EVLA Memo # 200
Dipole Alignment Tolerance for the JVLA's Low-Band System

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Abstract

Tolerances for the alignments of the P-band dipoles on the VLA antennas are determined. The most important requirements are that they be stable to better than 0.5 degrees over long periods of time, and that they be oriented to within 5 degrees of the antenna quadrupod legs. A more accurate orientation is preferred, but not required.

1 Introduction

The original P-band VLA receiver system utilized circular polarization. This was very convenient for the users, as the other bands also employed circular polarization, so no special changes in software, nor in calibration methodology were required.

However, it turned out that the original P-band receivers were incompatible with the EVLA electronics systems, so a new design was required. This provided an opportunity to design and implement a wide-band low-frequency system, covering roughly 50 to 500 MHz. Due to the wide bandwidth it was not possible to design circular polarizers which worked well enough, so an important decision was made to utilize the native linear polarization – putting this receiver system at odds with all the others, which retain circular polarizers.

There is no fundamental reason not to use a linear system – the same astronomical information is available in both linear and circular bases, provided all four correlations are available. The most significant advantage of the circular basis is the significantly easier calibration regimen – nearly all calibrators utilized for the high frequency bands are significantly linearly polarized, but have nearly zero circular polarization. For a 2-dimensional array such as the VLA, which often utilizes 'snapshot' observations, this is a major advantage, as the polarization state of the calibrators is (almost) immaterial.

The problem of linear polarized calibrators is almost certainly not an issue at low frequencies (below 500 MHz), as it appears that all the useable standard calibrators are depolarized at these frequencies. Thus, it was thought that there are few consequences, other than relatively minor changes to the post-processing software, needed to support the linear-polarized low-band system.

Unfortunately, this hope was not realized. Tests of the low-band response immediately showed a large – occasionally very large – cross-hand response from observations of strong, unpolarized sources. It was quickly realized that the most likely (almost certain) cause was large misalignments of the P-band dipoles. Visual inspection of the the array quickly determined that misalignments of more than 10 degrees were common. And there is good evidence that these misalignments are dynamic – rotation of the subreflector caused by high frequency observing is changing the orientation of the P-band dipoles.

Efforts to corrently orient and stabilize the P-band dipoles are now underway. The purpose of this memo is to show the consequences of dipole misalignment, and set useful limits on the degree of tolerable misalignment.

2 A Simple Model

I consider a simple model to illustrate the major points. Consider two alt-az mounted antennas with orthogonal dipoles responding to the linearly polarized components of the electric field. Presume the data are correctly calibrated (that is, the effects of system gain in the two signal channels are known and accounted for). Also presume that there are no 'leakage' terms coupling the othogonal channels' signals[1] The two antennas' dipoles are oriented at angles to

[1]This should be a reasonable assumption, as there is no polarizer in this system. I expect these couplings to be of order .01 or less.

the sky frame of Ψ_1 and Ψ_2, respectively. Then, the relation between the four calibrated correlations and the four Stokes visibilities can be written

$$\begin{pmatrix} R_{v1v2} \\ R_{v1h2} \\ R_{h1v2} \\ R_{h1h2} \end{pmatrix} = \begin{pmatrix} \cos\Delta\Psi & \cos\Sigma\Psi & \sin\Sigma\Psi & \sin\Delta\Psi \\ \sin\Delta\Psi & -\sin\Sigma\Psi & \cos\Sigma\Psi & -\cos\Delta\Psi \\ -\sin\Delta\Psi & -\sin\Sigma\Psi & \cos\Sigma\Psi & \cos\Delta\Psi \\ \cos\Delta\Psi & -\cos\Sigma\Psi & -\sin\Sigma\Psi & \sin\Delta\Psi \end{pmatrix} \begin{pmatrix} \mathcal{I}/2 \\ \mathcal{Q}/2 \\ \mathcal{U}/2 \\ i\mathcal{V}/2 \end{pmatrix} \tag{1}$$

where

$$\Delta\Psi = \Psi_1 - \Psi_2 \tag{2}$$
$$\Sigma\Psi = \Psi_1 + \Psi_2 \tag{3}$$

Subscripts 'v' and 'h' refer to the vertical and horizontal dipoles, respectively. Note that the Stokes visibilities $\mathcal{I}, \mathcal{Q}, \mathcal{U}$, and $\mathcal{V}$ are in general complex. For a calibrated, unresolved source, they are real valued.

Now assume that the dipoles for the first antenna are correctly oriented in the antenna (az-el) frame, while those for the second antenna are rotated by α. The relation becomes

$$\begin{pmatrix} R_{v1v2} \\ R_{v1h2} \\ R_{h1v2} \\ R_{h1h2} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \cos(2\Psi_p+\alpha) & \sin(2\Psi_p+\alpha) & -\sin\alpha \\ -\sin\alpha & -\sin(2\Psi_p+\alpha) & \cos(2\Psi_p+\alpha) & -\cos\alpha \\ \sin\alpha & -\sin(2\Psi_p+\alpha) & \cos(2\Psi_p+\alpha) & \cos\alpha \\ \cos\alpha & -\cos(2\Psi_p+\alpha) & -\sin(2\Psi_p+\alpha) & -\sin\alpha \end{pmatrix} \begin{pmatrix} \mathcal{I}/2 \\ \mathcal{Q}/2 \\ \mathcal{U}/2 \\ i\mathcal{V}/2 \end{pmatrix} \tag{4}$$

where Ψ_p is the antenna parallactic angle, assumed to be the same for both antennas.

Further simplication occurs if we assume the parallactic angle is zero[2]. Then we have

$$\begin{pmatrix} R_{v1v2} \\ R_{v1h2} \\ R_{h1v2} \\ R_{h1h2} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \cos\alpha & \sin\alpha & -\sin\alpha \\ -\sin\alpha & -\sin\alpha & \cos\alpha & -\cos\alpha \\ \sin\alpha & -\sin\alpha & \cos\alpha & \cos\alpha \\ \cos\alpha & -\cos\alpha & -\sin\alpha & -\sin\alpha \end{pmatrix} \begin{pmatrix} \mathcal{I}/2 \\ \mathcal{Q}/2 \\ \mathcal{U}/2 \\ i\mathcal{V}/2 \end{pmatrix} \tag{5}$$

This shows the linkages between the complex Stokes' visibilities and the (calibrated) output correlations.

Because the angle offset α is considered as an unknown, we can not correct for its effect. In such a case, we utilize the elementary inversion formulae to generate the four Stokes' visibilities. These are

$$\mathcal{I} = R_{v1v2} + R_{h1h2} \tag{6}$$
$$\mathcal{Q} = R_{v1v2} - R_{h1h2} \tag{7}$$
$$\mathcal{U} = R_{v1h2} + R_{h1v2} \tag{8}$$
$$\mathcal{V} = i(R_{v1h2} - R_{h1v2}) \tag{9}$$

Since the presence of the misalignment angle α is unaccounted for, these inversions will not be correct. Denote by primed quantities the reconstructed visibilities calculated from the formulae above, and by the unprimed quantities the correct visibility values. We then find, after insertion of equation (5) into equations (6) through (8)

$$\mathcal{I}' = \mathcal{I}\cos\alpha - i\mathcal{V}\sin\alpha \tag{10}$$
$$\mathcal{Q}' = \mathcal{Q}\cos\alpha + \mathcal{U}\sin\alpha \tag{11}$$
$$\mathcal{U}' = \mathcal{U}\cos\alpha - \mathcal{Q}\sin\alpha \tag{12}$$
$$\mathcal{V}' = \mathcal{V}\cos\alpha + i\mathcal{I}\sin\alpha \tag{13}$$

which shows that the recovered visibilities (primed quantities) are in all cases diminished from the true value by a factor $\cos\alpha$, and are augmented by a contribution ('leakage') from another polarization with amplitudes diminished by $\sin\alpha$. The correct values are recovered only if $\alpha = 0$.

The presence of a misalignment has the effect of mixing Stokes I with Stokes V, and Stokes Q with Stokes U. We note that:

- The effect on Stokes $\mathcal{I}$ is negligible for most sources if the misalignment angle is small, since $|\mathcal{V}| \ll |\mathcal{I}|$.

[2]The conclusions drawn are actually independent of the parallactic angle, but the mathematics are simpler if we make this assumption

- The effect on $|\mathcal{V}|$ can be very strong for the same reason. If we want the contamination to be less than 1%, the angle $\alpha < 0.01$, or 0.5 degrees.
- The linearly polarized intensity $|P| = \sqrt{\mathcal{Q}\mathcal{Q}^* + \mathcal{U}\mathcal{U}^*}$ is unaffected.
- The derived position angle of the polarized emission is rotated by angle $\alpha/2$. Hence, a 5 degree error in the dipole alignment leads to a 2.5 degree error in the position angle.

So – what limit is reasonable? We would like to determine the position angles of polarized emission with an accuracy of ∼2.5 degrees, for which an offset of ∼5 degrees is needed. Note that this offset in angle is not frequency dependent, so rotation measure measurements would not be affected.

A much more stringent limit arises if Stokes V imaging is desired. Here, because of the strong coupling from Stokes I, the accuracy of the alignment must be considerably better – a 1% accuracy in V requires alignment of 0.5 degrees, or better. At this level, other effects, ignored in our simple model, will arise – notably the orientation of the antennas. I have assumed they are all perfectly parallel, so that the orientation of the sky is identical to all. In truth, they are not, with polar offsets of many arcminutes.

Because the P-band dipoles are on the antenna's axis of symmetry, the beam squint which strongly affects Stokes V imaging for the Cassegrain bands should not occur for P-band. (This statement, based on principle, needs to be confirmed by observation).

Given the rarity of VLA Stokes V observations (other than masers), I am inclined to ignore the stringent condition this imposes on alignment, and utilize the linear polarization position angle argument.

3 But Wait – Can't We Calibrate For This Misalignment?

In a word, yes. And it's easy to do – we have to make adjustments for the parallactic angle rotation anyway, so the dipole misalignment can be made part of this correction. But each antenna will be different, and for this to be a simple calibration, each needs to be known in advance.

It is straightforwards to formalize this: Equation (1) can be inverted to show:

$$\begin{pmatrix} \mathcal{I} \\ \mathcal{Q} \\ \mathcal{U} \\ i\mathcal{V} \end{pmatrix} = \begin{pmatrix} \cos\Delta\Psi & \sin\Delta\Psi & -\sin\Delta\Psi & \cos\Delta\Psi \\ \cos\Sigma\Psi & -\sin\Sigma\Psi & -\sin\Sigma\Psi & -\cos\Sigma\Psi \\ \sin\Sigma\Psi & \cos\Sigma\Psi & \cos\Sigma\Psi & -\sin\Sigma\Psi \\ \sin\Delta\Psi & -\cos\Delta\Psi & \cos\Delta\Psi & \sin\Delta\Psi \end{pmatrix} \begin{pmatrix} R_{v1v2} \\ R_{v1h2} \\ R_{h1v2} \\ R_{h1h2} \end{pmatrix} \tag{14}$$

where $\Delta\Psi$ and $\Sigma\Psi$ are defined in equations (2) and (3). If we presume that the parallactic angles Ψ_p are the same for both antennas, we have

$$\Delta\Psi = \alpha_1 - \alpha_2 \tag{15}$$

$$\Sigma\Psi = 2\Psi_p + \alpha_1 + \alpha_2 \tag{16}$$

Thus, knowledge of the parallactic angle, and the dipole misalignments permits correct generation of the four complex visibilities from the four complex correlations.

The most important requirement is that the alignments be stable to better than one degree over long periods of time – preferably 'for ever'. Although the effects of misalignment can be corrected for in software, doing so adds complexity to the calibration regimen. Ideally, the orientations of the dipoles would be aligned with the antenna frame with an accuracy of better than 1 degree, but 5 degrees is considered sufficient.

This analysis presume correct calibration and insignificant cross-channel leakages. There would be significant complicating factors if the misorientations are significant, and the signal isolation between the two signal channels poor. Neither is expected to be the case. An upcoming memo will explore the ramifications of both.

4 Summary

Requirements for the dipole orientations are:

- Long-term stability of better than one degree.
- Orientation accuracy of better than 5 degrees.

It would be good if the orientations are considerably better than 5 degrees, but this is not a requirement, as we can correct for such misalignments in software.