EVLA Memo 83
Quantization Loss for a Sloped Passband

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Abstract

An analysis of the effect of quantization on the SNR of the correlation coefficient is presented, using a simple model for the aliasing of the input power by a quantizer. If the input spectral power density has a strong frequency dependence, the SNR of the derived correlation coefficient also has a strong frequency dependence. The degradation of the SNR is especially strong at the low spectral power end. The results provide a gauge to decide when gain equalization is to be employed.

1 Introduction

The process of quantizing an analog signal causes a well-known loss in its SNR. Studies of the process show the loss is a function of the number and spacing of the quantized levels. With optimum spacing, the loss for a 1-bit, 2-bit, 3-bit, and 4-bit quantizer is 0.637, .881, .960, and .988, respectively.

The analyses assume a flat power spectrum input to the quantizer. However, modern broad-band systems may have a strong slope in their power gain functions, so that the input power spectrum to the quantizer may have a strong frequency dependence. The purpose of this memo is to quantify the effect, assuming a simple model for the loss.

2 The model

We write $P_{\nu,s1}$ and $P_{\nu,s2}$ as the signal spectral power densities from antennas 1 and 2, and $P_{\nu,p1}$ and $P_{\nu,p2}$ for the spectral power densities of the uncorrelated noise from these antennas. The correlation coefficient is then defined as

$$\rho_{\nu,0} = \frac{\sqrt{P_{\nu,s1}P_{\nu,s2}}}{\sqrt{(P_{\nu,s1}+P_{\nu,r1})(P_{\nu,s2}+P_{\nu,r2})}}. \tag{1}$$

To simplify the expressions, we assume the signals from each antenna are identical, so

$$\rho_{\nu,0} = \frac{P_{\nu,s}}{P_{\nu,s}+P_{\nu,r}}. \tag{2}$$

Normally, $P_{\nu,s} \ll P_{\nu,r}$, so $\rho_{\nu,0} \ll 1$.

The process of quantization causes a reduction in the power spectral density (equally to the signal and the noise) by a factor q, as a portion of the analog input power is aliased out of the bandwidth. We assume that this fractional loss is the same for all frequencies within the input passband. Our simple model then assumes that the aliased power lost by the quantizer is spectrally homogenized, and uniformly added back onto the power spectrum within the bandpass. The process of incoherently adding the lost power back into the spectrum causes a reduction in the measured correlation coefficient.

Assume the input power spectral density to be a linear function of frequency: $P_\nu = P_0 + M(\nu - \nu_l)$. Here, M is the gain slope: $M = P_0(G-1)/\Delta\nu$; G is the gain ratio between the upper and lower ends of the band; $\Delta\nu = \nu_u - \nu_l$ is the bandwidth; and P_0 is the input spectral power density at ν_l, the lower frequency edge. The total power input to the sampler, P_T, is then the integral of the input spectral power density:

$$P_T = \int P_\nu d\nu = P_0 \Delta\nu (G+1)/2. \tag{3}$$

In our model, a fraction $1-q$ of this total power is spectrally homogenized, and added to the input power spectral density, which is attenuated by a factor q. The total added power is then $\delta P_T = (1-q)P_T$, where q is the quantizer loss fraction, given in the preceding section, so the spectral power density added to the input becomes

$$\delta P_\nu = \frac{\delta P_t}{\Delta\nu} = \frac{(1-q)P_0(1+G)}{2}. \tag{4}$$

At frequency ν, the measured correlation coefficient will thus be

$$\rho_\nu = \frac{qP_{\nu,s}}{q(P_{\nu,s}+P_{\nu,r}) + (1-q)P_0(1+G)/2}, \tag{5}$$

corresponding to a degradation in the measured correlation coefficient by a factor:

$$\frac{\rho_\nu}{\rho_{\nu,0}} = \frac{1}{1 + \left(\frac{1-q}{q}\right)\left(\frac{P_0}{P_{\nu,s}+P_{\nu,r}}\right)\left(\frac{1+G}{2}\right)}. \tag{6}$$

At the low-power end of the spectrum, $P_{\nu,s} + P_{\nu,r} = P_0$, and we find

$$\frac{\rho_{\nu_l}}{\rho_{\nu_l,0}} = \frac{1}{1 + \frac{1-q}{q}\frac{1+G}{2}} = \frac{2q}{1+G+q(1-G)} \tag{7}$$

while at the high-power end of the spectrum, $P_{\nu,s} + P_{\nu,r} = GP_0$, and we find

$$\frac{\rho_{\nu_h}}{\rho_{\nu_h,0}} = \frac{1}{1 + \frac{1-q}{q}\frac{1+G}{2G}} = \frac{2qG}{1+G-q(1-G)}. \tag{8}$$

The ratio of the correlation coefficient at the high power end to that at the lower power end (which gives the effective differential SNR loss) is:

$$\frac{\rho_{\nu_h}}{\rho_{\nu_l}} = \frac{1+G-q(1-G)]}{G[(1+G+q(1-G)]} \tag{9}$$

The relationships shown in Equations 7 through 9 are shown in Figures 1 and 2.

3 Discussion

It is seen from this simple analysis that a frequency dependence in the input power to the quantizer results in variation in the SNR of the derived correlation coefficient as a function of frequency which is particularly acute at the low spectral power density end of the passband. For the 3-bit case (of special importance to the EVLA, as the quantizers for the 5 GHz and higher bands will be the 3-bit ALMA samplers), the SNR loss at the low-spectral power end will be by 10% with a 6 dB input gain ratio, and by nearly 20% if the gain ratio is 10 dB. To prevent excessive differential loss in the accuracy of the correlation coefficient, gain equalization prior to the quantizers will be necessary.

The model adopted here is based on a combination of intuition and experience with detailed simulations. Although we feel this combination is sufficient to justify our conclusions, it is likely that our simple model will be in error at some level, so that a theoretical analysis of the process of aliasing in this context is highly desirable.

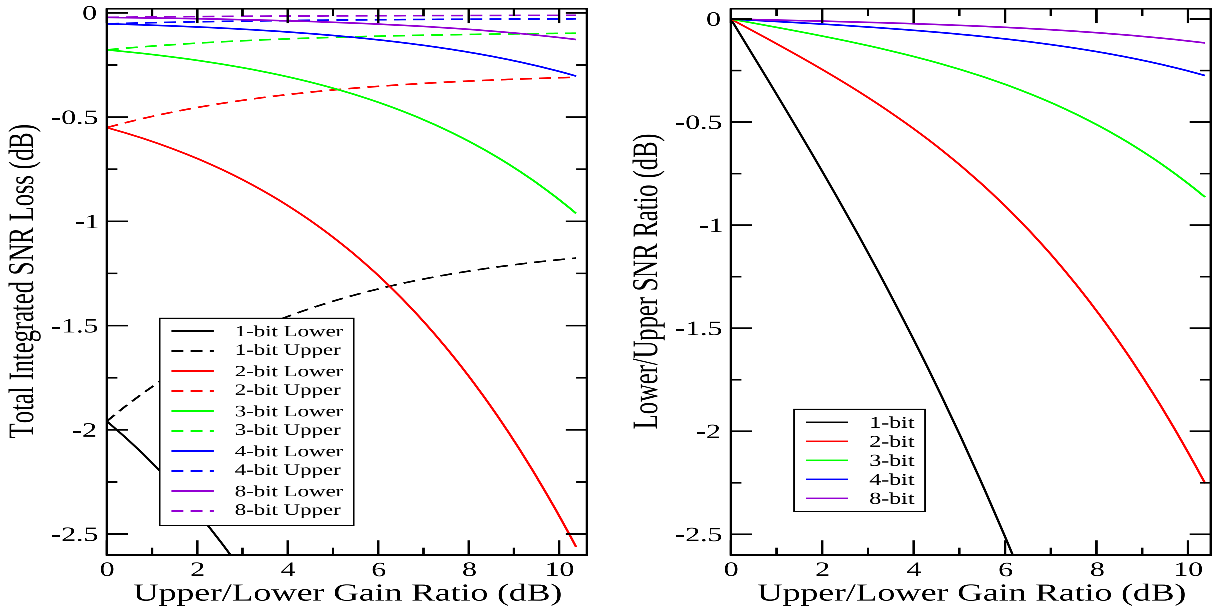

Figure 1: (Left) The SNR loss in the correlation coefficient, in dB, for the low gain (solid, Eqn.7) and high gain (dashed, Eqn.8) edges of the band, for various quantizer sampling schemes as a function of the power gain ratio, G, between the ends of the band. (Right) The correlation coefficient ratio, in dB, between the lower and upper ends of the band, for various quantizer sampler schemes, as a function of the power gain ratio between the lower and upper ends of the band. These curves are described by Eqn. 9.

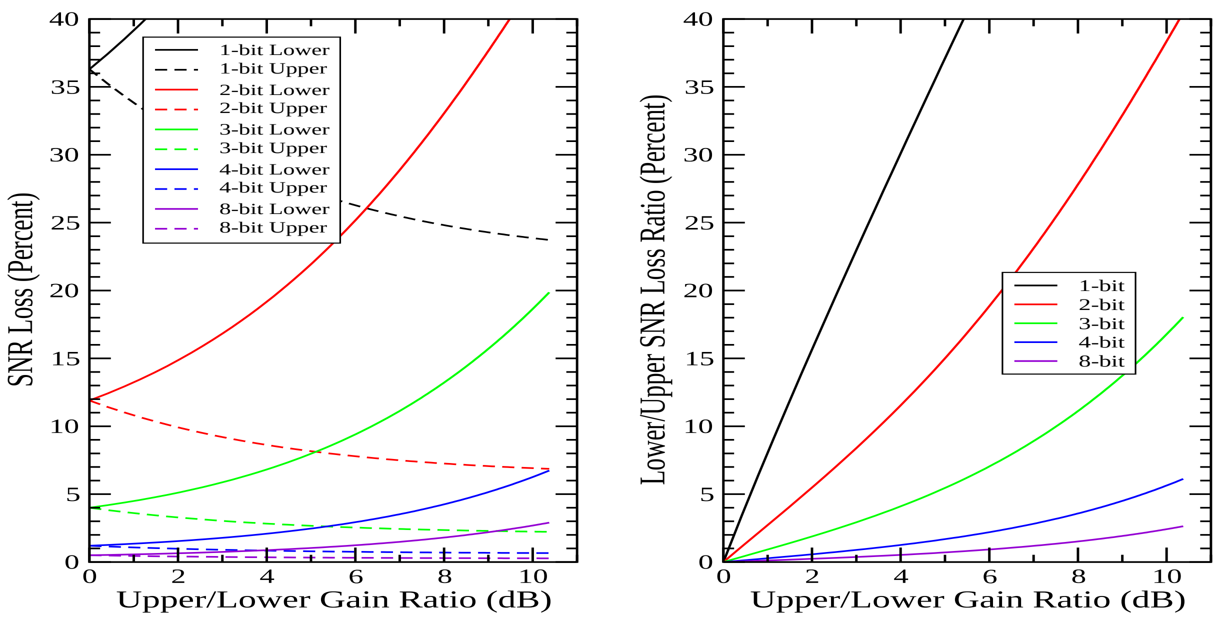

Figure 2: (Left) The percentage SNR loss in the correlation coefficient for the low gain (solid, Eqn.7) and high gain (dashed, Eqn.8) edges of the band, for various quantizer sampling schemes as a function of the power gain ratio, G, between the ends of the band. (Right) The correlation coefficient ratio, expressed as a percentage, between the lower and upper ends of the band, for various quantizer sampler schemes, as a function of the power gain ratio between the lower and upper ends of the band. These curves are described by Eqn. 9.