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Differential Measurements Of GBT Tipping Structure Retroreflector Vibrational Motions

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Abstract:

This note discusses a method to determine the vibrations of a rangefinder metrology target located on the tipping structure of the GBT telescope. The telescope is presumed to be stationary, that is undriven, while rangefinder observations of target vibrations are made. Using mean ranges and coordinates of the target measured from three ground based rangefinders, together with continuous observations of the target ranges one can obtain the differential displacement vector of the target versus time.

The differential time variations of the target's coordinates may be determined in both the ground reference and main reflector reference frames, and archived. In the future (Phase III of telescope operation), the archived main reflector frame motions are expected to be analyzed in a normal mode analysis, using an observed vibrational mode computational processor.

The Measurement Mission:

In the present mission we assume that the GBT telescope is not being driven; we assume that the antenna is stationary except for small vibrational motions. We assume that for time t in the time interval of observation,

(1) $AZ(t) = \text{constant} = AZ\,, \qquad EL(t) = \text{constant} = EL\,.$

We wish to determine the small scale vibrational displacement motions of a target retroreflector, T_j, mounted on the tipping structure. Let us call the ground reference frame coordinates of the fiducial reference point, T_j, of the target:

(2a) $X_j(t\,;\ T_j,\ AZ,\ EL) = X_j(t) = TX(t)\,,$
(2b) $Y_j(t\,;\ T_j,\ AZ,\ EL) = Y_j(t) = TY(t)\,,$
(2c) $Z_j(t\,;\ T_j,\ AZ,\ EL) = Z_j(t) = TZ(t)\,.$

Let us call the main reflector frame geometric coordinates of the fiducial reference point, T_j, of the target:

(3a) $X_{rj}(t\,;\ T_j,\ AZ,\ EL) = X_{rj}(t) = TX_r(t)\,,$
(3b) $Y_{rj}(t\,;\ T_j,\ AZ,\ EL) = Y_{rj}(t) = TY_r(t)\,,$
(3c) $Z_{rj}(t\,;\ T_j,\ AZ,\ EL) = Z_{rj}(t) = TZ_r(t)\,.$

We determine $X_j(t)$, $Y_j(t)$, $Z_{rj}(t)$, $X_{rj}(t)$, $Y_{rj}(t)$, $Z_{rj}(t)$ by range measurements to target T_j and a subsequent analysis. The analyzed measurement results are then transferred to an archive data base. At later times the archived results can be accessed and computations performed to resolve $X_{rj}(t)$, $Y_{rj}(t)$, $Z_{rj}(t)$ into characteristic vibrational modes and find the mode amplitude vectors.

Description of Measurements:

To perform the stated mission we measure ranges to target T_j from three ground-based rangefinders: R_1, R_2, R_3, which can simultaneously view T_j when the antenna is positioned at azimuth angle AZ and elevation angle EL.

Call the reference scan points of these rangefinders: S_1, S_2, S_3, respectively, and let their known ground reference frame coordinates be given:

(4) $$S_1 = (X_1, Y_1, Z_1), \quad S_2 = (X_2, Y_2, Z_2), \quad S_3 = (X_3, Y_3, Z_3).$$

These reference points (which are the scan axis intersection points on the rangefinder scanning mirrors) are spatially fixed and do not vary with time. Let the range distance measured from S_i to T_j at time t be $s_i(t)$, $\{i = 1, 2, 3\}$.

We assume that mean (time averaged) ranges averaged over times long compared to any vibrational mode periods are available. That is, range measurement results are available, averaged over suitably long time intervals, for the quantities:

(5) $$\overline{s_i} = \left(\frac{1}{t_2 - t_1}\right) \cdot \int_{t_1}^{t_2} s_i(t)\, dt \ , \qquad i = 1, 2, 3.$$

We also assume that mean coordinates of the target retroreflector are available:

(6) $$\overline{T_j} = \left(\overline{X_j}, \overline{Y_j}, \overline{Z_j}\right) \ , \qquad i = 1, 2, 3,$$

from trilateration coordinate adjustments using mean ranges measured to T_j.

We also assume that the range fluctuations with time are always small compared to the mean ranges. Let us call

(7) $$\delta s_i(t) = s_i(t) - \overline{s_i} \, , \qquad i = 1, 2, 3.$$

Then we assume that

(8) $$0 \leq |s_i(t) - \overline{s_i}| << \overline{s_i} \, .$$

Let the ground reference frame coordinates of the target fiducial point coordinates be:

(9) $$\begin{bmatrix} X_j(t) \\ Y_j(t) \\ Z_j(t) \end{bmatrix} = \begin{bmatrix} \overline{X_j} + \delta X_j(t) \\ \overline{Y_j} + \delta Y_j(t) \\ \overline{Z_j} + \delta Z_j(t) \end{bmatrix} .$$

Time-averaged target coordinates: $\overline{X_j}$, $\overline{Y_j}$, $\overline{Z_j}$ are assumed to be available. The differential coordinate changes $\delta X_j(t)$, $\delta Y_j(t)$, $\delta Z_j(t)$, are computed from the range relations:

$$
(10)\quad
\begin{aligned}
(\overline{s_i})^2 &= (\overline{X_j}-X_i)^2+(\overline{Y_j}-Y_i)^2+(\overline{Z_j}-Z_i)^2\,, \quad i=1,2,3 \quad \text{and}\\
(\overline{s_i}+\delta s_i(t))^2 &= (\overline{X_j}+\delta X_j(t)-X_i)^2+(\overline{Y_j}+\delta Y_j(t)-Y_i)^2+(\overline{Z_j}+\delta Z_j(t)-Z_i)^2\,.
\end{aligned}
$$

Subtracting the mean range equation from the time-dependent range equation above, for each of the three observed ranges, and neglecting second order differential range and coordinate terms, using assumption (7), we get a set of three first order linear equations in the differential coordinates:

$$
(11)\quad (-1)\cdot\,\overline{s_i}\cdot\delta s_i(t) = (\overline{X_j}-X_i)\cdot\delta X_j(t)+(\overline{Y_j}-Y_i)\cdot\delta Y_j(t)+(\overline{Z_j}-Z_i)\cdot\delta Z_j(t)\ ;\ i=1,2,3.
$$

These equations are more transparent when written explicitly, in matrix notation:

$$
(12)\quad
\begin{bmatrix} \overline{s_1}\cdot\delta s_1(t)\\ \overline{s_2}\cdot\delta s_2(t)\\ \overline{s_3}\cdot\delta s_3(t)\end{bmatrix}
=
\begin{bmatrix}
(X_1-\overline{X_j}) & (Y_1-\overline{Y_j}) & (Z_1-\overline{Z_j})\\
(X_2-\overline{X_j}) & (Y_2-\overline{Y_j}) & (Z_2-\overline{Z_j})\\
(X_3-\overline{X_j}) & (Y_3-\overline{Y_j}) & (Z_3-\overline{Z_j})
\end{bmatrix}
\cdot
\begin{bmatrix}\delta X_j(t)\\ \delta Y_j(t)\\ \delta Z_j(t)\end{bmatrix}.
$$

To solve them compute the determinants:

$$
(13.4)\quad DET4 = \begin{vmatrix}
(X_1-\overline{X_j}) & (Y_1-\overline{Y_j}) & (Z_1-\overline{Z_j})\\
(X_2-\overline{X_j}) & (Y_2-\overline{Y_j}) & (Z_2-\overline{Z_j})\\
(X_3-\overline{X_j}) & (Y_3-\overline{Y_j}) & (Z_3-\overline{Z_j})
\end{vmatrix},
$$

$$
(13.1)\quad DET1 = \begin{vmatrix}
\overline{s_1}\cdot\delta s_1(t) & (Y_1-\overline{Y_j}) & (Z_1-\overline{Z_j})\\
\overline{s_2}\cdot\delta s_2(t) & (Y_2-\overline{Y_j}) & (Z_2-\overline{Z_j})\\
\overline{s_3}\cdot\delta s_3(t) & (Y_3-\overline{Y_j}) & (Z_3-\overline{Z_j})
\end{vmatrix},
$$

$$
(13.2)\quad DET2 = \begin{vmatrix}
(X_1-\overline{X_j}) & \overline{s_1}\cdot\delta s_1(t) & (Z_1-\overline{Z_j})\\
(X_2-\overline{X_j}) & \overline{s_2}\cdot\delta s_2(t) & (Z_2-\overline{Z_j})\\
(X_3-\overline{X_j}) & \overline{s_3}\cdot\delta s_3(t) & (Z_3-\overline{Z_j})
\end{vmatrix},
$$

$$
(13.3)\quad DET3 = \begin{vmatrix}
(X_1-\overline{X_j}) & (Y_1-\overline{Y_j}) & \overline{s_1}\cdot\delta s_1(t)\\
(X_2-\overline{X_j}) & (Y_2-\overline{Y_j}) & \overline{s_2}\cdot\delta s_2(t)\\
(X_3-\overline{X_j}) & (Y_3-\overline{Y_j}) & \overline{s_3}\cdot\delta s_3(t)
\end{vmatrix}.
$$

The solution is:

$$
(14)\quad \delta X_j(t) = \frac{DET1}{DET4}\,,\quad \delta Y_j(t) = \frac{DET2}{DET4}\,,\quad \delta Z_j(t) = \frac{DET3}{DET4}\,.
$$

Alternatively, the equations (12) can be solved by inverting the matrix of coordinate differences and computing the matrix product

$$(12.1)\quad \begin{bmatrix} \delta X_j(t) \\ \delta Y_j(t) \\ \delta Z_j(t) \end{bmatrix} = \begin{bmatrix} (X_1 - \overline{X_j}) & (Y_1 - \overline{Y_j}) & (Z_1 - \overline{Z_j}) \\ (X_2 - \overline{X_j}) & (Y_2 - \overline{Y_j}) & (Z_2 - \overline{Z_j}) \\ (X_3 - \overline{X_j}) & (Y_3 - \overline{Y_j}) & (Z_3 - \overline{Z_j}) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \overline{s_1} \cdot \delta s_1(t) \\ \overline{s_2} \cdot \delta s_2(t) \\ \overline{s_3} \cdot \delta s_3(t) \end{bmatrix} .$$

The time variations of the main reflector frame coordinates of a tipping structure target can be calculated via the rotation matrix relating ground frame displacement components to main reflector frame displacement components:

$$(15)\quad \begin{bmatrix} \delta X_{rj}(t) \\ \delta Y_{rj}(t) \\ \delta Z_{rj}(t) \end{bmatrix} = \begin{bmatrix} \cos AZ & -\sin AZ & 0 \\ \sin AZ \cdot \sin EL & \cos AZ \cdot \sin EL & -\cos EL \\ \sin AZ \cdot \cos EL & \cos AZ \cdot \cos EL & \sin EL \end{bmatrix} \cdot \begin{bmatrix} \delta X_j(t) \\ \delta Y_j(t) \\ \delta Z_j(t) \end{bmatrix} .$$

An Extension Of The Method:

A situation may arise where target T_j may not have lines of sight available simultaneously to three ground based rangefinders. In some cases of this type, the target may have lines of sight to only two ground-based rangefinders but may also be located near a second tipping structure retroreflector target, T_k, which has identical vibrational displacements as T_j, and can also be sighted by two ground-based rangefinders. For example, ball retroreflector targets are paired at locations on the right and left sides of the vertical and horizontal feed arms. For bending or rocking oscillations of the feed arm one expects equal displacements of targets symmetrically located on the right and left sides of the feed arm. One would not expect this to be so for torsional oscillations of the feed arm about its vertical axis. In the former case, the previously discussed method of obtaining the target vibrational displacements versus time can be extended to give the vibrational motions.

The extended method, outlined below, also applies to the example of targets mounted on the alidade structure underneath the elevation bearings. A pair of retroreflector targets is mounted undeneath each of the two GBT elevation bearing weldment structures. The motions of the target pair under each weldment follow one another. Two ground-based rangefinders can sight one target of the pair under a given weldment, while two other rangefinders can sight the other target under the same weldment. The two targets are close to one another and are rigidly connected to one another and to the weldment. Availability of lines of sight to either target by three rangefinders simultaneously is not a usual occurrence. In this situation the procedure given below can be used.

In these cases one sets up modified equations (12) using range measurements to T_j from rangefinders S_1 and S_2 and range measurements to T_k from rangefinder S_3. One assumes,

(16) $\delta X_k(t) = \delta X_j(t)$, $\delta Y_k(t) = \delta Y_j(t)$, $\delta Z_k(t) = \delta Z_j(t)$ and solves

$$(12m) \quad \begin{bmatrix} \overline{s_1} \cdot \delta s_1(t) \\ \overline{s_2} \cdot \delta s_2(t) \\ \overline{s_3} \cdot \delta s_3(t) \end{bmatrix} = \begin{bmatrix} (X_1 - \overline{X_j}) & (Y_1 - \overline{Y_j}) & (Z_1 - \overline{Z_j}) \\ (X_2 - \overline{X_j}) & (Y_2 - \overline{Y_j}) & (Z_2 - \overline{Z_j}) \\ (X_3 - \overline{X_k}) & (Y_3 - \overline{Y_k}) & (Z_3 - \overline{Z_k}) \end{bmatrix} \cdot \begin{bmatrix} \delta X_j(t) \\ \delta Y_j(t) \\ \delta Z_j(t) \end{bmatrix} .$$

The assumption of equal displacement for the two sets of targets are checked by using the time-sampled ranges from the fourth rangefinder, S_4, to T_k using equation (11) applied to this ranger-target pair, to verify that the range fluctuation is correctly given from the solved coordinate displacements of (12m) by

$$(11m) \quad \delta s_4(t) = (\frac{1}{\overline{s_4}}) \cdot [(X_4 - \overline{X_k}) \cdot \delta X_j(t) + (Y_4 - \overline{Y_k}) \cdot \delta Y_j(t) + (Z_4 - \overline{Z_k}) \cdot \delta Z_j(t)] .$$

Measurement Procedure And Scheduling:

We denote the present measurement procedure mission as follows:

"Tipping Structure Retroreflector Coordinate Variations
Due To Vibrational Motion, Static."

In this mission the telescope sits at fixed arbitrary azimuth and elevation. A set of time-averaged ground frame and reflector frame coordinates of a target on the tipping structure are determined. This is done either by trilateration of ranges which have been time averaged over suitably long time intervals, or lookup from an archive data base. One does not need extremely accurate mean values to get good values of the vibrational displacement components.

The ranges to the target from three ground-based rangefinders are measured as functions of time. The measurements are archived and time sampled. For desired sample times, the target coordinate displacements from the mean values are computed by solving the linear equations (12), using (13) and (14). The time sample solutions are then archived for later use in a suitable data base.

Discussion:

In this note we described a procedure for measuring vibrations of rangefinder targets on the GBT tipping structure when the structure is not actively driven by its motors. It was assumed that suitable mean values of ranges and target coordinates are available for use as coefficients in linear equations determining differential target displacements. Actually, the range measurements are of time-sampled rangefinder signal return-phase, integrated over a short sample time and referenced to the observation mid-time, and are not really continuous in time. But the ranges are sampled at rates rapid enough to sample many times over an oscillation period of the target, and one has in effect time-continuous range samples.

The method was developed for the case that each target is viewed simultaneously by three rangefinders. It was extended to the case of two individual targets known to have identical vibrational displacements but each target can be simultaneously viewed by only two rangefinders.

These methods may be applied usefully to several interesting situations. First, they can be used to study wind-driven motions of the feed arm and alidade structures. They can also be used to compute differential target displacements directly while the temperature of the telescope changes. They can also be used to study jerk-induced vibrations of the tipping and alidade structures due to start and stop drive motions of the telescope.

When these methods are employed, the results should be archived in appropriate data base files. It is expected that suitable computer code will be written to carry out the linear algebra required to compute the vibration components, and later to perform normal mode analyses and and suitably display any information generated on computer monitor screens.