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## Confirmation Of S. Srikanth's Pointing Coefficients For GBT Single Subreflector Optics.

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### ABSTRACT

This note presents independent ray trace calculations to check the values of GBT pointing coefficients for GBT single subreflector optics given in a note of March 6, 1991 by S. Srikanth. These are pointing offsets caused by translations, rotations, or focal length change of the GBT optical elements (main reflector, subreflector, Gregorian focus feed horn, prime focus feed horn). The ray trace derived error coefficients agree in sign and magnitude with those of Srikanth to a few percent.

### 1 Introduction

Dated March 6, 1991 a short memorandum was filed by S. Srikanth: "Pointing Coefficients for Single Subreflector Optics." This memorandum, Green Bank archive document GBRN 0006995, presented the results of calculations of pointing offsets caused by perturbations in the optical elements forming the Green Bank Telescope. Included in the memorandum was a comparison with the results of pointing error coefficient calculations by R. Levy of Jet Propulsion Laboratories (dated January 23, 1990). The two sets of calculations agreed in some coefficients but differed by factors of two in others. In neither case were details of the calculations presented.

Recently, John Payne questioned whether those results had been independently validated. It was decided that an optical ray trace program should be run to check the coefficients presented in the above memorandum. This has been carried out using the BEAM 3 optical ray trace codes (Stellar Software, Berkeley, CA). The results are presented in Table 2. The geometric assumptions underlying the present calculations are given in the next section of this document. The setup of the ray tracing files is discussed in a subsequent section. The ray tracing files themselves and a copy of Srikanth's memorandum are appended.

## 2 The Design Telescope Optical Geometry

The starting point for our discussion is the geometry of the reference optical telescope. The design of the GBT optics is given in GBT Memo 155, "A Summary of the GBT Optics Design" by R. Norrod and S. Srikanth, March 1996. A detailed analysis of this design is given in GBT Memo 165, "GBT Coordinates And Coordinate Transforms" by M.A. Goldman, February 1997.

For the purposes of the present document, a summary of the relevant features of the reference optical telescope (which we will also refer to as the design telescope) are given below.

The primary reflector surface is an offset portion of a paraboloid of revolution of focal length  $f_p = 60$  meters. We call this paraboloid the "design paraboloid" or the "parent paraboloid." The plane parabola which generates this surface has curvature at its vertex point,  $R_d$ , which can be shown to be:  $C(R_d) = 1 / (2 \cdot f_p) = 0.00833333 \text{ m}^{-1}$ . (This curvature is needed to set up the ray tracing files.)

The geometry of the reference optical telescope is described in relation to the telescope's main reflector reference frame coordinate system. This is a right hand Cartesian coordinate system whose origin is located at the vertex point of the parent paraboloid. We assume that it is rigidly embedded in the tipping structure of the telescope. We will call the coordinate axes  $X_{rd}, Y_{rd}, Z_{rd}$ . The  $Z_{rd}$  axis points from the vertex to the focal point of the parent paraboloid. The plane of the  $Y_{rd}$  and  $Z_{rd}$  axes defines the plane of symmetry of the design telescope. The  $Y_{rd}$  axis points towards the primary reflector which is located in the half-space  $Y_{rd} > 0$ . The  $X_{rd}$  axis is normal to the telescope's plane of symmetry, is parallel to the elevation axis of the telescope, and points towards the right side of the telescope (towards the man lift). The subscript "rd" on the axis designates that the coordinates refer specifically to the main reflector coordinate system for the design telescope.

The secondary reflector is a surface patch on a prolate ellipsoid of revolution, the "design ellipsoid" or "parent ellipsoid." Let us call the focal points of this ellipsoid  $F_0$  and  $F_1$ . The design focal spacing is  $|F_0 F_1| = 11 \text{ meter} \equiv 2 \cdot f_e$ . The design eccentricity is  $e = 0.528$ . These values produce calculated values of the ellipsoid semi-axis lengths:  $a = 10.416667 \text{ meter}$  and  $b = c = 8.846296 \text{ meter}$ . The ellipsoid focus  $F_0$  is defined to coincide with the parent paraboloid focus, and is defined to be the prime focus of the design telescope. The major axis of the ellipsoid lies in the telescope plane of symmetry and is defined to make an angle  $\beta = 5.570^\circ$  with the parent paraboloid axis. The focal point  $F_1$  is defined to be the Gregorian focal point of the design telescope. (Figure 1 and Figure 2).

The subreflector surface patch is symmetric with respect to the design telescope's plane of symmetry. It has an approximate center point,  $I_1$ , which lies in the plane of symmetry. The point  $I_1$  together with the point  $F_1$  define an optical axis for the design telescope. The design telescope receiver room is located so that the phase center of each (non-offset) receiver room feed horn will be at the Gregorian focus point  $F_1$  and the horn's electromagnetic axis will lie along the line  $F_1 I_1$ . The plane through  $F_1$  perpendicular to line  $F_1 I_1$  is the Gregorian focal plane of the design telescope. For the design telescope, an optical ray which starts at  $F_1$  towards  $I_1$  will reflect specularly through  $F_0$ , reflect specularly from the parent paraboloid and emerge parallel to the paraboloid axis.

Specifically, the location of the point  $I_1$  is defined by:

$$(2.1) \quad \angle F_0 F_1 I_1 = \alpha = 17.899^\circ,$$

and it follows by simple trigonometry that:

$$(2.2) \quad \angle F_0 I_1 F_1 = \gamma = 36.127028^\circ ,$$

and the ray  $F_0 I_1$  makes an angle with the  $+Z_{rd}$  axis of

$$(2.3) \quad \alpha + \gamma - \beta = 48.456028^\circ ,$$

$$(2.4) \quad |F_0 I_1| = r_2 = 5.7341748 \text{ m} .$$

To set up the ray trace files we will need to reference the parent ellipsoid to one of its mathematical vertices and compute the curvature of its generating ellipse at this vertex. We will reference the parent ellipsoid to the upper vertex point on its major axis, which we will call  $V_{el}$ . It can be shown that the curvature at a vertex of the generating ellipse is  $C(V_{el}) = \pm(a/b^2)$ . We also need to compute a “shape factor” for the ellipsoid, defined by:

$$(2.5) \quad \text{Ellipsoid shape factor} = 1 - e^2 \quad (= 0.721216 \text{ for the parent ellipsoid}).$$

In Table 1 we list the coordinates of the design telescope reference points, in the main reflector coordinate system.

Table 1. Design Telescope Reference Point Coordinates (Meters).

Point	$X_{rd}$	$Y_{rd}$	$Z_{rd}$
$R_d$	0	0	0
$F_0$	0	0	$f_p = 60$
$F_1$	0	$-2f_e \cdot \sin \beta = -1.0676797$	$f_p - 2f_e \cdot \cos \beta = 63.802874$
$I_1$	0	$-r_2 \cdot \sin(\alpha + \gamma - \beta) = -4.2917258$	$f_p + r_2 \cdot \cos(\alpha + \gamma - \beta) = 63.802874$
$V_{el}$	0	$(a - f_e) \cdot \sin \beta = 0.4772204$	$f_p + (a - f_e) \cdot \cos \beta = 64.893452$

### 3 Ray Trace Computations Of Error Pointing Coefficients

The Beam 3 ray trace codes are set up to compute an optical system in the following way. An optics (.OPT) file is generated which lists sequentially the properties of the optical surfaces and optical indexes to be traversed sequentially. A ray (.RAY) file is also generated, which lists an initial point and direction for each ray which is to traverse the optical system. Rays are characterized by  $X, Y, Z$  Cartesian coordinates and direction cosines  $U, V, W$  to the respective  $X, Y, Z$  coordinate axes. The optical code files are configured so that code  $X, Y, Z$  axes correspond to telescope  $X_{rd}, Y_{rd}, Z_{rd}$  axes respectively. Aspheric surfaces of revolution generated by rotation of a plane conic curve, are described by: a vertex point location, the curvature of the generating conic at its vertex, a shape factor if the surface is ellipsoidal, and local tilt and pitch of the surface’s axis of symmetry to the coordinate axes. The ray trace calculation generates a vertex list (.VXL file) giving the coordinates of the intersection point of each ray with the optical surfaces, sequentially, and the direction cosines of the ray on exit from the surface.

Ray trace computations of the design telescope were carried out initially to confirm the setup of the optics and ray files. A ray from the Gregorian focus point  $F_1$  to the parent ellipsoid subreflector’s reference point  $I_1$  should pass through the prime focus point  $F_0$  of the parent paraboloid and, after reflection from the paraboloid, be directed parallel to the paraboloid’s axis. This was confirmed for the

files GBT.OPT and GBT.RAY which were configured to model the design telescope.

Optics files are generated to study the behavior of a reference ray from the design telescope Gregorian focus towards the design subreflector reference point when the telescope is perturbed from its design configuration by motions of single optical elements. The deviation of the exit ray direction, after reflection from the perturbed telescope's paraboloid, is examined when the subreflector is translated (by changing a coordinate of the ellipsoid vertex) or rotated (by changing the tilt or pitch of the ellipsoid). It is examined when the paraboloid is tilted or pitched or translated, by similar means. For these perturbations the original file GBT.RAY is used together with a separate optics file GBT\_n.OPT which configures the perturbed optical telescope. To study the effect of feed horn translation (or feed room translation when the active feed horn is fixed relative to the receiver room) the original GBT.OPT optics file is used, together with a separate ray file GBT02.RAY which translates the origin of the reference ray but preserves its initial direction. The exit ray direction angle shift is computed per unit optical element shift in coordinate or orientation.

A right hand sign convention is used for rays rotating about the positive coordinate axes. A positive rotation of an exit ray about the  $+X_{rd}$  axis corresponds to the ray leaving with an increased elevation.

The perturbations of the telescope configuration are small, and the small angle approximation:  $\sin(\text{angle}) = \text{angle}$ ,  $\cos(\text{angle}) = 1$  holds for deviations of the exit reference ray angle to the  $+Z_{rd}$  axis. If either the  $U$  or the  $V$  direction cosine of the exit ray vanishes, and the other of these two cosines is small, one may read the output file directly to find the ray deviation angle. For such cases the rotation, in radians, of the exit ray about the  $+X_{rd}$  axis is equal to  $-V_{final}$  when  $U_{final}$  vanishes; the rotation of the exit ray to the  $+Y_{rd}$  axis is equal to  $+U_{final}$  when  $V_{final}$  vanishes. Here  $U_{final}, V_{final}, W_{final}$  are the direction cosines of the ray after leaving the final optical surface.

In our study of perturbed telescope configurations we have restricted perturbations of optical component orientation to the following cases. When the paraboloid surface alone is perturbed, by a change in orientation, we leave the paraboloid vertex fixed at the point  $R_d$ . That is, the perturbed telescope's paraboloid vertex point remains unmoved with respect to the subreflector and Gregorian feed, although the paraboloid becomes tilted about the  $X_{rd}$  or the  $Y_{rd}$  axis. When the subreflector surface alone is perturbed, by a change of tilt or pitch, we leave the subreflector reference point  $I_1$  fixed with respect to the paraboloid and feed. The Srikanth and Levy results did not explicitly cite the invariant point of these perturbations. It is not clear, for example, whether the ellipsoid reference point or the ellipsoid vertex was held fixed in perturbing the telescope configuration in those results.

As an example of how the optics file is reconfigured for a perturbed telescope, let us give the example of the configuration where the subreflector is tilted by a small angle,  $\theta$  radians, about the  $+X_{rd}$  axis, while leaving the subreflector reference point  $I_1$  fixed. The ellipsoid vertex point transforms from point  $V_{el}$  to a new point,  $V'_{el}$ . Under this perturbation of the telescope configuration:  $R_d \rightarrow R_d, F_0 \rightarrow F_0, F_1 \rightarrow F_1, I_1 \rightarrow I_1$ . Coordinates of the latter points remain fixed. The coordinates of the ellipsoid vertex of the perturbed telescope are given by:

$$(3.1) \quad \begin{bmatrix} X_{rd}(V'_{el}) \\ Y_{rd}(V'_{el}) - Y_{rd}(I_1) \\ Z_{rd}(V'_{el}) - Z_{rd}(I_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} X_{rd}(V_{el}) \\ Y_{rd}(V_{el}) - Y_{rd}(I_1) \\ Z_{rd}(V_{el}) - Z_{rd}(I_1) \end{bmatrix}.$$

The small angle approximation is valid and (3.1) simplifies to:

$$(3.1) \quad \begin{bmatrix} X_{rd}(V'_{el}) \\ Y_{rd}(V'_{el}) \\ Z_{rd}(V'_{el}) \end{bmatrix} = \begin{bmatrix} X_{rd}(V_{el}) \\ Y_{rd}(V_{el}) - \theta \cdot (Z_{rd}(V_{el}) - Z_{rd}(I_1)) \\ Z_{rd}(V_{el}) + \theta \cdot (Y_{rd}(V_{el}) - Y_{rd}(I_1)) \end{bmatrix}.$$

The optics file parameter TILT is modified from  $\beta$  degrees to  $\beta + \theta \cdot (180/\pi)$  degrees. The optics

file data entries are modified to indicate the new ellipsoid vertex coordinates and ellipsoid tilt.

The optics files GBT\_n.OPT each describe the changes made to the design telescope GBT.OPT file to reconfigure the file to generate the individual perturbations of the telescope configuration. The final direction cosine entries for each ray in the GBT\_n.VXL ray trace output files provide the exit ray deviation angles when evaluated as described above.

## 4 Results

The ray trace results are summarized and listed in Table 2, together with a comparison of Levy's 1990 JPL results and Srikanth's 1991 results. The coefficients have also been converted to units of arcseconds per millimeter, using the conversion 1 radian/meter = 206.265 arcsec/mm.

Table 2. Differential Error Pointing Coefficients

Element Shift	Pointing Coefficient				arcsec / mm	
	BEAM3, 1999	Srikanth 1991	JPL	unit	1999	1991
Subrefl. Rot. about an axis parallel to $+X_{rd}$ axis	+0.159 (EL)	+0.1504	+0.0657	mrad/mrad		
Subreflector $+Y_{rd}$ Shift	+0.0143 (EL)	+0.0141	+0.0120	mrad/mm	+2.950	+2.908
Subreflector $+Z_{rd}$ Shift	+0.0115 (EL)	+0.0103	+0.0089	mrad/mm	+2.372	+2.125
Feed $+Y_{rd}$ Shift	-0.0051 (EL)	-0.0051	-0.0031	mrad/mm	-1.052	-1.052
Feed $+Z_{rd}$ Shift	-0.0011 (EL)	-0.0012	-0.0005	mrad/mm	-0.227	-0.248
Parab. Rot. about $+X_{rd}$ axis	+1.552 (EL)	+1.5490	+1.5453	mrad/mrad		
Paraboloid $+Y_{rd}$ Shift	-0.0092 (EL)	-0.0091	-0.0090	mrad/mm	-1.898	-1.877
Paraboloid $+Z_{rd}$ Shift	-0.0104 (EL)	-0.0092	-0.0087	mrad/mm	-2.145	-1.898
Prime Focus Beam Deviation Factor in $(Y_{rd}, Z_{rd})$ -plane	(EL)	+0.9280		mrad/mrad		
Subrefl. Rot. about an axis parallel to $+Y_{rd}$ axis	+0.131 (AZ)	+0.1336		mrad/mrad		
Subreflector $+X_{rd}$ Shift	-0.0191 (AZ)	-0.0183		mrad/mm	-3.940	-3.775
Feed $+X_{rd}$ Shift	+0.0053 (AZ)	+0.0051		mrad/mm	+1.093	+1.052
Parab. Rot. about $+Y_{rd}$ axis	+1.832 (AZ)	+1.7710		mrad/mrad		
Paraboloid $+X_{rd}$ Shift	+0.0139 (AZ)	+0.0130		mrad/mm	+2.867	+2.681
$f_p$ change ( $+ \rightarrow$ increase)	-0.0104 (EL)	-0.00924		mrad/mm	-2.145	-1.906
Prime Focus Beam Deviation Factor in $(X_{rd}, Z_{rd})$ -plane	(AZ)	+0.9400		mrad/mrad		

## 5 Pointing Error Coefficients From Exit Ray Deviations

The ray trace program results provide exit ray deviation angles per unit displacement or rotation of an optical element of a conceptual telescope, the design telescope. We assume that these results will be used to provide error estimates of pointing errors of the as-built GBT telescope, given the errors of placement of the real optical elements. These error estimates would be used to provide *a-priori* correction terms in the telescope pointing series. To do this, calculated rotations of the reference exit ray from the  $+Z_{rd}$  axis must be converted to error correction terms of the form  $\Delta EL_{corr}$  and  $(\cos EL) \cdot \Delta AZ_{corr}$ . Exit ray rotations are sufficiently small, for realistic optical element perturbations, that error terms generated by individual optical element perturbations are separately additive. We next derive pointing error and correction terms generated by small exit ray rotations about the  $+X_{rd}$  and  $+Y_{rd}$  axes.

We start with the reference exit ray from the design telescope. This ray starts at the design Gregorian focus  $F_1$  and initially travels to the parent ellipsoid reference point  $I_1$ . After reflections from the parent ellipsoid and parent paraboloid it exits the telescope parallel to the  $+Z_{rd}$  axis. We denote a unit vector in the direction of this design telescope exit ray by  $\hat{r}_d$ .

When the reference telescope is oriented at azimuth angle  $AZ$  and elevation angle  $EL$ , the main reflector frame unit basis vectors can be represented with respect to the ground coordinate frame of the GBT by unit vectors:

$$(4.1) \quad \begin{aligned} \hat{X}_{rd} &= (\cos AZ) \cdot \hat{E} + (-\sin AZ) \cdot \hat{N} , \\ \hat{Y}_{rd} &= (\sin EL \cdot \sin AZ) \cdot \hat{E} + (\sin EL \cdot \cos AZ) \cdot \hat{N} + (-\cos EL) \cdot \hat{Zen} , \\ \hat{Z}_{rd} &= (\cos EL \cdot \sin AZ) \cdot \hat{E} + (\cos EL \cdot \cos AZ) \cdot \hat{N} + (\sin EL) \cdot \hat{Zen} = \hat{r}_d . \end{aligned}$$

We use the notation  $\hat{N}$  (North),  $\hat{E}$  (East),  $\hat{Zen}$  (Zenith), for the ground frame base vectors because we have previously assigned  $X, Y, Z$  as base vectors for the ray trace codes.

Let us assume that the telescope configuration is perturbed by a single optical element shift or rotation, described by a telescope design parameter  $p$  so that the exit ray is rotated by a small angle  $\theta(p)$  about the  $+X_{rd}$  axis. The exit ray then rotates to the direction of the vector

$$(4.2) \quad \vec{r} = \hat{r}_d + \Delta \vec{r}(p) = \hat{r}_d + \theta(p) \cdot [\hat{X}_{rd} \times \hat{Z}_{rd}] \quad \text{which gives}$$

$$(4.3) \quad \Delta \vec{r} = -\theta(p) \cdot \hat{Y}_{rd} .$$

The rotated ray remains in the median plane of the design telescope; it has only rotated in elevation. The  $+X_{rd}$  axis is the elevation axis of the design telescope; the perturbation rotates the exit ray by an angle  $(\Delta EL(p))_{exit\_ray} = +\theta(p)$  in elevation. This may be considered to be an error  $EL_{err}(p)$  in the pointing elevation angle, which requires a pointing correction  $EL_{corr}(p) = -EL_{err}(p)$ . That is,

$$(4.4) \quad EL_{corr}(p) = -EL_{err}(p) = -(\Delta EL(p))_{exit\_ray} = -\theta(p) .$$

If  $\theta$  is a smooth function of  $p$  (when other telescope parameters are held fixed), then for small changes in  $p$  we have

$$(4.5) \quad (\Delta EL)_{exit\_ray} = \left(\frac{d\theta}{dp}\right) \cdot \Delta p .$$

The value of  $\left(\frac{d\theta}{dp}\right)$  found by the ray tracing then can be considered as a differential elevation pointing error rate coefficient when a small change in parameter  $p$  produces a small exit ray rotation about the  $+X_{rd}$  axis.

Let us now consider the case that a small change in a general telescope parameter  $q$  rotates the exit ray by a small angle  $\phi(q)$  about the  $+Y_{rd}$  axis, where  $\phi$  is a smooth function of  $q$ . In this case the perturbation rotates the exit ray from the direction of  $\hat{r}_d$  to the direction of the vector

$$(4.6) \quad \vec{\rho} = \hat{r}_d + \Delta \vec{\rho} = \hat{r}_d + (\phi \cdot \hat{Y}_{rd}) \times \hat{Z}_{rd} = \hat{Z}_{rd} + (\phi \cdot \hat{X}_{rd}) .$$

The vector increment  $\Delta \vec{\rho}$  is always horizontal since  $\hat{X}_{rd}$  has no component in zenith; the exit beam is thus rotated in azimuth, but not in elevation. The exit beam's azimuth shift,  $(\Delta AZ(q))_{exit\_ray} = \Delta AZ$  is found as follows. When  $\hat{r}_d$  is rotated in azimuth from  $AZ$  to  $AZ + \Delta AZ$  its increment,  $\Delta \vec{\rho}$ , can be calculated using the last equation of (4.1). We have,

$$(4.7) \quad \phi \cdot \hat{X}_{rd} = (\cos EL \cdot \cos AZ \cdot \Delta AZ) \cdot \hat{E} + (-\cos EL \cdot \sin AZ \cdot \Delta AZ) \cdot \hat{N} = (\cos EL) \cdot (\Delta AZ) \cdot \hat{X}_{rd} .$$

This gives, by arguments similar to those presented above,

$$(4.8) \quad (\cos EL) \cdot AZ_{corr}(q) = -(\cos EL) \cdot AZ_{err}(q) = -(\cos EL) \cdot (\Delta AZ(q))_{exit\_ray} = -\phi(q) ,$$

$$(4.5) \quad (\Delta AZ)_{exit\_ray} = \left(\frac{d\phi}{dq}\right) \cdot \Delta q .$$

We see that the exit ray deviation angle rate  $\left(\frac{d\phi}{dq}\right)$  provides a differential azimuth pointing error rate for the case of exit ray rotations about the  $+Y_{rd}$  axis.

## 6 Summary

The ray trace computations carried out using the BEAM 3 codes agree well with the 1991 results of Srikanth. Agreement is typically within 3%, at worst 8%. The differences may depend on the precise values of telescope parameters used in the 1991 and 1999 computations, and the precise location of the axis about which the subreflector rotates. In all cases, the signs of the coefficients were in agreement. The results confirm those of Srikanth, rather than those of Levy, where they disagree.

The coefficients also directly give error pointing coefficient rates. Some care should be taken in interpretation of the coefficients'signs to be used in applying the results. Here, the viewpoint taken is that when an optical component is moved from its design telescope configuration, the exit ray (which can also be considered as the reverse of incoming sky ray) is shifted to a wrong position on the sky. The shift rates in elevation angle, and azimuth angle multiplied by cosine of elevation angle have the same signs as the respective coefficients computed by Srikanth. If one wishes correct the as-built telescope in pointing by making use of statically measured optical component shifts from their design values, or measured under dynamical observing conditions, one should examine precisely how the available information on component shifts is to be used. The physical significance of the ray shifts should be examined, before entering calculated component error pointing correction terms into the telescope pointing equations.



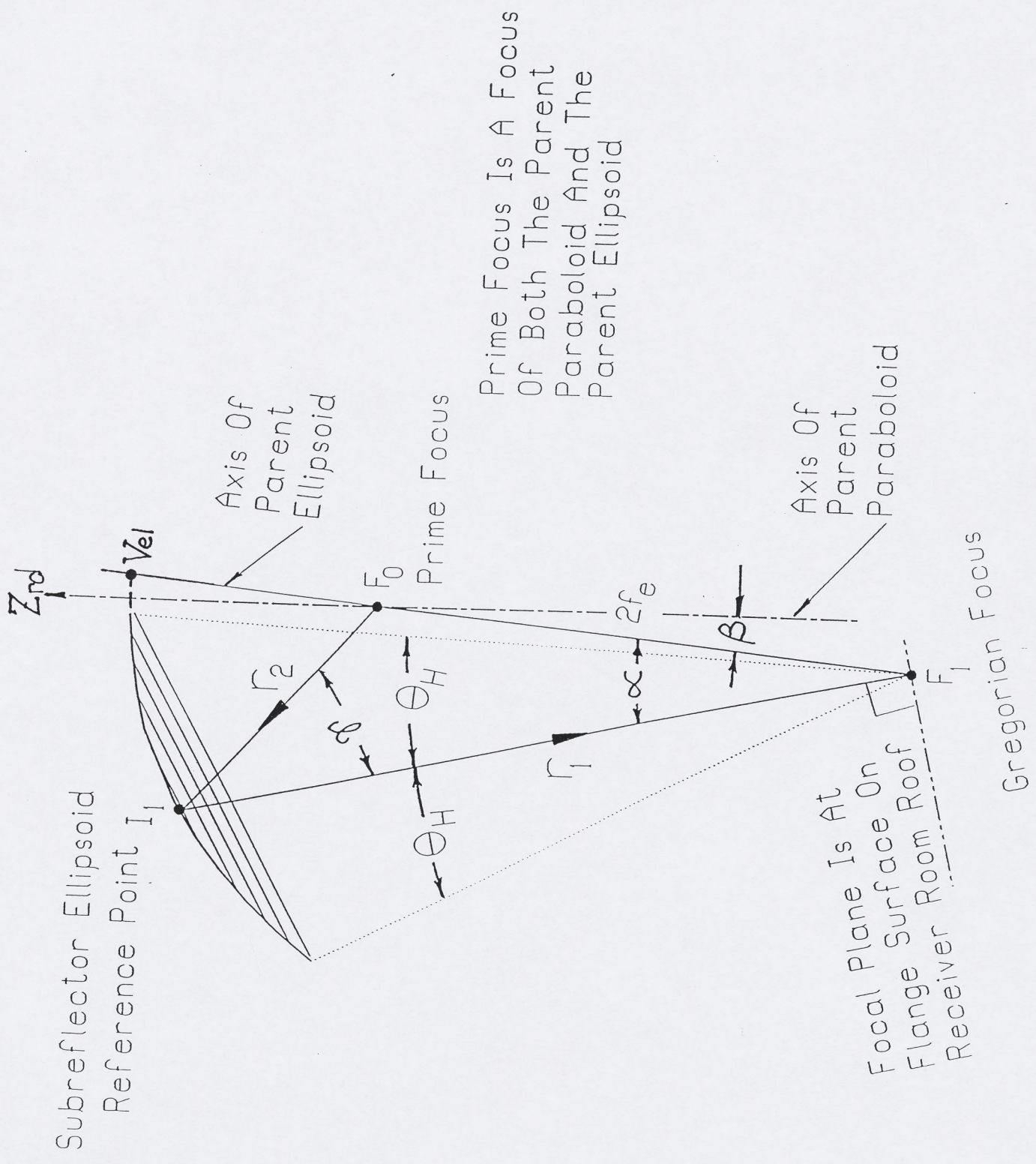
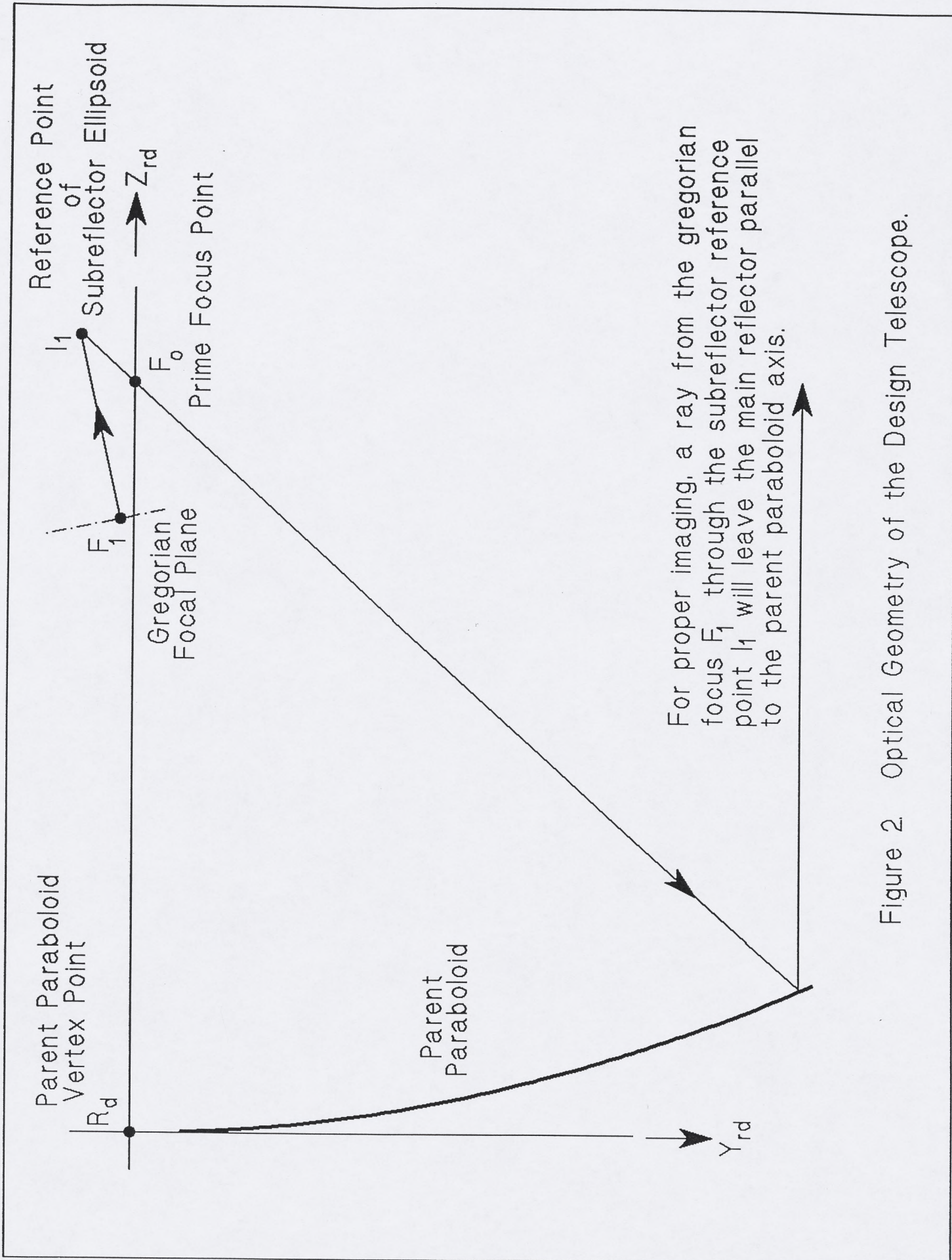


Figure 1. The Subreflector Ellipsoid Reference Geometry



For proper imaging, a ray from the Gregorian focus  $F_1$  through the subreflector reference point  $I_1$  will leave the main reflector parallel to the parent paraboloid axis.

Figure 2 Optical Geometry of the Design Telescope.

4 surfaces				GBT.OPT	M.A. Goldman	November 28, 1999		
Zvertex	Yvertex	Xvertex	Shape	Curvature	Mir/Lens	Tilt	Pitch	
49.051938	-1.067679	0.000000	0.000000	0.000000000	iris	12.329	0.0	
64.893452	0.477220	0.000000	0.721216	-0.13310852	mirror	-5.57	0.0	
0.0000000	0.0000000	0.000000	0.000000	0.0083333333	mirror	0.0	0.0	
70.0	0.0	0.0	:	:	iris	:	:	

This ray file is a model of the GBT optics. Distances are in meters. This file models the unperturbed GBT optical telescope geometry. Refractive indexes default to 1.000. The ray tracing is done using the BEAM 3 ray tracing program (Stellar Software, Berkeley, CA).

Surface 1 is the Gregorian focal plane through the Gregorian focal point. The listed vertex point is at the design location of the Gregorian focus point.

Surface 2 is the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$  where  $e$  is its eccentricity ( $= 0.528$ ). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 3 is the parent paraboloid primary mirror. Its vertex point curvature is  $1/(2f)$ , where  $f=60$  meter.

The file will be used in the following manner. In an associated ray file: GBT.RAY, a single ray will be traced from the Gregorian focal point, at an angle 12.329 degrees to the paraboloid axis and in the median plane, towards the ellipsoidal secondary reference point I1. For the unperturbed telescope geometry, it should emerge parallel to the paraboloid axis.

In subsequent optics files: GBT\_n.OPT the coordinates and orientations of the mirrors will be shifted, one at a time, and the direction of the exit ray will be computed. In this manner the partial derivatives of the pointing angles on the sky with telescope length and angle parameters will be obtained.

In those .OPT files for which the secondary mirror is tilted and/or pitched in orientation, the mirror vertex point location will be recomputed to leave the mirror reference point I1 at a fixed location [Zrd(I1)= 63.802874, Yrd(I1)=-4.2917258, Xrd(I1)= 0.0 meter]. The subscript rd indicates that coordinates are in the main reflector coordinate system and are derived values from the telescope design configuration. The BEAM 3 ray trace program X,Y,Z coordinates are the same as the corresponding main reflector system coordinates.

```

1 rays   GBT.RAY   November 28, 1999
  X0      Y0        Z0        U0        V0        W0        Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0   :-0.2135248:0.9769376:
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

This ray file is used with the BEAM 3 optics files GBT.OPT and GBT\_n.OPT to examine GBT pointing when the optical geometry of the telescope is perturbed.

Output ray trace file : GBT.VXL  
Page 1 of 1.

Input Files: GBT.OPT , GBT.RAY  
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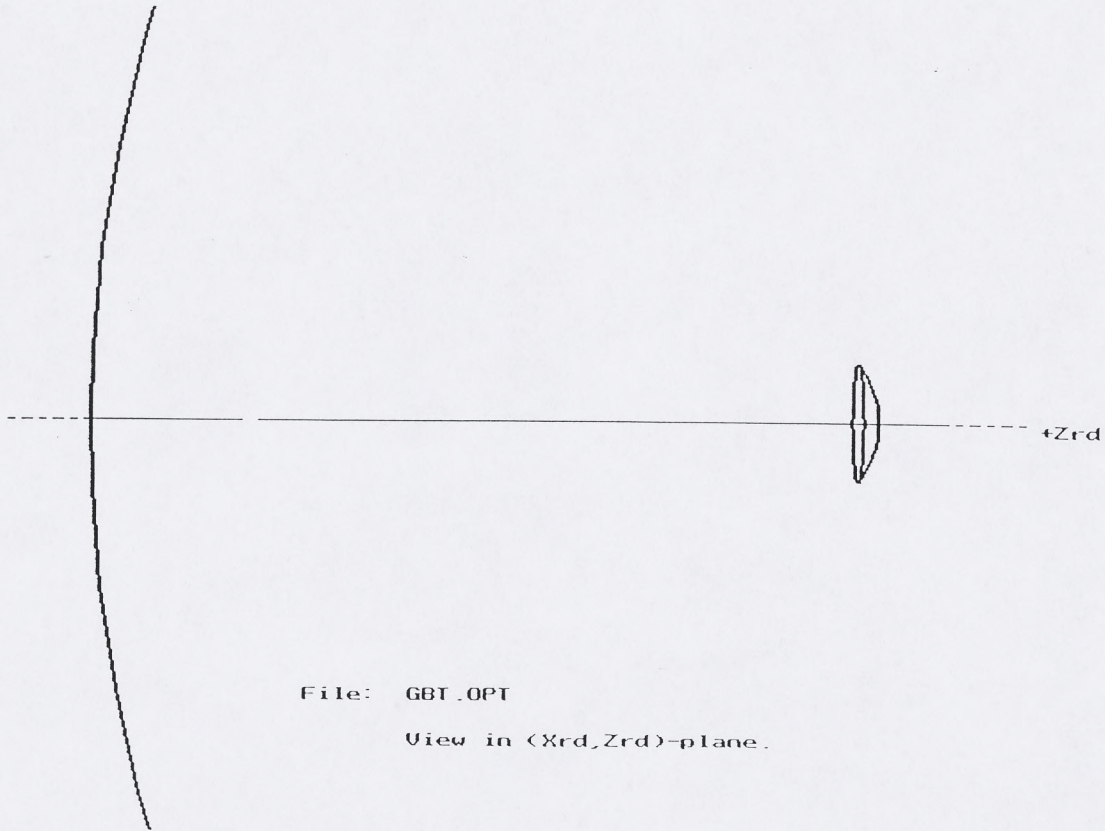
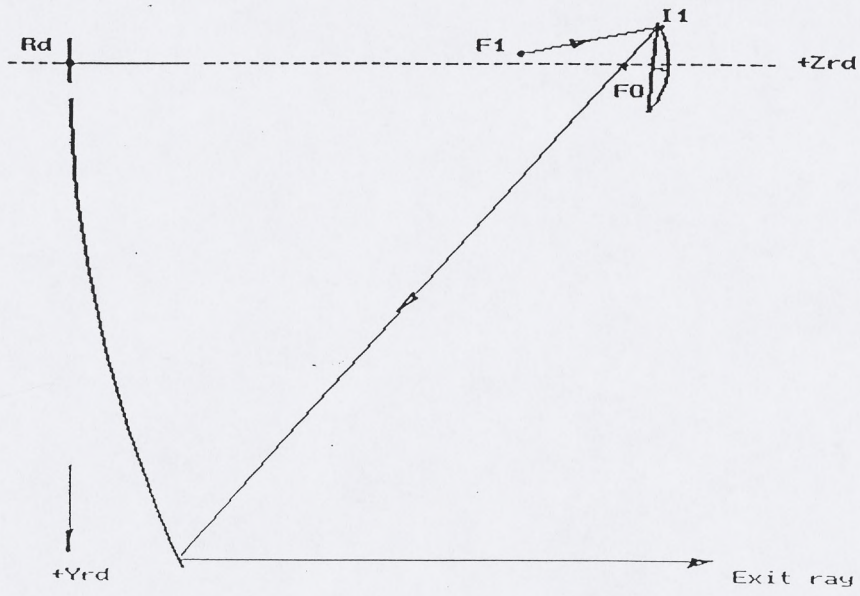
Ray1

	X	Y	Z	U	V	W
S0	0.000000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S1	0.000000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S2	0.000000	-4.291724	63.802875	0.000000	0.748447	-0.663195
S3	0.000000	54.000637	12.150287	0.000000	0.000000	1.000000
S4	0.000000	54.000638	70.000000	0.000000	0.000000	1.000000
optical path =			151.06131029			

Comment:

This ray trace confirms that the ray traced for the design geometry starts at the Gregorian focus F0, passes through the subreflector reference point I1, then passes through the prime focus F0, and after reflection by the design parent paraboloid leaves parallel to the paraboloid axis.

File: GBT.OPT  
View in (Yrd,Zrd)-plane.



File: GBT.OPT  
View in (Xrd,Zrd)-plane.

3 surfaces			GBT02.OPT	M.A. Goldman	November 28, 1999		
Zvertex	Yvertex	Xvertex	Shape	Curvature	Mir/Lens	Tilt	Pitch
64.893452	0.477220	0.000000	0.721216	-0.13310852	mirror	-5.57	0.0
0.0000000	0.000000	0.000000	0.000000	0.0083333333	mirror	0.0	0.0
70.0	0.0	0.0	:	:	iris	:	:

This ray file is a model of the GBT optics. Distances are in meters. This file models the unperturbed GBT optical telescope geometry. Refractive indexes default to 1.000. The ray tracing is done using the BEAM 3 ray tracing program (Stellar Software, Berkeley, CA).

Surface 1 is the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$  where  $e$  is its eccentricity ( $= 0.528$ ). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 2 is the parent paraboloid primary mirror. Its vertex point curvature is  $1/(2f)$ , where  $f=60$  meter.

Surface 3 is a phantom, entered to continue the ray trace output to include the exit rays emerging from surface 2.

The file will be used in the following manner. In an associated ray file: GBT02.RAY, a ray will be traced from the Gregorian focal point, at an angle 12.329 degrees to the paraboloid axis and in the median plane, towards the ellipsoidal secondary reference point I1. For the unperturbed telescope geometry, it should emerge parallel to the paraboloid axis.

Six additional rays parallel to the first ray are also traced. These rays start parallel to the first ray and are displaced by 10 mm distance in the directions of the plus and minus main reflector frame axes. The final exit rays emerge along the respective directions that an axial ray from the feed horn would take, had the receiver room been displaced by 10mm along the respective (+/-) directions of the main reflector coordinate axes.

Additional files GBT\_n.OPT and GBT\_n.RAY will be subsequently generated, to examine ray pointing when the telescope optical elements are perturbed in various ways.

In some later .OPT files for which the secondary mirror is tilted and/or pitched in orientation, the mirror vertex point location will be recomputed to leave the mirror reference point I1 at a fixed location [Zrd(I1)= 63.802874, Yrd(I1)=-4.2917258, Xrd(I1)= 0.0 meter].

The subscript rd indicates that coordinates are in the main reflector frame coordinate system and are derived values from the telescope design configuration. The BEAM 3 ray trace program X,Y,Z coordinates correspond respectively to the Xrd,Yrd,Zrd main reflector system coordinates.

```

7 rays      GBT02.RAY      November 28, 1999
X0          Y0            Z0            U0            V0            W0            Ufinal        Vfinal
-----:-----:-----:-----:-----:-----:-----:-----:-----
0.0 :-1.067679:49.051938:0.0 :-0.2135248:0.9769376:
0.010:-1.067679:49.051938:0.0 :-0.2135248:0.9769376:
-0.010:-1.067679:49.051938:0.0 :-0.2135248:0.9769376:
0.0 :-1.057679:49.051938:0.0 :-0.2135248:0.9769376:
0.0 :-1.077679:49.051938:0.0 :-0.2135248:0.9769376:
0.0 :-1.067679:49.061938:0.0 :-0.2135248:0.9769376:
0.0 :-1.067679:49.041938:0.0 :-0.2135248:0.9769376:
: : : : : : : :

```

This file describes exit rays starting at and near the Gregorian focus point of the GBT. The rays all are directed perpendicular to the Gregorian focus plane. Ray 1 leaves the Gregorian focal point and travels towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray leaving the n'th optical surface. These are the direction cosines to the Xrd,Yrd,Zrd axes respectively (and X,Y,Z axes respectively for the ray tracing code) which are the main reflector frame coordinates for the design GBT telescope.

The other rays are initially parallel to the ray leaving from the Gregorian focal point F1, but pass through points whose Xrd, Yrd or Zrd coordinates are displaced by (+/-)10 mm. These simulate rays which would leave a receiver room roof feed horn at Gregorian focus if the feed room were displaced by (+/-)10 mm in the Xrd, Yrd or Zrd directions respectively.

The direction of the exit ray (specified by Ufinal,Vfinal,Wfinal) leaving the final optical surface is opposite to that of a ray from the sky arriving at the telescope and which would travel to the center of feed horn center, and arrive along the horn axis, when the receiver room was mounted on the telescope displaced by (+/-) 10 mm in the Xrd, Yrd, or Zrd direction respectively.

This ray file is used with the BEAM 3 optics file GBT.OPT, to examine GBT pointing when the optical geometry of the telescope is perturbed.

- Ray1 simulates ray path with receiver room in design position.
- Ray2 simulates ray path with receiver room displaced by dXr=+10mm.
- Ray3 simulates ray path with receiver room displaced by dXr=-10mm.
- Ray4 simulates ray path with receiver room displaced by dYr=+10mm.
- Ray5 simulates ray path with receiver room displaced by dYr=-10mm.
- Ray6 simulates ray path with receiver room displaced by dZr=+10mm.
- Ray7 simulates ray path with receiver room displaced by dZr=-10mm.

Displacement of the receiver room by dXr=+1mm moves the receiver room away from the telescope median plane, towards the man lift.

Displacement of the receiver room by dYr=+1mm moves the receiver room in the median plane, away from the main reflector surface.

Displacement of the receiver room by dZr=+1mm moves the room along the design paraboloid axis, away from the main reflector surface.

Output ray trace file: GBT02.VXL  
Page 1 of 2.

Input Files: GBT02.OPT , GBT02.RAY  
November 28, 1999

Ray1

	X	Y	Z	U	V	W
S0	0.000000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S1	0.000000	-4.291724	63.802875	0.000000	0.748447	-0.663195
S2	0.000000	54.000637	12.150287	0.000000	0.000000	1.000000
S3	0.000000	54.000638	70.000000	0.000000	0.000000	1.000000
	optical path =		151.06131029			

Ray2

	X	Y	Z	U	V	W
S0	0.010000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S1	0.010000	-4.291723	63.802869	-0.002406	0.748445	-0.663192
S2	-0.177407	54.000629	12.150414	0.000053	0.000000	1.000000
S3	-0.174362	54.000630	70.000000	0.000053	0.000000	1.000000
	optical path =		151.06130570			

Ray3

	X	Y	Z	U	V	W
S0	-0.010000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S1	-0.010000	-4.291723	63.802869	0.002406	0.748445	-0.663192
S2	0.177407	54.000629	12.150414	-0.000053	0.000000	1.000000
S3	0.174362	54.000630	70.000000	-0.000053	0.000000	1.000000
	optical path =		151.06130570			

Ray4

	X	Y	Z	U	V	W
S0	0.000000	-1.057679	49.051938	0.000000	-0.213525	0.976938
S1	0.000000	-4.282859	63.808068	0.000000	0.746885	-0.664953
S2	0.000000	53.827402	12.072455	0.000000	0.000051	1.000000
S3	0.000000	53.830383	70.000000	0.000000	0.000051	1.000000
	optical path =		151.06130591			

Ray5

	X	Y	Z	U	V	W
S0	0.000000	-1.077679	49.051938	0.000000	-0.213525	0.976938
S1	0.000000	-4.300587	63.797670	0.000000	0.750003	-0.661434
S2	0.000000	54.174034	12.228441	0.000000	-0.000051	1.000000
S3	0.000000	54.171065	70.000000	0.000000	-0.000051	1.000000
	optical path =		151.06130591			



Output ray trace file: GBT02.VXL  
Page 2 of 2.

Input Files: GBT02.OPT , GBT02.RAY  
November 28, 1999

Ray6

	X	Y	Z	U	V	W
S0	0.000000	-1.067679	49.061938	0.000000	-0.213525	0.976938
S1	0.000000	-4.289787	63.804011	0.000000	0.748106	-0.663580
S2	0.000000	53.962760	12.133248	0.000000	0.000011	1.000000
S3	0.000000	53.963411	70.000000	0.000000	0.000011	1.000000

optical path = 151.05154070

Ray7

	X	Y	Z	U	V	W
S0	0.000000	-1.067679	49.041938	0.000000	-0.213525	0.976938
S1	0.000000	-4.293661	63.801739	0.000000	0.748787	-0.662810
S2	0.000000	54.038522	12.167341	0.000000	-0.000011	1.000000
S3	0.000000	54.037873	70.000000	0.000000	-0.000011	1.000000

optical path = 151.07107946

Comment:

These exit ray rays correspond to the following tilt rates about the +Yrd and +Xrd axes respectively:

$$d(\text{Tilt\_Yrd})/d(\text{X\_feed}) = + 0.0053 \text{ milliradians / millimeter}$$

$$d(\text{Tilt\_Xrd})/d(\text{Y\_feed}) = - 0.0051 \text{ milliradians / millimeter}$$

$$d(\text{Tilt\_Xrd})/d(\text{Z\_feed}) = - 0.0011 \text{ milliradians / millimeter .}$$

3 surfaces			GBT03.OPT	M.A. Goldman	November 28, 1999		
Zvertex	Yvertex	Xvertex	Shape	Curvature	Mir/Lens	Tilt	Pitch
64.893452	0.477220	0.000000	0.721216	-0.13310852	mirror	-5.57	0.0
0.0000000	0.000000	0.000000	0.000000	0.0083333333	mirror	0.0572957	0.0
70.0	0.0	0.0	:	:	iris	:	:

This ray file is a model of the GBT optics. Distances are in meters. This file models a perturbed GBT optical telescope geometry. The paraboloid has been rotated about the +Xrd axis by 1 milliradian = 0.0572957 degrees. The paraboloid vertex, prime focus, Gregorian focus, and subreflector reference point lie at their design configuration coordinates.

Surface 1 models the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$ , where  $e$  is its eccentricity ( $= 0.528$ ). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 2 models the paraboloid primary mirror. Its vertex lies at the design position relative to the feed room and subreflector but it is tilted towards the feed room and subreflector.

Surface 3 is a phantom, entered to continue the ray trace output to include the exit rays emerging from surface 2.

The file will be used in the following manner. In an associated ray file: GBT.RAY, a ray will be traced from the Gregorian focal point, at an angle 12.329 degrees to the design paraboloid's axis (+Xrd) and in the telescope median plane, towards the ellipsoidal secondary reference point I1. The axis of the paraboloid for this ray trace (which is no longer the design parent paraboloid) is tilted by one milliradian about the +Xrd axis (The main reflector is tilted towards the feed room and subreflector). Its focal length has been held to the design value of 60 meters.

The pointing of the emergent ray from the telescope is indicated by the direction cosines  $U_f$ ,  $V_f$ ,  $W_f$ .

```

1 rays   GBT.RAY   November 28, 1999
  X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0   :-0.2135248:0.9769376:   0.00000:   -0.001552
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

This ray file is used with the BEAM 3 optics files GBT.OPT and GBT\_n.OPT to examine GBT pointing when the optical geometry of the telescope is perturbed.

```

Output ray trace file:  GBT03.VXL      Input Files:  GBT03.OPT , GBT.RAY
Page 1 of 1.           November 28, 1999.

```

```

Ray1
  X      Y      Z      U      V      W
S0  0.000000  -1.067679  49.051938  0.000000  -0.213525  0.976938
S1  0.000000  -4.291724  63.802875  0.000000   0.748447 -0.663195
S2  0.000000  53.956149  12.189707  0.000000 -0.001552  0.999999
S3  0.000000  53.866433  70.000000  0.000000 -0.001552  0.999999
      optical path = 150.96251993

```

Comment:  
 The exit ray direction cosines indicate that a +1 milliradian tilt of the paraboloid about the +Xrd axis gives a rotation of the exit ray by 1.552 milliradian in the same sense.

3 surfaces			GBT04.OPT	M.A. Goldman	November 28, 1999		
Zvertex	Yvertex	Xvertex	Shape	Curvature	Mir/Lens	Tilt	Pitch
64.893452	0.477220	0.000000	0.721216	-0.13310852	mirror	-5.57	0.0
0.0000000	0.0000000	0.0000000	0.0000000	0.0083333333	mirror	0.0	0.0572957
70.0	0.0	0.0	:	:	iris	:	:

This ray file is a model of the GBT optics. Distances are in meters. This file models a perturbed GBT optical telescope geometry. The paraboloid has been rotated about the +Yrd axis by 1 milliradian = 0.0572957 degrees. The paraboloid vertex, prime focus, Gregorian focus, and subreflector reference point lie at their design configuration coordinates.

Surface 1 models the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$ , where  $e$  is its eccentricity (= 0.528). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 2 models the paraboloid primary mirror. Its vertex lies at the design position relative to the feed room and subreflector but it is pitched by 1 milliradian relative to the median plane of the telescope.

Surface 3 is a phantom, entered to continue the ray trace output to include the exit rays emerging from surface 2.

The file will be used in the following manner. In an associated ray file: GBT.RAY, a ray will be traced from the Gregorian focal point, at an angle 12.329 degrees to the design paraboloid's axis (+Xrd) and in the telescope median plane, towards the ellipsoidal secondary reference point I1. The axis of the paraboloid for this ray trace (which is no longer the design parent paraboloid) is pitched by one milliradian about the +Yrd axis (The main reflector right half is lowered in elevation and the left half is raised by the pitching rotation). The main reflector paraboloid's focal length is held to the design value of 60 meters.

The pointing of the emergent ray from the telescope is indicated by the direction cosines  $U_f$ ,  $V_f$ ,  $W_f$ .

1 rays GBT.RAY November 28, 1999

X0	Y0	Z0	U0	V0	W0	Ufinal	Vfinal
0.0	-1.067679	49.051938	0.0	-0.2135248	0.9769376	0.00183	0.000000
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT04.VXL  
Page 1 of 1.

Input Files: GBT04.OPT , GBT.RAY  
November 28, 1999

Ray 1

	X	Y	Z	U	V	W
S0	0.000000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S1	0.000000	-4.291724	63.802875	0.000000	0.748447	-0.663195
S2	-0.000000	54.000632	12.150291	0.001832	0.000000	0.999998
S3	0.105957	54.000654	70.000000	0.001832	0.000000	0.999998

optical path = 151.06139620

Comment:

The exit ray direction cosines indicate that a +1 milliradian pitch of the paraboloid about the +Yrd axis gives a rotation of the exit ray about that axis by 1.832 milliradian in the same sense.

```

3 surfaces      GBT05.OPT      M.A. Goldman      November 28, 1999
Zvertex      Yvertex      Xvertex Shape      Curvature      Mir/Lens      Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----
64.893452: 0.477220:0.00000:0.721216:-0.13310852 : mirror :-5.57 :0.0
0.0000000: 0.000000:0.00000:0.000000:0.0083319447: mirror : 0.0 :0.0
70.0      : 0.0      :0.0      :      :      : iris      :      :

```

This ray file is a model of the GBT optics. Distances are in meters . This file models a perturbed GBT optical telescope geometry. The paraboloid focal length has been increased by +10 millimeters. The paraboloid vertex, prime focus, Gregorian focus, and subreflector reference point lie at their design configuration coordinates.

Surface 1 models the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$ , where  $e$  is its eccentricity ( $= 0.528$ ). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 2 models the paraboloid primary mirror. Its vertex lies at the design and orientation position relative to the feed room and subreflector. Its focal length has been increased from 60 meter to 60.01 meter. The vertex curvature,  $(1/2*f)$ , changes to 0.0083319447 reciprocal meter.

Surface 3 is a phantom, entered to continue the ray trace output to include the exit rays emerging from surface 2.

The file will be used in the following manner. In an associated ray file: GBT.RAY, a ray will be traced from the Gregorian focal point, at an angle 12.329 degrees to the design paraboloid's axis (+Xrd) and in the telescope median plane, towards the ellipsoidal secondary reference point I1. The main reflector paraboloid's focal length is increased from the design value of 60 meters to 60.01 meters.

The pointing of the emergent ray from the telescope is indicated by the direction cosines  $U_f$ ,  $V_f$ ,  $W_f$ .

```

1 rays   GBT.RAY   November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal   Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0   :-0.2135248:0.9769376:   0.00000:   0.000104
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point II. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT05.VXL  
Page 1 Of 1.

Input Files: GBT05.OPT , GBT.RAY  
November 28, 1999

```

Ray1
   X           Y           Z           U           V           W
S0  0.000000   -1.067679   49.051938   0.000000   -0.213525   0.976938
S1  0.000000   -4.291724   63.802875   0.000000   0.748447   -0.663195
S2  0.000000   54.002152   12.148944   0.000000   0.000104   1.000000
S3  0.000000   54.008153   70.000000   0.000000   0.000104   1.000000
      optical path =      151.06467812

```

Comment:

An increase of paraboloid focal length from 60 meter to 60.010 meter produces a tilt of the exit ray of -0.104 milliradian about the +Xrd axis. (A unit vector in the (Y,Z)-plane which is nearly along the Z axis and has a positive projection (V > 0) on the +Y axis will appear to be rotated by an angle -V radians about the X axis, in the small angle approximation.

The tilt rate of the exit ray per unit increase of paraboloid focal length is then -0.104 milliradian / 10 millimeter, or  $d(\text{Tilt\_Xrd})/df = -0.0104$  milliradian / millimeter.

3 surfaces			GBT06.OPT	M.A. Goldman	November 28, 1999		
Zvertex	Yvertex	Xvertex	Shape	Curvature	Mir/Lens	Tilt	Pitch
64.898221	0.476129	0.000000	0.721216	-0.13310852	mirror	-5.5127043	0.0
0.000000	0.000000	0.000000	0.000000	0.0083333333	mirror	0.0	0.0
70.0	0.0	0.0	:	:	iris	:	:

This ray file is a model of the GBT optics. Distances are in meters. This file models a perturbed GBT optical telescope geometry. Refractive indexes default to 1.000. The ray tracing is done using the BEAM 3 ray tracing program (Stellar Software, Berkeley, CA).

Surface 1 is the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$  where  $e$  is its eccentricity ( $= 0.528$ ). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 2 is the parent paraboloid primary mirror. Its vertex point curvature is  $1/(2f)$ , where  $f=60$  meter.

Surface 3 is a phantom, entered to continue the ray trace output to include the exit rays emerging from surface 2.

The file will be used in the following manner. In an associated ray file: GBT.RAY, a ray will be traced from the design Gregorian focal point, at an angle 12.329 degrees to the paraboloid axis and in the median plane, towards the ellipsoidal secondary reference point I1. (For the unperturbed telescope geometry, it emerges parallel to the parent paraboloid axis (+Zrd axis)).

In this file for the secondary mirror is tilted by +1 milliradians about a line through the subreflector reference point and parallel to the +Xrd axis (This axis is parallel to the elevation axis and passes through the design ellipsoid reference point and points to the right side of the paraboloid). The ellipsoid secondary mirror's upper vertex point location (on the ellipsoid major axis) has been recomputed so as to leave the mirror reference point I1 at a fixed location. [Zrd(I1)= 63.802874, Yrd(I1)=-4.2917258, Xrd(I1)= 0.0 meter].

The BEAM 3 parameter TILT becomes, for the additionally tilted ellipsoidal secondary surface:  
 $TILT = -5.570 \text{ degrees} + 1*(0.0572957)\text{degrees} = -5.5127043 \text{ degrees}.$

The vertex of the additionally tilted subreflector surface is calculated to move to V'el where:  $Xrd(V'el)=0$ ,  $Yrd(V'el)= -0.4761294 \text{ m}$ ,  $Zrd(V'el)=64.898221 \text{ m}.$



```

1 rays   GBT.RAY   November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal   Vfinal
-----:-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0      :-0.2135248:0.9769376:  0.00000:  -0.000159
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

```

Output ray trace file:  GBT06.VXL      Input Files:  GBT06.OPT , GBT.RAY
Page 1 of 1.           November 28, 1999

```

```

Ray1
   X           Y           Z           U           V           W
S0  0.000000   -1.067679   49.051938   0.000000   -0.213525   0.976938
S1  0.000000   -4.291724   63.802875   0.000000   0.749772   -0.661696
S2  0.000000   54.156508   12.220531   0.000000   -0.000159   1.000000
S3  0.000000   54.147332   70.000000   0.000000   -0.000159   1.000000
      optical path =      151.06129775

```

Comment:

The rate of tilt of the exit ray about the +Xrd axis per unit of subreflector tilt about the axis X' parallel to the +Xrd axis and passing through the fixed subreflector reference point I1 is:

$$d(\text{Tilt\_Xrd})/d(\text{Tilt\_X'}) = 0.159 \text{ milliradians / milliradian}$$

```

3 surfaces      GBT07.OPT      M.A. Goldman      November 28, 1999
Zvertex  Yvertex  Xvertex  Shape      Curvature      Mir/Lens  Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----:-----
64.893452: 0.477220:0.00000:0.721216 :-0.13310852 : mirror :-5.57 :0.0
0.0100000: 0.000000:0.00000:0.000000 :0.00833333333: mirror : 0.0 :0.0
70.0      : 0.0      :0.0      :      :      : iris   :      :

```

This file models the GBT optics with the main reflector surface paraboloid moved +10mm along the direction of the paraboloid axis (+Zrd axis) towards the receiver room and subreflector.

```

1 rays      GBT.RAY      November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0      :-0.2135248:0.9769376: 0.00000: -0.000159
:      :      :      :      :      :      :
:      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT07.VXL      Input Files: GBT07.OPT , GBT.RAY  
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```

Ray1
      X      Y      Z      U      V      W
S0  0.000000  -1.067679  49.051938  0.000000  -0.213525  0.976938
S1  0.000000  -4.291724  63.802875  0.000000  0.748447  -0.663195
S2  0.000000  53.993152  12.156919  0.000000  0.000104  1.000000
S3  0.000000  53.999154  70.000000  0.000000  0.000104  1.000000
      optical path = 151.04467826

```

Comment: The tilt rate for the exit ray about the +Xrd axis per unit translation of the main reflector paraboloid in the +Zrd direction is:

$$d(\text{Tilt}_{\text{Xrd}})/dZ = -0.0104 \text{ milliradian / millimeter.}$$

```

3 surfaces      GBT08.OPT      M.A. Goldman      November 28, 1999
Zvertex  Yvertex  Xvertex Shape      Curvature      Mir/Lens      Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----:-----
64.893452: 0.477220:0.00000:0.721216 :-0.13310852 : mirror :-5.57 :0.0
0.0000000: 0.010000:0.00000:0.000000 :0.00833333333: mirror : 0.0 :0.0
70.0      : 0.0      :0.0      :      :      : iris      :      :

```

This file models the GBT optics with the main reflector surface paraboloid translated +10mm away from the upper feed arm (that is, moved in the direction of the +Yrd axis).

```

1 rays      GBT.RAY      November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0      :-0.2135248:0.9769376: 0.00000: -0.000159
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT08.VXL      Input Files: GBT08.OPT , GBT.RAY  
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```

Ray1
      X      Y      Z      U      V      W
S0  0.000000  -1.067679  49.051938  0.000000  -0.213525  0.976938
S1  0.000000  -4.291724  63.802875  0.000000  0.748447  -0.663195
S2  0.000000  54.004005  12.147302  0.000000  0.000092  1.000000
S3  0.000000  54.009324  70.000000  0.000000  0.000092  1.000000
      optical path = 151.06879470

```

Comment: The tilt rate for the exit ray about the +Xrd axis per unit translation of the main reflector paraboloid in the +Yrd direction is:

$$d(\text{Tilt\_Xrd})/dY = -0.0092 \text{ milliradian / millimeter.}$$

```

3 surfaces      GBT09.OPT      M.A. Goldman      November 28, 1999
Zvertex      Yvertex      Xvertex Shape      Curvature      Mir/Lens      Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----:-----
64.893452: 0.477220:0.00000:0.721216 :-0.13310852 : mirror :-5.57 :0.0
0.0000000: 0.000000:0.01000:0.000000 :0.0083333333 : mirror : 0.0 :0.0
70.0      : 0.0      :0.0      :      :      : iris      :      :

```

This file models the GBT optics with the main reflector surface paraboloid translated +10mm to the right of the design telescope median plane (that is, translated towards the man lift, along the direction of the +Xrd axis).

```

1 rays      GBT.RAY      November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0      :-0.2135248:0.9769376: 0.00000: -0.000159
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT09.VXL      Input Files: GBT09.OPT , GBT.RAY  
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```

Ray1
X      Y      Z      U      V      W
S0 0.000000 -1.067679 49.051938 0.000000 -0.213525 0.976938
S1 0.000000 -4.291724 63.802875 0.000000 0.748447 -0.663195
S2 0.000000 54.000637 12.150287 0.000139 0.000000 1.000000
S3 0.008018 54.000638 70.000000 0.000139 0.000000 1.000000
optical path = 151.06131015

```

Comment: The tilt rate for the exit ray about the +Yrd axis per unit translation of the main reflector paraboloid in the +Xrd direction is:

$d(\text{Tilt\_Yrd})/dX = +0.0139$  milliradian / millimeter.

```

3 surfaces      GBT10.OPT      M.A. Goldman      November 28, 1999
Zvertex      Yvertex      Xvertex      Shape      Curvature      Mir/Lens      Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----:-----
64.903452: 0.477220:0.00000:0.721216 :-0.13310852 : mirror :-5.57 :0.0
0.0000000: 0.0000000:0.000000:0.000000 :0.0083333333: mirror : 0.0 :0.0
70.0 : 0.0 :0.0 : : : iris : :

```

This file models the GBT optics with the subreflector surface translated +10mm along the direction of the paraboloid axis (+Zrd axis), away from the receiver room and main reflector surface.

```

1 rays      GBT.RAY      November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0 :-0.2135248:0.9769376: 0.00000: -0.000159
: : : : : : :
: : : : : : :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT10.VXL      Input Files: GBT10.OPT , GBT.RAY  
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```

Ray1
X      Y      Z      U      V      W
S0  0.000000  -1.067679  49.051938  0.000000  -0.213525  0.976938
S1  0.000000  -4.293661  63.811739  0.000000  0.748787  -0.662810
S2  0.000000  54.046009  12.170713  0.000000  -0.000115  1.000000
S3  0.000000  54.039360  70.000000  0.000000  -0.000115  1.000000
optical path = 151.07770747

```

Comment: The tilt rate for the exit ray about the +Xrd axis per unit translation of the subreflector ellipsoid in the +Zrd direction is:

$$d(\text{Tilt\_Xrd})/dZ = +0.0115 \text{ milliradian / millimeter.}$$

```

3 surfaces      GBT11.OPT      M.A. Goldman      November 28, 1999
Zvertex      Yvertex      Xvertex Shape      Curvature      Mir/Lens      Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----
64.893452: 0.487220:0.00000:0.721216 :-0.13310852 : mirror :-5.57 :0.0
0.0000000: 0.000000:0.00000:0.000000 :0.00833333333: mirror : 0.0 :0.0
70.0      : 0.0      :0.0      :      :      : iris      :      :

```

This file models the GBT optics with the subreflector surface translated +10mm along the direction of the +Yrd axis (way from the vertical feed arm and towards the main reflector and elevation axle).

```

1 rays      GBT.RAY      November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0      :-0.2135248:0.9769376: 0.00000: -0.000159
:      :      :      :      :      :      :
:      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT11.VXL  
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Input Files: GBT11.OPT , GBT.RAY  
November 28, 1999

Ray1

	X	Y	Z	U	V	W
S0	0.000000	-1.067679	49.051938	0.000000	-0.213525	0.976938
S1	0.000000	-4.290587	63.797670	0.000000	0.750003	-0.661434
S2	0.000000	54.180648	12.231427	0.000000	-0.000143	1.000000
S3	0.000000	54.172389	70.000000	0.000000	-0.000143	1.000000
	optical path =		151.05167050			

Comment: The tilt rate for the exit ray about the +Xrd axis per unit translation of the subreflector ellipsoid in the +Yrd direction is:

$d(\text{Tilt}_{\text{Xrd}})/dY = +0.0143$  milliradian / millimeter.

```

3 surfaces      GBT12.OPT      M.A. Goldman      November 28, 1999
Zvertex      Yvertex      Xvertex Shape      Curvature      Mir/Lens      Tilt      Pitch
-----:-----:-----:-----:-----:-----:-----:-----:-----
64.893452: 0.477220:0.01000:0.721216 :-0.13310852 : mirror :-5.57 :0.0
0.0000000: 0.000000:0.00000:0.000000 :0.00833333333: mirror : 0.0 :0.0
70.0      : 0.0      :0.0      :      :      : iris : :

```

This file models the GBT optics with the subreflector surface translated +10mm from the telescope median plane to the right (translation parallel to the +Xrd axis, in the direction towards the man lift).

```

1 rays      GBT.RAY      November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal      Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0      :-0.2135248:0.9769376: 0.00000: -0.000159
:      :      :      :      :      :      :
:      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT12.VXL      Input Files: GBT12.OPT , GBT.RAY  
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```

Ray1
X      Y      Z      U      V      W
S0  0.000000  -1.067679  49.051938  0.000000  -0.213525  0.976938
S1  0.000000  -4.291723  63.802869  0.002406  0.748445  -0.663192
S2  0.187407  54.000617  12.150424  -0.000191  0.000000  1.000000
S3  0.176344  54.000629  70.000000  -0.000191  0.000000  1.000000
optical path = 151.06128140

```

Comment: The tilt rate for the exit ray about the +Yrd axis per unit translation of the subreflector ellipsoid in the +Xrd direction is:

$$d(\text{Tilt\_Yrd})/dX = -0.0191 \text{ milliradian / millimeter.}$$

3 surfaces			GBT13.OPT	M.A. Goldman	November 28, 1999		
Zvertex	Yvertex	Xvertex	Shape	Curvature	Mir/Lens	Tilt	Pitch
64.893452	0.477220	0.0010905	0.721216	-0.13310852	mirror	-5.57	0.0572957
0.0000000	0.0000000	0.0000000	0.000000	0.0083333333	mirror	0.0	0.0
70.0	0.0	0.0	0.0	0.0	iris	0.0	0.0
:	:	:	:	:	:	:	:

This file models a perturbed GBT optical telescope geometry where the subreflector is pitched about an axis which passes through the subreflector reference point and is directed parallel to the Yrd axis.

Surface 1 is the parent ellipsoid secondary mirror. Its shape parameter is  $(1 - e^2)$  where  $e$  is its eccentricity ( $= 0.528$ ). Its vertex curvature is  $-a/(b^2)$ . Its vertex point for the calculation is the upper vertex on the major axis.

Surface 2 is the parent paraboloid primary mirror. Its vertex point curvature is  $1/(2f)$ , where  $f=60$  meter.

Surface 3 is a phantom, entered to continue the ray trace output to include the exit rays emerging from surface 2.

This file is used in the following manner. In an associated ray file: GBT.RAY, a ray will be traced from the design Gregorian focal point, at an angle 12.329 degrees to the paraboloid axis and in the median plane, towards the ellipsoidal secondary reference point I1.

The secondary mirror is pitched by +1 milliradian about a line through the subreflector reference point and parallel to the +Yrd axis. The ellipsoid secondary mirror's vertex point location has been recomputed so as to leave the mirror reference point I1 at a fixed location.

[Zrd(I1)= 63.802874, Yrd(I1)=-4.2917258, Xrd(I1)= 0.0 meter].

For the subreflector mirror, the BEAM 3 parameter PITCH is:  
 PITCH = 0.0572957degrees ( $\approx$  1 milliradian).

The vertex of the pitched subreflector surface is calculated to move to V"el where: Xrd(V"el)= 0.0010905 m, Yrd(V"el)= 0.447220 m, Zrd(V"el)= 64.893452 m.



```

1 rays   GBT.RAY   November 28, 1999
X0      Y0      Z0      U0      V0      W0      Ufinal   Vfinal
-----:-----:-----:-----:-----:-----:-----:-----
0.0:-1.067679:49.051938:0.0   :-0.2135248:0.9769376:  0.00000:  -0.000159
:      :      :      :      :      :      :      :
:      :      :      :      :      :      :      :

```

This file describes an exit ray leaving the Gregorian focus point of the GBT, leaving perpendicular to the gregorian focus plane, and heading initially towards the ellipsoidal subreflector's design reference point I1. It leaves at an angle 12.3290 degrees to the Zrd-axis, and lies in the (Yrd,Zrd)-plane. Un,Vn,Wn are the direction cosines of the ray with respect to the Xrd,Yrd,Zrd axes (the X,Y,Z axes respectively for the ray tracing code).

Output ray trace file: GBT13.VXL  
Page 1 of 1.

Input Files: GBT13.OPT , GBT.RAY  
November 28, 1999

```

Ray1
   X           Y           Z           U           V           W
S0  0.000000   -1.067679   49.051938   0.000000   -0.213525   0.976938
S1  0.000000   -4.291724   63.802876  -0.001649   0.748446  -0.663193
S2  -0.128443   54.000664   12.150368   0.000131   0.000000   1.000000
S3  -0.120861   54.000667   70.000000   0.000131   0.000000   1.000000
      optical path =      151.06130283

```

Comment:

The rate of tilt of the exit ray about the +Yrd axis per unit of subreflector tilt about an axis parallel to the +Yrd axis and passing through the fixed subreflector reference point I1 is:

$$d(\text{Tilt\_Yrd})/d(\text{Tilt\_Y}) = +0.131 \text{ milliradians / milliradian}$$