EVLA Memo 199
Considerations for JVLA Frequency and Time Averaging

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Abstract

The VLA's correlator, operating in its default 'continuum' modes, generates more data than most people actually need. We record here the relations governing chromatic and time aberrations in imaging, and make recommendations for post-correlation frequency and time decimation.

1 Introduction

The WIDAR correlator makes a lot of data. Too much data in most cases. At high frequencies especially, when observing for 'continuum' projects, there is no need to channelize at a 2 MHz and 1 second increment (the current basic operating mode). A judicious choice of time and channelwidth averaging will result in dramatic improvements in required disk storage – and equally importantly, data processing times. This memo reviews the relevant factors, and makes recommendations for on-line (and off-line) data decimation.

2 Bandwidth, Delay Error, and Time Averaging Expressions

In the following, we utilize the analyses by Perley in EVLA Memo #64, (2004) and by Bridle and Schwab in Chapter 18 in the NRAO Synthesis Imaging Summer School book (vol. 180 in the ASP Conference Series, 1999).

2.1 Chromatic Aberration – a.k.a. Bandwidth Broadening

Averaging the visibilities over frequency introduces a well-understood image aberration – a reduction in the peak brightness, and a radial broadening of the image such that the total flux is preserved.

For a baseline of length B, and source offset by angle θ from the delay tracking position, the visibility amplitude is reduced by a factor of (Perley, 2004)

$$R_{\Delta\nu} = \mathrm{sinc}\left(\frac{B\Delta\nu}{c}\theta\right). \tag{1}$$

A convenient, and very stringent condition, can be set by asking that this reduction be less than 10% for the longest baseline. This condition is met when the factor within the parentheses is less than 0.25, from which we derive for the maximum bandwidth:

$$\Delta\nu = \frac{c}{4B\theta} \tag{2}$$

In more convenient units, we get for the maximum channelwidth in MHz:

$$\Delta\nu_M = \frac{26}{B_{Km}\theta_{arcmin}} \tag{3}$$

where B_{Km} is the maximum baseline in kilometers, and θ_{arcmin} is the source offset in arcminutes.

This expression is most readily applicable at high frequencies, where the object of interest is likely the only significant source of bright emission within the primary beam. But at lower frequencies, the presence of background sources may present a problem in imaging – the effect of chromatic aberration will make these more difficult to suitably manage if their residual brightnesses are well above the noise. In this case, a more suitable offset to utilize is the position of the first null of the primary beam. Roughly, this is given by $\theta \sim \lambda/D$, where D is the antenna diameter. Using this offset as the criterion, we find for the maximum channelwidth:

$$\Delta\nu = \frac{\nu D}{4B} \tag{4}$$

where ν is the observing frequency. Converting to more practical units, , we get a condition for the maximum channelwidth in MHz, $\Delta\nu_M$

$$\Delta\nu_M = 6.25\frac{\nu_G}{B_{km}} \tag{5}$$

where ν_G is the frequency in GHz and B_{km} is the maximum baseline in kilometers. Note that this condition is independent of observing band.

This is an extremely stringent condition, hardly ever necessary in normal observing, where the scientific targets are located near the center of the antenna beam, and where the background sources can usually be counted on to be of low brightness compared to the noise.

An alternate approach is to base the criterion on the reduction in amplitude response of the array to a point source at a given offset. For the case of a square bandpass and effective circular Gaussian taper, Bridle and Schwab provide in their eqn. 18-24 the imaging response:

$$R = \frac{\sqrt{\pi}}{\gamma\beta}\mathrm{erf}\frac{\gamma\beta}{2} \tag{6}$$

where $\gamma \equiv 2\sqrt{\ln 2}$, and $\beta = \Delta\nu\theta_o/\nu_0\theta_{syn}$ Here, $\Delta\nu$ is the channel-width, ν_0 is the frequency, θ_o is the angular offset of the object, and θ_{syn} is the synthesized beam's FWHM. For a small loss, we can write, for the fractional loss:

$$L \sim 0.23\beta^2. \tag{7}$$

Setting a condition of 5% drop in peak brightness at a position of the half power offset at the primary beam, we find

$$\Delta\nu_M = \frac{25\nu_G}{B_{km}} \tag{8}$$

which is four times the channelwidth of the more stringent criterion.

The following table summarizes the results. The 'Min' column is the channelwidth set by the first criterion – 10% loss on the maximum baseline at the primary beam first null, the 'Decent' column is set by the more relaxed criterion – loss of brightness of 5% at the primary beam half power.

Channelization Limits in MHz								
	A-Conf		B-Conf		C-Conf		D-Conf	
Band	Min	Decent	Min	Decent	Min	Decent	Min	Decent
L	0.26	1.0	0.82	3.2	2.6	10	8.2	32
S	0.53	2.1	1.7	6.6	5.3	21	17	66
C	1.07	4.3	3.4	14	10.7	43	34	140
X	1.8	7.2	5.7	23	18	72	57	230
U	2.7	11	8.5	35	27	110	85	350
K	4.1	16	13	51	41	160	130	510
A	6.1	24	19	76	61	240	190	760
Q	8.0	32	25	100	80	320	250	1000

Note that averaging beyond 16 MHz width is not recommended. The VLA's correlator has a basic spectral window maximum width of 128 MHz. Averaging to 16 MHz reduces the number of channels to eight – of which the first and last will likely not be useful due to edge effects. Thus, in the table above, any value greater than 16 should be replaced by 16 MHz.

Channel averaging of greater than 16 MHz can be executed in post-processing, once the bandpass and delay calibration are applied. The spectral data can then be re-organized so the entire IF band is considered as a single spectral window (the analog of the program 'NOIFS' in AIPS).

2.2 Delay Losses

If the delays are not properly set prior to spectral averaging, there will be an amplitude loss caused by the subsequent phase slope across the averaging width. The resulting normalized amplitude is given by:

$$R = \mathrm{sinc}(\Delta t \Delta \nu) \tag{9}$$

where Δt is the delay error in seconds, $\Delta\nu$ is the averaging width in Hz, and 'sinc' is the sinc function. The following table shows the delay error in nsec for losses of 1, 5, and 10%.

Delay Limits			
DelayBandwidth	Ampl. Loss		
	1%	5%	10%
$\Delta t_{nsec}\Delta\nu_M$	75	175	250

Even with a large delay error of 5 nsec, the loss over 16 MHz is only 1.1%. Only a minuscule fraction of (parallel-hand) delay errors are as large as this. Note that the same criterion applies to the cross-hand delays – so it is important to monitor these on a regular basis as part of Operations.

2.3 Time Averaging Loss

Although not part of our charge, we include this effect for completeness. Time averaging of the visibilities from an object offset from the fringe-tracking center causes a loss of amplitude similar to the bandwidth effect discussed above. The reduction in amplitude is by a factor

$$R_t = \mathrm{sinc}(ft) \tag{10}$$

where f is the fringe frequency of the offset source, with respect to the delay tracking center. For a baseline of length B, and source offset of θ radians from the delay center, the maximum fringe freqency is

$$f_{max} = \frac{B\nu\omega\theta}{c} \tag{11}$$

where $\omega = 7.27 \times 10^{-5}$ is the angular rotation rate of the earth.

We adopt the same strict criterion as for the chromatic aberration case – reduction by 10% on the longest baseline from an object located at offset angle θ. In this case, the averaging time Δt_{min} is limited to

$$\Delta t_{min} = 1/4 f_{min} \tag{12}$$

or, in useful units,

$$\Delta t = \frac{3560}{B_{km}\theta_{arcmin}\nu_G} \tag{13}$$

seconds.

Again, this works fine for higher frequencies, where background sources are not expected to be an issue. For cases where background sources are important, we adopt the condition that the longest baseline's amplitude not be reduced by more than 10% for an object located at the primary beam first null. The time averaging condition becomes

$$\Delta t = \frac{D}{4\omega B} \tag{14}$$

where ω is the angular rotation rate of the earth. Using the VLA's primary antenna diamter of 25 meters, we find $\Delta t = 89/B_{km}$ seconds. This leads to a limit of 2.5 seconds in 'A' configuration. This is a very strict criterion, and EVLA Memo #64 suggests a relaxation by a factor of four.

Bridle and Schwab provide an analysis based on image response. The effect time averaging on an image is not simple, and symbolic analysis is only possible for objects at the NCP. However, this is considered representative of other declinations, so we utilize equation 18-43 from Bridle and Schwab for the reduction in point-source response:

$$R = 1 - 1.22 \times 10^{-9} \left(\frac{\theta_o}{\theta_{Syn}}\right)^2 \Delta t^2 \tag{15}$$

where Δt is the time averaging in seconds. We again consider a realistic case of a 5% loss at the primary beam half-power offset, to find (in useful units)

$$B_{km}\Delta t = 320 \tag{16}$$

These relations are displyed in the following table, where 'Min' refers to the minimum averaging time, sufficient to reduce the longest baseline's visibility amplitude from a source at the antenna first null by less than 10%, while 'Decent' is sufficient to keep time averaging losses for an object at the primary beam's half power to less than 5%. A more stringent condition of a 10% loss in amplitude at the first null of the primary beam results in

Time Average Limits		
Config	Min	Decent
	seconds	seconds
A	2.5	9.1
B	8	29
C	25	91
D	80	290

a limit of 6.5 seconds for 'A' configuration.

3 Discussion

For 'continuum-science' observations, the only viable reasons to not implement both channel and time averaging are the effects of RFI and time variability. For the latter, a finer time division is required to track atmospheric phase variations, at least until these can be calibrated and removed. In periods of fine weather, and when in the more compact configurations, on-line averaging of 10 seconds or more is sensible.

Editing RFI is a significant issue for a wide-band instrument such as the JVLA. Averaging the spectra will increase the fraction of 'good' channels contaminated by RFI. Fortunately, at high frequencies (say, above 8 GHz) the fraction of the bandpass occupied by RFI is fairly low, while above 18 GHz, the RFI contamination problem becomes negligible. Hence the loss of sensitivity caused by increasing the fraction of the spectrum contaminated by RFI through frequency averaging should be negligible.

At lower frequencies, the situation is not so simple, as the fraction occupied by RFI can exceed 50%. The output spectra produced by the JVLA's correlator exhibit 'Gibbs' phenomenum ringing' – due to the abrupt truncation of the lag spectrum. Simple boxcar averaging of the original output spectra will not adequately remove the RFI contamination in channels outside those directly affected by the RFI. Hence, for these lower frequency bands, simple frequency averaging is not recommended, and a more sophisticated approach is needed. The usual recourse is to apply a smoothing kernal to the output spectra which minimizes the frequency response outside the central window – 'Hanning' smoothing is a simple and effective means to do this, but other, more sophisticated functions which can be applied to the lag spectrum prior to Fourier transform are well known and should be implemented.