Sources of Error in Position Observations with the NRAO Interferometer

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Positions of several point sources have been determined by the method outlined in Ref.(1). (See Ref. (2).) Sufficient information is available from this investigation to make an analysis of the sources of error and their relative importance.

The philosophy behind any determination of position with an interferometer is that the instantaneous phase of the fringes determines the projection of the source position on a great circle in the direction of resolution. If, by observing at a second hour angle, the projection of the source position on a second direction of resolution is obtained, then the source position is determined by these two observations. In the method outlined in Ref. (1), an arbitrary instrumental phase is also allowed, so that two observations determine a locus rather than a point, so a second pair of points is needed to determine the position.

The accuracy of a position observation naturally depends on the angle with which the two loci cross. The best case -- that in which the loci cross perpendicularly -- is obtained by using three observations in which the directions of resolution differ by 90°.

To get an idea of the magnitude of this effect, let us return to the analytical notation of Ref. (1). If the hour angles are distributed uniformly about some arbitrary 0, extending from -H to H, then, if the errors in each measurement are random and uncorrelated, the error in the cos (H) term is, as derived from the equations of page 9 of Ref. 1

$$\sqrt{\mu^2 \, w_x^{-1}} = \sqrt{\mu^2 \, \frac{1}{\frac{1+p}{2} - q^2}}$$

where $$p = \frac{\sin 2H}{2H} \;, \quad q = \frac{\sin H}{H}$$

The error in the sine term is similar,

$$\sqrt{\mu^2 \, w_y^{-1}} = \sqrt{\mu^2 \, \frac{2}{1-p}}$$

These two functions are plotted in Fig. 1. These functions have relevence, not only for the random, uncorrelated phase scatter, but also indicate the order of magnitude of the effects of slow drifts in the phase, such that the errors are correlated over the period of observation.

That this effect appears in practice as well as in theory is seen in Fig. 2, in which the position differences of the individual day's observations from the mean of all observations is plotted as a function of the time difference between the beginning and ending times of the observation.

This is the most striking effect in the observed data. However, several other effects may be noted. Firstly, the scatter of the phases should be related to the scatter in the amplitudes by

$$\Delta\phi \approx \frac{\Delta A}{A} \;,$$

Thus predicting that the weak sources, when the signal is comparable to noise, should have much higher phase scatters than the strong sources. However, instead of a simple $\Delta\phi \alpha \frac{1}{A}$ law, the actual law is approximately $\Delta\phi \approx \frac{10}{A} + 3$, where A is in flux units and $\Delta\phi$ in degrees. Thus the phase scatter is almost uninfluenced by thermal noise until the source is weaker than 2 flux units. The present methods of observation should easily suffice to derive accurate positions for sources of 1 flux unit. As is well known, over an appreciable time, the rms deviations of the amplitudes are also much larger than thermal noise on the strong sources, because of gain changes, inaccurate delay tracking, etc. A scatter diagram was made of $\Delta\phi$ against $\frac{\Delta A}{A}$, and these excessive scatters were found to be essentially uncorrelated, although records with a 30% or more rms deviation in the amplitudes also had a high phase scatter.

An attempt was made to find the time dependence of the phase variations by plotting the rms phase deviation against the number of minutes in the sample and against the hour angle range in the sample. The phase deviation is nearly uncorrelated with both. This implies that there is very little correlated phase drift with typical times between 2 hours and 12 hours. The components longer than 12 hours may be seen by the day to day variations of phase at the same hour angle. There must be present components with typical times between 1 minute and 2 hours in order to explain the discrepancy between the computed errors in the least squares fitting and the errors deduced from the day-to-day agreement, which are an order of magnitude higher.

Since the strongest effect on the position errors is the hour angle range, and since this also enters strongly into the computed errors of the rms fitting one might hope that the computed errors would be an accurate guide to the actual errors. However, the correlation is not very close, though for small errors -- up to about .1 or.2" error in the fitting parameter, the actual errors are about 10 times the quoted fitting errors. For fitting errors in excess of 1" the correlation is to some extent restored, but with the actual errors about 6 times the quoted fitting errors.

The observed fitting errors may be affected by celestial phenomena, as well as by instrumental and atmospheric ones. In the case of several sources the rms errors are higher than would be expected for the source intensity, and in three cases this arises from a repeatable phase variation with hour angle which may only be attributed to resolution of the source. A closely related phenomenon would be the effect of a weak confusing source in the beam at the same time as the source under study.

The simplest method of measuring the position of a resolved source is to perform an aperture synthesis, and measure the center of gravity of the synthesized source. (This necessarily implies an absolute calibration of phase, the determination of the B_3 term.) For sources only slightly resolved, it might be possible to extract the 4 third moments as well as the 2 first moments, though solving for so many parameters would undermine the accuracy of all of them. This essentially means solving for the second and third harmonics of the once/day sine wave presently solved for. The difficulties of resolving the phase ambiguities and of weighting the observations properly make this method rather difficult to apply in practice, though in theory it should be applicable to as far as the region of the first minimum in the visibility function. It would require essentially continuous observations for the time the source is above the horizon.

The effects of confusion are also difficult to estimate since a confusing source may not only shift the apparent position of a source within a fringe but, if the observations are not sufficiently complete, also make possible a lobe shifting of the source position. A single, weak point source at a large distance from the source under investigation merely imposes a rapid sinusoidal variation on the phase, and does not affect the derived position if the observations extend over several periods of the rapid variation. An evaluation of the probability of a source occurring within a given area is complicated by the fact that nothing is known of the angular size distribution of the weak sources, as the contribution that the source makes to the confusion is its power times its visibility function. A rough estimate of confusion may be made by multiplying the sky surface brightness due to sources (about 1000 flux units/steradian, scaled with an average spectrum of 0.75 from the 0.5 flux unit average deflection in an 800 sq. min beam at 179 Mc/s reported by Hewish, M.N. 123) by the solid angle of the synthesized beam, determined by the reciprocal of the area swept out in the u,v plane. A typical position measurement sweeps out a track 200 λ wide by 3000 λ long, so the uncertainty due to confusion is small for sources stronger than 1 flux unit.

There are several different sources of error in the phases, which have different dependencies on baseline, so they may be separable by this factor.

1. Instrumental phase changes, such as changes in the paramp phase delay, phase changes as a function of hour angle due to the twisting of the local oscillator lines, phase changes in the local oscillator amplifiers, and phase changes in the local oscillator doublers and independent of baseline.

2. LO transmission phase changes, due to differential changes in the electrical lengths of the LO lines to each antenna vary more slowly than directly proportional to the baseline, the exact rate depending to the correlation length of the disturbences -- it must be smaller than the baseline in order to produce any differential effect. If it is very short compared to the baseline, the effect is a random walk, going as square root of baseline length.
3. Atmospheric phase changes should be independent of baseline length when the baseline is much smaller than the typical lengths of the turbulent eddies, and are directly proportional to baseline length when the baseline is much longer than the turbulence.

A critical examination of phase errors over a range of baselines with the same systems may result in a sorting out of these various effects.

References

1. C. M. Wade, "Interferometer Calibration and Source Position Measurement," NRAO Internal Report, December 1964

2. C. M. Wade, B. G. Clark and D. E. Hogg, "Interferometer Calibration and Source Position Measurement on Baseline 2," NRAO Internal Report, March 1965.

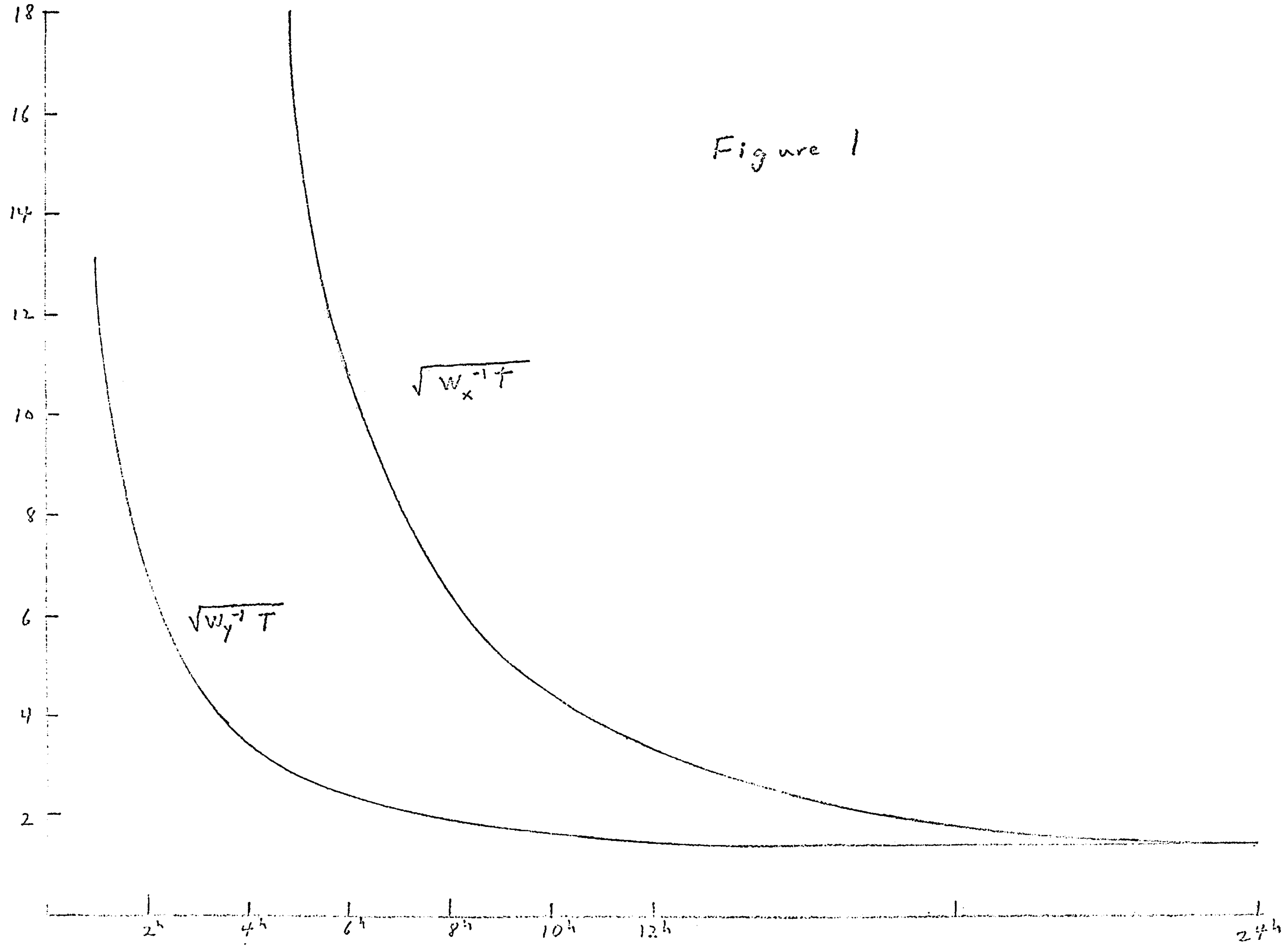
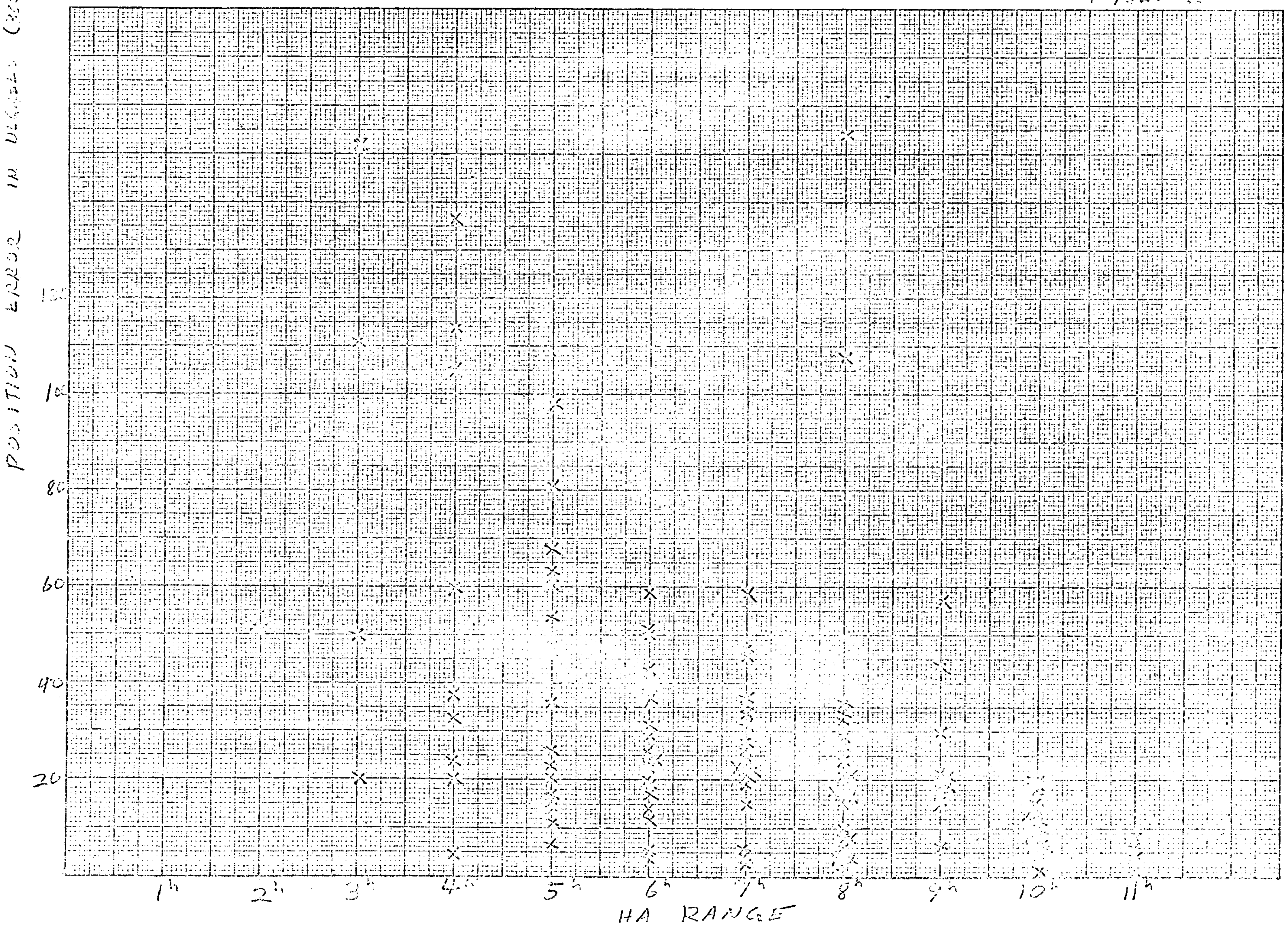


Figure 1
$\sqrt{W_x^{-1} T}$
$\sqrt{W_y^{-1} T}$
18
16
14
12
10
8
6
4
2
2^h
4^h
6^h
8^h
10^h
12^h
24^h

Figure 2


POSITION ERROR IN DEGREES
100
80
60
40
20
1h
2h
3h
4h
5h
6h
7h
8h
9h
10h
11h
HA RANGE