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Noise Contribution of 90° Hybrids Before Low Noise Amplifiers

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Abstract: Analysis and measurement indicates that the noise contribution of cooled hybrids will be acceptable for use in radio astronomy receivers.

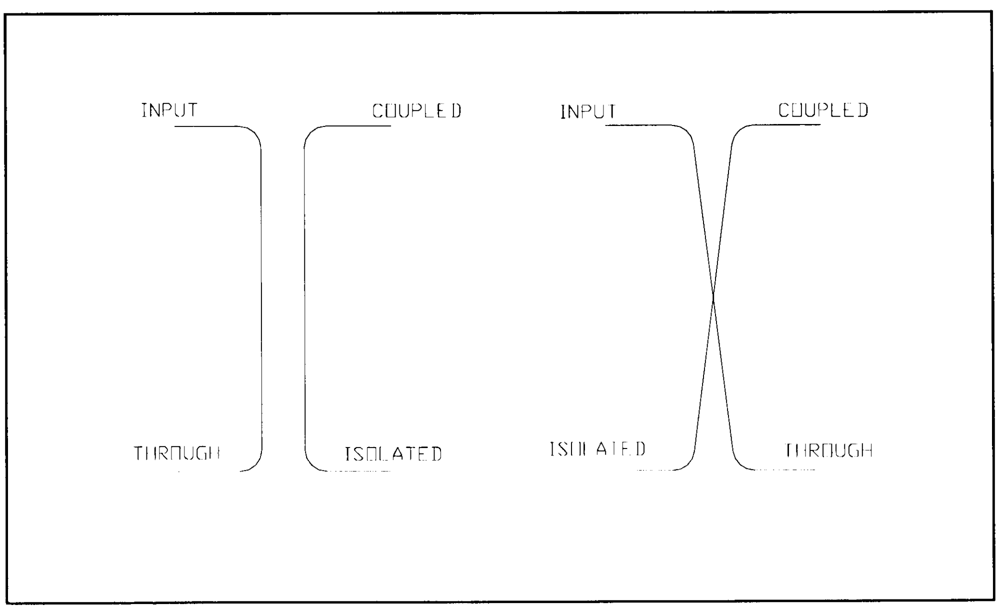
3-dB hybrids are used to convert between orthogonal linear and orthogonal circular polarizations. In broadband systems it is desireable to do this before amplification.

This note discusses some characteristics of hybrids, the increase in noise temperature caused by the dissipation of the hybrid, a method of measuring that dissipation, and results of a typical measurement.

I. Some background on hybrids (see, for example, J. A. G. Malherbe, *Microwave Transmission Line Couplers,* Artech House 1988):

A "3-dB hybrid" or "90° hybrid" is a directional coupler with nominally -3 dB coupling. It is made of a pair of electromagnetically coupled transmission lines. Fig. 1 illustrates typical configurations. The second, crossed-over, configuration is more convenient in many applications, and is more common. The device is symmetrical, and any port can be considered the "input" port with the other ports then being designated as shown.

Fig. 1



For a directional coupler with midband coupling k^2 and all ports matched, the following relations hold (from Malherbe, equations (4.9) and (4.10)):

$$\frac{V_{THROUGH}}{V_{INPUT}} = \frac{\sqrt{(1-c^2)}}{\sqrt{(1-c^2)}Cos\theta + jSin\theta} \quad \text{or} \quad \frac{|V_{THROUGH}|^2}{|V_{INPUT}|^2} = \frac{1-c^2}{1-c^2Cos^2\theta} \equiv 1-k^2$$

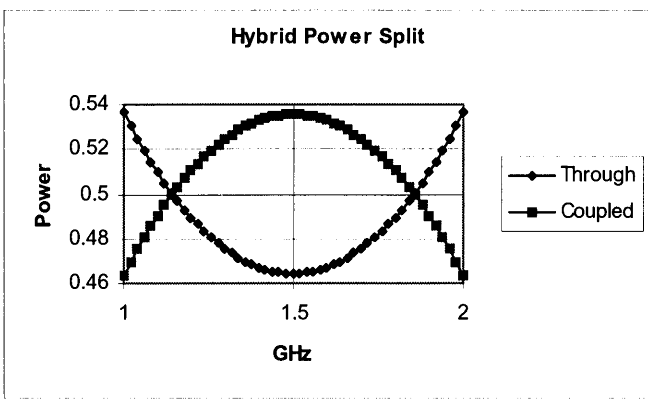
$$\frac{V_{COUPLED}}{V_{INPUT}} = \frac{jcSin\theta}{\sqrt{(1-c^2)}Cos\theta + jSin\theta} \quad \text{or} \quad \frac{|V_{COUPLED}|^2}{|V_{INPUT}|^2} = \frac{c^2Sin^2\theta}{1-c^2Cos^2\theta} \equiv k^2$$

$$\frac{V_{ISOLATED}}{V_{INPUT}} = 0$$

θ, the electrical length of the coupled section, is 90° at midband, and goes from 60° to 120° in covering an octave bandwidth.

In the usual "3-dB" hybrid, c^2 is chosen to be 0.536..., or –2.71... dB, which minimizes the power unbalance over an octave bandwidth:

Fig. 2



II. Effect of loss on noise temperature. Single channel:

To measure the effective noise temperature, T_N, of an amplifier, we apply a matched termination of physical temperature T_H ("hot load") and then T_C ("cold load") tc the amplifier's input and measure the resulting output powers, $P_H = G(T_H + T_N)$ and $P_C = G(T_C + T_N)$, where G is the gain[1] of the amplifier. Then we solve for T_N:

$$T_N = \frac{T_H P_C - T_C P_H}{P_H - P_C} \quad \text{or} \quad T_N = \frac{T_H - YT_C}{Y-1} \quad \text{where} \quad Y \equiv \frac{P_H}{P_C} \text{ (the famous "Y - factor")}$$

[1] G includes Boltzmann's constant and the measurement bandwidth, converting temperature to power.

If an attenuator of gain[2] g at physical temperature T_P is inserted before the amplifer and we measure as before, we will get

$$P_H = G[gT_H + (1-g)T_P + T_N] \quad \text{and} \quad P_C = G[gT_C + (1-g)T_P + T_N]$$

Using these values in the equation above, we will get an effective noise temperature of

$$T_N' = \frac{T_N}{g} + T_P\frac{(1-g)}{g} \quad \text{or} \quad T_N' = T_N + (T_N + T_P)\frac{(1-g)}{g}$$

That is, the effective noise temperature has been increased by an amount

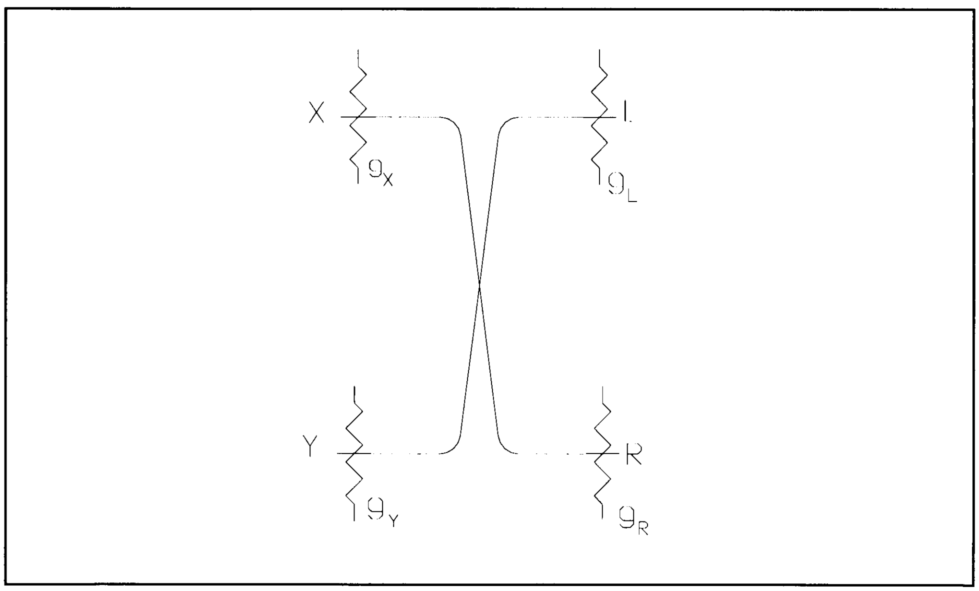
$$\Delta T_N = (T_N + T_P)\frac{(1-g)}{g}$$

III. Lossy hybrid before amplifier:

Linear polarization inputs X and Y are connected to the "input" and "isolated" ports of the hybrid. The output from the "coupled" and "through" ports will be fed to the LCP and RCP amplifiers. We will label the ports X, Y, L, and R from now on.

If the hybrid were lossless and P_X and P_Y were injected into ports X and Y, the power output to port L would be $k^2P_X + (1-k^2)P_Y$ and that to port R would be $(1-k^2)P_X + k^2P_Y$. To account for the losses of a real hybrid, we then assume that attenuators g_k (where k = X, Y, L, R) (all at physical temperature T_P) are attached to the corresponding ports.

Fig. 3



[2] "gain" and "g" are simply a power ratios, e.g. 1 dB loss means a gain of −1 dB and g = 0.7943...

We measure T_N' of (say) the LCP channel by applying two identical terminations at temperature T_H to ports X and Y, and then two at temperature T_C. We measure the resulting output powers P_H and P_C of the LCP amplifier. For P_H we will have

$$P_H = G\{k^2 g_L[T_H g_X + T_P(1-g_X)] + (1-k^2)g_L[T_H g_Y + T_P(1-g_Y)] + T_P(1-g_L) + T_N\}$$

where T_N is the noise temperature of the LCP amplifier.

For P_C we will have the same expression, replacing "H" with "C".

Since the terminations on the X and Y ports are independent thermal noise sources, their output voltages are completely uncorrelated and we can simply add their average powers.

Plugging these expressions into the Y-factor equation above, and doing some algebra, we can arrive at

$$T_N' = T_N + (T_N + T_P)\frac{(1 - g_L[k^2 g_X + (1-k^2)g_Y])}{g_L[k^2 g_X + (1-k^2)g_Y]} \quad \text{or}$$

$$\Delta T_N = (T_N + T_P)\frac{(1 - g_{eff})}{g_{eff}}$$

where $g_{eff} \equiv g_L[k^2 g_X + (1-k^2)g_Y]$ is the effective gain ahead of the LCP amplifier.

Comparing with the previous result, we see that the effect of the lossy hybrid on noise temperature is the same as that of a simple attenuator of gain g_{eff}.

The expression for the RCP channel is the same, with R replacing L, and with X and Y interchanged.

Because the hybrid is physically symmetric, we may reasonably assume $g_X = g_Y$ and $g_L = g_R$ so that

$$g_{eff} = g_X g_L$$

IV. Measuring g_{eff}

How can we measure g_{eff}? The hybrid is a 4-port device and a complete characterization by a network analyzer would be quite time-consuming. Also, the quantities we wish to measure will be small fractions of a decibel, which will be difficult to separate from the ~3 dB coupling loss included in each port-to-port measurement.

Instead, we make the measurement as follows: Put pure reflective terminations (shorts) on ports L and R, and measure the device from X to Y as a two-port. From the S11 and S21 measurements we can calculate

$$g_{eff} = \sqrt{|S_{11}|^2 + |S_{21}|^2}$$

Analysis:

Power input to port X will be split, with a fraction c^2 going to port L and a fraction $(1-k^2)$ to Port R. All the power reaching port L will be reflected, and split again with k^2 going back to port X and $(1-k^2)$ to Port Y. Likewise, the power reflected from port R will be split $(1-k^2)$ to X and k^2 to Y.

Including the gains g_k, the net gain from X to Y by one path (call it path XLY) is $g_{XLY} = g_X k^2 g_L g_L (1-k^2) g_Y$. The other path from X to Y ("path XRY") has the gain $g_{XRY} = g_X (1-k^2) g_R g_R k^2 g_Y$. Since these paths both originate at a single source, the voltages are coherent and we must consider their relative phases when combining them.

It is a fundamental property of the hybrid coupler that there is a 90° relative phase shift between the "k^2" path and the "$(1-k^2)$" path at every frequency in the band. Since each path considered here involves one "k^2" and one "$(1-k^2)$", each path will have the same 90° phase shift, and the two voltages will be in phase and can be added directly.

$$V_Y/V_X = \sqrt{g_X g_Y g_L^2 k^2 (1-k^2)} + \sqrt{g_X g_Y g_R^2 k^2 (1-k^2)} = (g_L + g_R)\sqrt{g_X g_Y k^2 (1-k^2)}$$

so that $$g_{YX} \equiv |V_Y|^2 / |V_X|^2 = (g_L + g_R)^2 g_X g_Y k^2 (1-k^2) = |S_{21}|^2 .$$

Then for $g_X = g_Y$ and $g_R = g_L$, we have

$$|S_{21}|^2 = g_{YX} = 4 g_X^2 g_L^2 k^2 (1-k^2)$$

Consider also the paths whereby the power input at X is reflected back to X. These paths, "XLX" ($g_{XLX} = g_X k^2 g_L g_L k^2 g_X$) and "XRX" ($g_{XRX} = g_X (1-k^2) g_R g_R (1-k^2) g_X$), differ from each other in that one involves two "k^2" splits and the other involves two "$(1-k^2)$". Thus there will be a 180° phase difference between the two and they will tend to cancel.

The net "reflected" gain is $|S_{11}|^2 = g_{XX} = g_X^2 g_L^2 [(1-k^2) - k^2]^2$.

It can be seen that $|S_{21}|^2 + |S_{11}|^2 = g_{YX} + g_{XX} = g_X^2 g_L^2 = g_{eff}^2$

Note that we do not need phase information for S11 and S21; scalar measurements suffice.

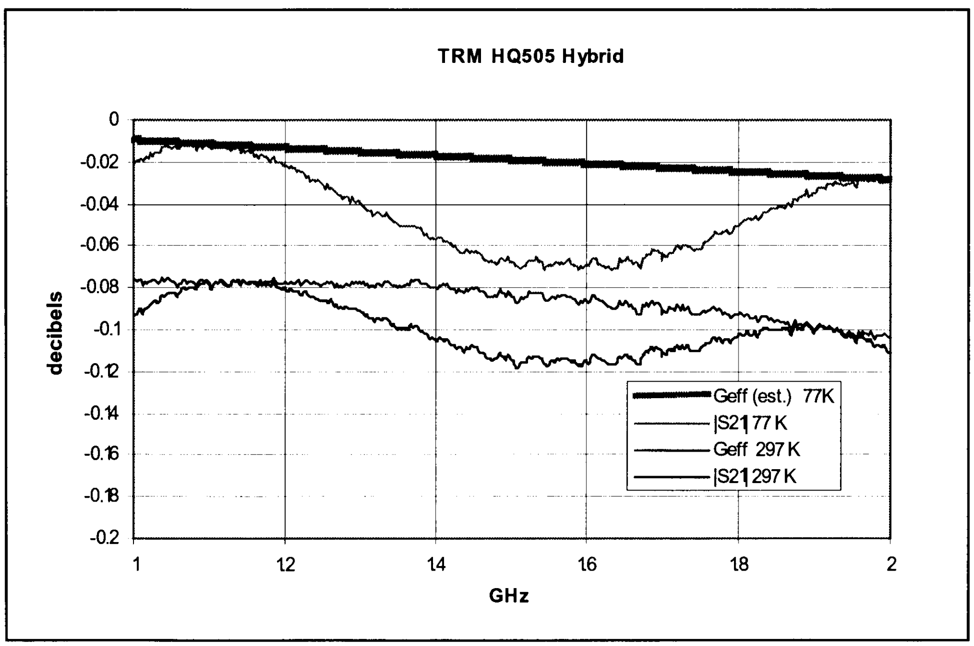
If only $|S_{21}|$ data is available, note that $|S_{11}|^2$ is zero and $|S_{21}|^2$ is maximum where $(1\text{-}k^2) = k^2$ (i.e., the power split is exactly even.) At these frequencies, g_{eff} is just $\sqrt{|S_{21}|^2}$ or |S21| and so an estimate over the whole frequency range can be made by drawing a line through the maxima of |S21|.

A more conservative approximation would be to simply use |S21| everywhere since |S11| is small.

V. Measurements

Measurements were made by E. Szpindor on a TRM HQ-505 1-2 GHz, 3 dB hybrid. The unit uses stripline transmission lines with teflon dielectric. Tests were made at room temperature and at 77 K.

Fig. 4



The bottom two traces are room temperature measurements. The lower is |S21| and the one above it is g_{eff} (from S11 and S21 data).

The upper two traces are for 77 K, with the lower of these being |S21| and the upper a straight-line estimate of g_{eff}, since |S11| data were unavailable for this temperature.

These data show that g_{eff} is close to –0.09 dB at room temperature and better than -0.07 dB at 77 K (using the more conservative |S21| for the 77K data).

Assuming a physical temperature of 15 K and an LNA noise temperature of 2 K, a loss of 0.07 dB would produce an increase in noise temperature of 0.29 K.

Note that |s21| itself is better than or equal to –0.07 dB across the band. Where |S21| is at its worst, equal to –0.07 dB, g_{eff} (in dB) is > |S21|, so that g_{eff} is better than –0.07 dB across the entire band.

Since g_{eff} is better than -0.07 dB, and since it would improve still further on cooling from 77 K to 15 K, the estimate of 0.29 K noise contribution by this hybrid at 15 K can be said to be quite conservative.

Using the G_{eff} (est) 77K curve, which goes from about –0.01 dB at 1 GHz to –0.03 dB at 2 GHz, we would estimate the increase in noise temperature, due to the hybrid at 15 K, to be between 40 and 120 mK across the band. This would represent an increase of less than 1% in sytstem temperature, which would be acceptable.