculation requires that the phase model is decimated by a factor of 8 relative to the astronomical datastream. This mode of operation is known as "phase serialization". Some characterization of serialization is given in the WIDAR document "Requirements and functional specification: Recirculation controller FPGA" - hereafter called the Recirculation RFS. Page 50 gives some analysis of this effect. This document also notes that the phase serialization and deserialization is handled in such a way that there is a predictable sample-rate dependent phase offset introduced. To account for this, the phase model should be calculated for 3.5 sample times later than it otherwise would be calculate for (page 60).

Tests and analysis

To investigate the effect, we have performed observations of a strong continuum point source at an observing frequency of 5 GHz. A total of 25 antennas were available. Baseline board stacking ('static recirculation') achieving 256 channels across a 2 MHz band. We also simultaneously observed 128 channels across the same 2 MHz band - a setting that does not require any form of recirculation or phase model decimation. Several observations were performed with the so-called $f_{\rm shift}$ fundamental frequency set to different values, namely $f_0 = 2$, 3, 4, 4.1 and 6 kHz. The $f_{\rm shift}$ frequency for antenna i (i running from 1 to 27) is approximately equal to $(32 - i) \times f_0$. Thus the $f_{\rm shift}$ frequencies for the tests were in the range of about 10 to 180 kHz. The actual $f_{\rm shift}$ frequencies were 100 Hz multiplied by a prime number (in the analysis that follows the actual $f_{\rm shift}$

frequencies were used rather than the approximation). Note that these f_{shift} frequency setting are significantly larger than would normally be used for an observation of this type. This was intentionally done to make the effects more readily measurable. The natural fringe rates for this observation are insignificant compared with the f_{shift} frequencies[1] and have been ignored in computing the phase models.

In the reduction of the data, antenna amplitude and phase calibration was determined for the data observed with no recirculation. These calibrations were then applied to the data observed with recirculation. A second phase-only calibration was then performed on the data observed with recirculation. Apart from correcting for small delay errors across the band, no bandpass calibration was performed on any data.

Amplitude response

For the observation with no recirculation, the data showed good amplitude response. There were not apparent closure amplitude errors, although the scatter in closure amplitudes were somewhat larger than predicted. The observations with recirculation showed clear amplitude decorrelation. For these data, Fig. 1 plots the real part of the RR vs LL correlations on each baseline for all f_{shift} settings. These were formed by averaged data over the central 70% of the 2 MHz band. These data have been normalized by the flux density of the no-recirculation data. Clearly there is nearly 20% decorrelation on some baselines, and the RR and LL experience the same decorrelation - as expected.

Decimation of the phase model will introduce a sawtooth waveform error in the phase model used with the data. For an otherwise perfect correlator, this phase error will result in amplitude decorrelation by a factor [2]

$$\mathrm{sinc}(\frac{8f'}{f_s})$$

where f_s is the sampling rate (4 MHz for our tests) and f' is an "effective" baseline phase rate. The effective baseline phase rate comes about when combining the model phase errors of the two antennas. If the model phase errors were statistically independent, then one would combine the individual antenna phase rates in quadrature to get the baseline phase rate, i.e. for our observations with f_{shift} frequencies of f_x and f_y,

$$f' = (f_x^2 + f_y^2)^{\frac{1}{2}}.$$

However the phase model errors will not be statistically independent - particularly for a continuum source. Clearly there is a timing relationship between the errors in two model phase streams. It is unclear whether this timing relationship stays in lock step throughout a correlator integration. For a source which is spectrally unresolved, the spectrum is formed by summing over many lags. In this case, the phase model errors being statistically independent is likely a good representation. However for a continuum source, where there is a single dominant lag cell, it is unclear whether combining the phase rates in quadrature is an appropriate representation. One could imagine a case where the f_{shift} frequencies are nearly the same on two antennas, where the error in the phase models are in lock step so that the phase errors cancel out at the zero lag. In this case there would be little or no decorrelation regardless of f_{shift}. This argument suggests, for the zero lag value,

$$f' = f_x - f_y.$$

This gives a significantly smaller effective phase rate and less decorrelation. Given the WIDAR lag ladder, this argument suggests that the decorrelation will be a function of lag.

[1] The observation was in B array, which gave natural fringe rates no more than a few tens of Hertz.

[2] Note the decorrelation equation on page 50 of the Recirculation RFS has an error of 2 within the sinc function argument.

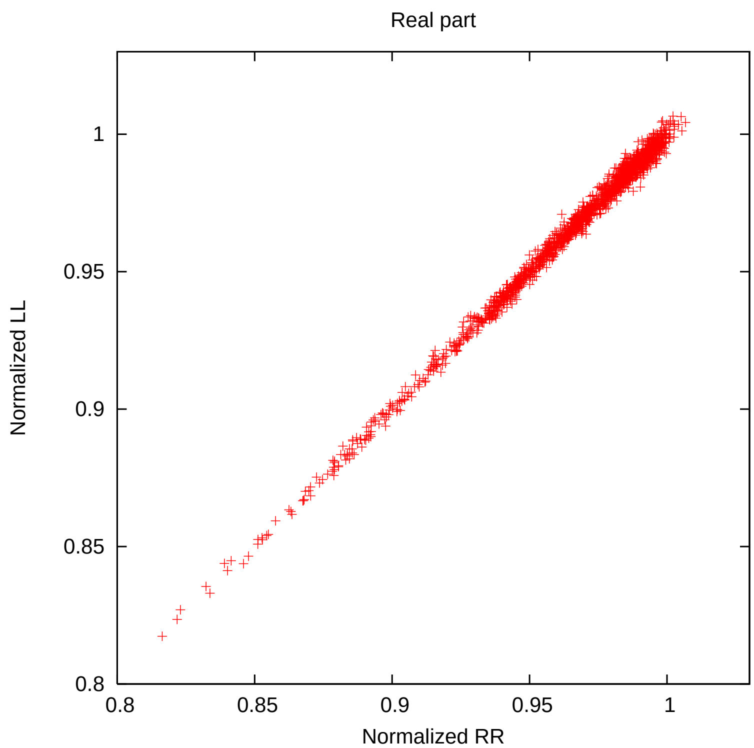

Figure 1: Comparison of RR and LL flux densities of a baseline of the data observed with recirculation. These have been normalized by the flux density of the flux density of the no-recirculation data. The plot includes data from all f_{shift} settings.

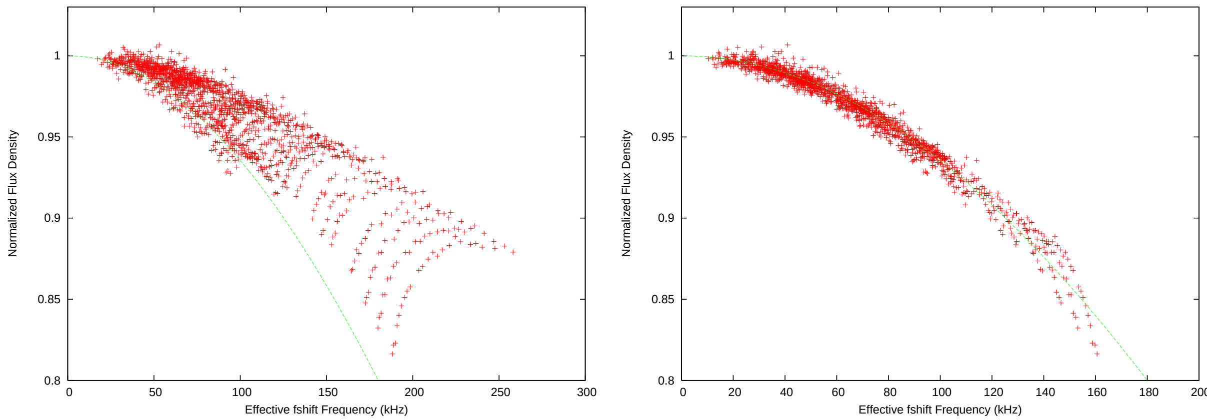

Figure 2: Comparison of predicted (line) and observed (crosses) decorrelation for all test data. The effective phase rate is computed in two different manners. The left panel computes the effective phase rate as antenna phase rates combined in quadrature, whereas the right panel uses the empirical relationship $f' = 0.88f_x - 0.16f_y$.

The observed data poorly matches both quadrature combination or differencing as a way to determine the effective phase rate. Empirically we find that

$$f' = 0.88f_x - 0.16f_y$$

is a tolerable match to the data. Here f_x and f_y are the larger and smaller of the antenna phase rates respectively. This form is purely empirical: we offer no justification or suggestion it might be an appropriate fit with other tests. This form gives less decorrelation than that predicted by quadrature combination but more than by pure differencing. Figure 2 compares the observed data with both the quadrature combination and this empirical relationship as a way to determine the effective phase rate. These are for the tests with all $f_{\rm shift}$ settings.

Figure 3 shows the amplitude of the lag spectrum for three baselines for the $f_{\rm shift}$ settings where the fundamental is $f_0 = 6$ kHz. Because of its low phase rates, baseline 26-27 will show essentially no decorrelation. Note lags +1 and -1 show little or no decorrelation on all the baselines, whereas the baseline with the most decorrelation varies between lags 0, -2 and +2. Thus the amount of decorrelation is a function of lag. As a consequence the amount of decorrelation a source will experience will be a function of its location within the field. Indeed it will vary during the course of a long observation. If some form of correction for the decorrelation were to be attempted, this suggests it would need to be implemented in the lag domain.

We note that the above models of the decorrelation is for an otherwise perfect correlator. However there are other sources of some form of phase noise in the WIDAR correlator. The WIDAR phase model is only 4 bits in size, and there is a 3-level fringe rotator associated with each lag cell. These both introduce a phase noise whose decorrelating effects are already accounted for in the sensitivity of the system with no phase model decimation. The added phase noise from decimation will have less of an effect in a system which already has phase noise than in a perfect system.

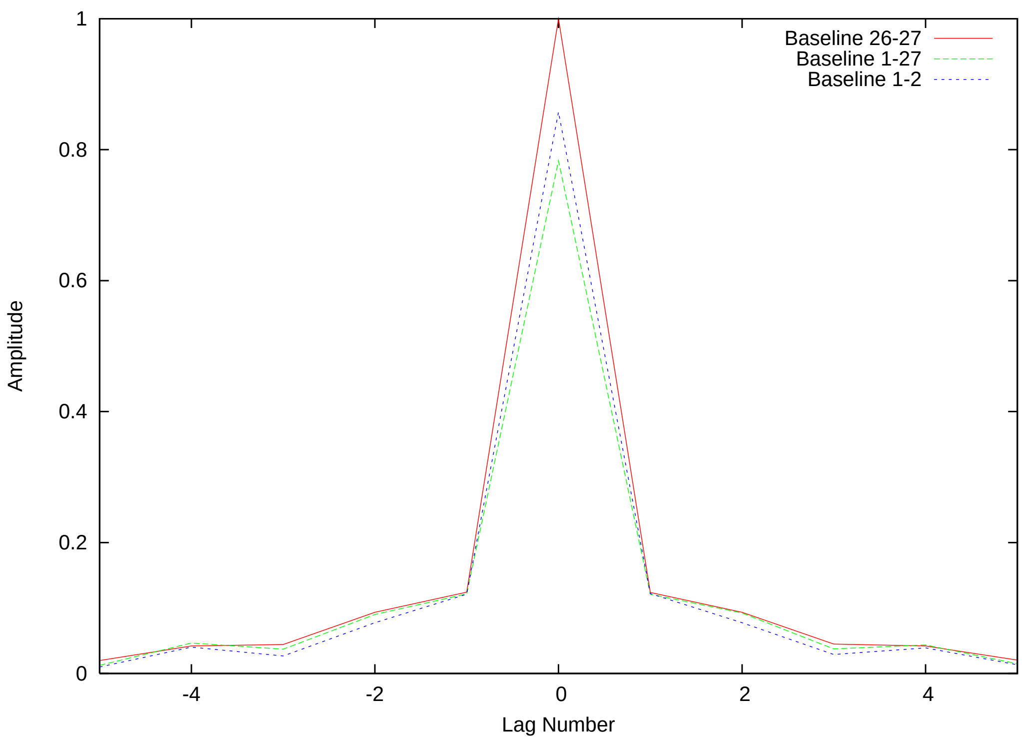

Figure 3: Lag spectra of three baselines showing different amounts of decorrelation with lag number. The f_{shift}fundamental for these data is $f_0 = 6$ kHz.

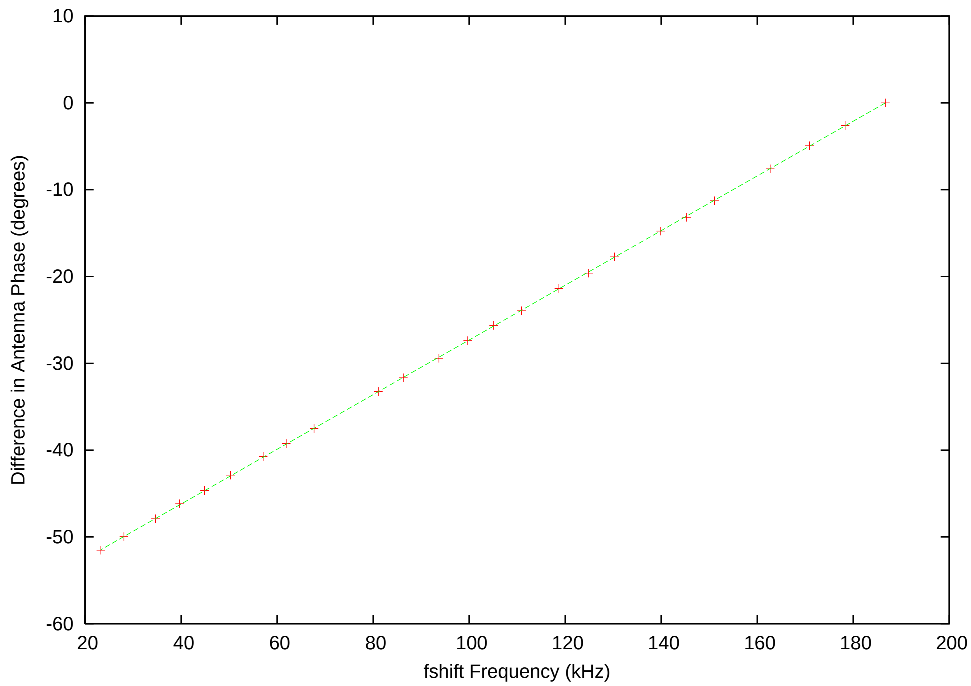

Figure 4: The phase difference in antenna gains, as a function of f_{shift}, with and without recirculation. The crosses are the measured difference whereas the line is the expected difference.

Phase response

As noted above, the Recirculation RFS states that the phase model should be computed with an offset of 3.5 samples when using phase serialization. This would result in the mean of the sawtooth phase error being 0 and would result in the antenna phase response being the same with and without recirculation. However currently this offset of the phase model is not performed in the VLA on-line system. Hence we expect an antenna phase difference between observations with and without recirculation that is equivalent to a delay of 3.5 samples. Figure 4 plots the difference in the antenna phases with and without recirculation as a function of the f_{shift} frequency of that antenna. These data are for the f_{shift} fundamental of $f_0 = 6$ kHz. Also plotted is the expected phase given that the 3.5 sample offset is not yet implemented. Agreement is very good: we do not note any deviation of the phase response from that expected (antenna phase is a relative quantity: the expected phases were offset to make them zero at the reference antenna).

Although the phase offsets are significant, they may not be of large practical importance. It is likely only an issue if there is a desire to transfer calibration from modes with and without phase serialization or different f_{shift} settings. The phase offsets are readily computed and could be simply applied off-line, although this might be undesirable for practical reasons.

Conclusion

This memo investigated the effects of phase model decimation in the WIDAR modes where recirculation is achieved with so-called "phase serialization". Note, at the expense of other WIDAR resources, other recirculation modes can avoid phase serialization and so avoid the issues investigated here.

We find broad agreement with the simple model of the amplitude decorrelation that will be experienced. The measured decorrelation is less than that suggested by the Recirculation RFS. This is because the antenna-based phase noise resulting from decimation is not independent on different antennas and so the phase noise on a baseline is less. However the phase noise and decorrelation will be lag dependent. This means that any attempt to correct the decorrelation would need to work in the lag domain.

In terms of phase characteristics, the observations show that the phase model computation needs to be offset by 3.5 samples when phase serialization is invoked. This shortcoming had been appreciated before these tests and may not be of much practical importance.