# NATIONAL RADIO ASTRONONY OBSERVATORY Green Bank, WV 

From: Roger D. Norrod
Subj: Increase in Distance between Large and Small Subreflectors

We have been asked to increase the distance between the large (M1) and small (M2) subreflectors, to allow M2 to be pivoted into place without moving Ml. Figure 1 shows the current design. Table 1 gives numerical parameters for both Ml and M2.

The parameter $M_{d}$ (see Table 1) is given by:

$$
M_{d}=\frac{\overline{F_{1} I_{1}}}{\overline{F_{0} I_{1}}}
$$

so,

$$
\begin{aligned}
& \overline{\mathrm{F}_{0} \mathrm{I}_{1}}=\frac{15.099}{2.633}=5.735 \text { meters } \\
& \overline{\mathrm{F}_{0} \mathrm{I}_{2}}=\frac{13.143}{4.332}=3.034 \text { meters }
\end{aligned}
$$

and,

$$
\begin{aligned}
\overline{I_{1} I_{2}}=5.735-3.034 & =2.701 \text { meters } \\
& =106 \text { inches }
\end{aligned}
$$

Lee King requested we increase $\overline{I_{1} I_{2}}$ by at least 30 inches. This memo will discuss how this could be done and point out the repercussions.

The design equations for offset reflector geometry can be arranged in various forms. The equations $I$ use were taken from [1] and relate physical antenna parameters in a reasonable way.

The equation for the paraboloid in Figure 2 is given by:

$$
\begin{equation*}
x^{2}+y^{2}=-4 f(z-f) \tag{1}
\end{equation*}
$$

$R$ is usually chosen to achieve a desired aperture size, $f$ on the basis of electrical performance, and $Y_{c}$ at the minimum for no blockage. Once these three parameters are set, the coordinates of $I_{L}$ and $I_{H}$ can be determined from (1), and the angles $\theta_{0}$ and $\theta^{*}$ found.

It has been shown that the offset dual reflector geometry can be arranged such that circles of rays transmitted by the secondary feed are transformed by the dual reflectors to concentric circles on the paraboloid aperture, and that a rotationally symmetric feed pattern produces a rotationally symmetric aperture distribution. This also guarantees that a feed with no cross-pol gives rise to an aperture field with no cross-pol. Figure 3 shows the required geometry. The subreflector axis ( $F_{0} F_{1}$ ) is tilted at an angle $\beta$ with respect to the paraboloid axis ( $F_{0} V_{0}$ ), and $\beta$ can be found by solving

$$
\begin{equation*}
Y_{C}=4 f e \sin \beta /\left(1+e^{2}-2 e \cos \beta\right) \tag{2}
\end{equation*}
$$

where $e$ is the eccentricity of the subreflector. In order to illuminate the main reflector, the feed axis must be tilted by the angle $\alpha$ with respect to the ellipsoid axis. For the transformation yielding zero cross-pol to hold, $\alpha$ and $\beta$ must satisfy:

$$
\tan \alpha=\frac{\left(1-e^{2}\right) \sin \beta}{\left(1+e^{2}\right) \cos \beta-2 e}
$$

$\theta_{\mathrm{B}}$ is the cone angle from $\mathrm{F}_{1}$ defining the rays which illuminate the edge of the subreflector and then are reflected to the edge of the main reflector.

$$
\tan \left(\theta_{\mathrm{H}} / 2\right)=\mathrm{R} \frac{\left(1+\mathrm{e}^{2}-2 \mathrm{e} \cos \beta\right)}{2 \mathrm{f}\left(1-\mathrm{e}^{2}\right)}
$$

Note that $C$, the ellipsoid interfocal distance, does not enter into these equations. It serves as a scaling factor and can be used to vary the size of the subreflector, but does not change the angles $\alpha, \beta$, or $\theta_{\mathrm{H}}$. A program has been written in Pascal for the IBM-PC that will solve these equations. The program's input parameters are $e, Y_{C}, f, R$, and $C$. Table 1 was generated by this program, and it makes exploring options fairly painless.

Table 2 varies the system parameters to try to increase the distance $\overline{I_{1} I_{2}}$. Cases 1 and 2 are our current M1 and M2-1 designs. Let us assume that M1 is not changed because to move it further from $F_{0}$, it would get larger, which is undesirable economically. Cases 3 and 4 illustrate what happens when $e$ is varied to bring M2 closer to $F_{0}$. Case 4 achieves the 30 inch ( 0.76 meter) increase in $\overline{I_{1} I_{2}}$ desired. Case 3 is an intermediate step that achieves a 17 inch increase.

The subreflectors of cases 3 and 4 are smaller and the effective focal lengths longer, both desirable trends. The undesirable trend is the decrease in $\theta_{\mathrm{H}}$. The feed diameters change approximately as the inverse ratio of $\theta_{\mathrm{H}}$, and the feed length approximately by the square of that ratio.

Tables 3A-3D list representative feed sizes for the four values of $\theta_{\mathrm{H}}$ from Table 2. These sizes are approximate, and do not include corrugations, flanges, etc., but are useful for comparisons. Comparing tables 3B and 3D, case 4, wideband feeds at 3 cm are 1.9 feet dia. $\times 6.1$ feet long, whereas case 2 feeds are 1.5 feet dia. $\times 3.7$ feet long. At 6 cm the feeds are $3.8 \times 12.1$ feet versus 3.0 x 7.4 feet. On balance, case 4 does not look particularly objectionable, especially if we are allowed to do all wavelengths longer than about 3 cm with M1.

As mentioned above, the subreflector can be reduced in size by decreasing $C$, without changing $e, \alpha, \beta$, or $\theta_{\mathrm{H}}$. Table 2, case 5 , achieves the 17 inch $\bar{I}_{1} I_{2}$ increase of case 3 by moving the M2 feed 1.6 meters in front of the M1 feeds. Case 6 achieves the 30 inch increase by decreasing $C$ to 8.2 meters. The attractive feature is that the subreflector has been made smaller and moved closer to $F_{0}$ without making the feeds larger. A practical problem with this approach is that the M2 feeds and receivers will block the M1 feeds and would probably have to be physically moved when using the Ml feeds. Also, one working idea for supporting the prime focus feeds might have interference problems if the subreflector feeds are moved closer to the subreflectors.

It is possible to effectively reduce $C$ using curved reflectors, while keeping the feeds in convenient locations, as sketched in Figure 4. Theoretically, if edge diffraction is negligible, mirrors M4 and M3 can transfer a perfect image of the feed pattern from F2' to F2. Also, theory indicates that the system M2, M3, and M4 could be designed to achieve zero cross-pol in the main reflector aperture. However, the cost of two extra curved reflectors is probably not negligible, and we would have to be concerned with the feed spillover, possible baseline effects, and other hidden traps. I do not feel qualified to bet the farm on such a system without serious analysis and the opinions of electromagnetic experts.

There is a possible performance penalty when $C$ is reduced. As was mentioned in GBT Memo 29, offset feeds do not properly illuminate the main reflector if the subreflector is not enlarged. This effect probably gets worse as $C$ is reduced, but we have no numerical analysis yet to indicate the seriousness of the problem. However, the problem depends on the absolute offset angle, not the number of beamwidths scanned, so I suspect it is not a serious concern for the frequency range of M2.

In summary, it appears feasible to reduce the distance $\overline{F_{0} I_{2}}$, and $M 2$ to approximately 3 meters, by reducing $\theta_{\mathrm{B}}$ or C , or a combination of both.

## REFERENCE

[1] Reflector and Lens Antennas, C. J. Sletten, Editor. Artech House, 1988.

TABLE 1
Parameters for Current M1 and M2-1.

GREGORIAN OPFSET ANTENNA $02 / 22 / 90 \quad 21: 47: 46$
Current M1

| 1) Rccentricity $=$ | 0.528 | 2) | Yc $=$ |
| :--- | :--- | ---: | ---: |
| 3) | Focal Length $=$ | 60.000 | 4) |
| Radius $=$ | 50.000 |  |  |
| 5) Bllipsoid Focal Length $=$ | 11.000 |  |  |


| Alpha | $=$ | 17.898781 | Beta $=$ |
| ---: | ---: | ---: | ---: |
| Mag | $=$ | 5.569959 |  |
| B | $=379.970705$ | Feed Semi-Angle | $=$ |
| Theta $*$ | $=39.992858$ |  |  |
| Then | $=$ | 14.228535 |  |
|  |  |  | 42.823536 |

Subreflector 7.553 by 7.948 Y Range -7.262 to -0.329
Main Reflector 100.000 by $109.659 \quad$ F1 to I1 $=15.099$
Theta $\mathrm{C}=48.455491 \quad$ Rho $C=72.150 \quad \mathrm{Md}=-2.633200$
Equivalent Paraboloid: $\quad \mathbf{f 0}=-189.985$ i0 $=\mathbf{- 0 . 0 0 0 0 0 0}$

GREGORIAN OFFSET ANTENNA $02 / 22 / 90 \quad 21: 48: 08$
Current M2-1


Parameters to Increase the Distance $I_{1} I_{2}$

| Case | e | C |  | $\begin{aligned} & \text { eflector } \\ & \text { Size } \\ & \hline \end{aligned}$ | $\mathrm{F}_{1} \mathrm{I}_{1}$ | $\mathrm{F}_{0} \mathrm{I}_{1}$ | $\mathrm{F}_{\mathbf{e}}$ | $\theta_{\mathrm{H}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.528 | 11 | 7.55 | $\times 7.95$ | 15.1 | 5.74 | 190 | 14.99 |
| 2 | 0.680 | 11 | 4.07 | $\times 4.30$ | 13.1 | 3.03 | 313 | 9.15 |
| 3 | 0.714 | 11 | 3.5 | x 3.7 | 12.8 | 2.59 | 357 | 8.00 |
| 4 | 0.740 | 11 | 3.1 | x 3.3 | 12.6 | 2.27 | 400 | 7.16 |
| 5 | 0.680 | 9.4 | 3.5 | x 3.7 | 11.2 | 2.59 | 313 | 9.15 |
| 6 | 0.680 | 8.2 | 3.0 | x 3.2 | 9.8 | 2.27 | 313 | 9.15 |

FRED DESIGN

| INPUTS: <br> fRED HALF BEAM ANGLR, a | 14.99 | $=$ | 0.26 | RADIANS |
| :---: | :---: | :---: | :---: | :---: |
| Representative Feed Sizes, Wide-band Corrugated HomsSpherical Phase Error, DRL $=0.75$ |  |  |  |  |
|  |  |  |  |  |
| THETA * / THETA $f=$ | 0.80 |  |  |  |
| Flare Angle, THETA $\mathrm{f}=$ | 0.33 | $=$ | 18.74 | DEGRRES |
| FEED DIA / LAMBDA = | 9.09 |  |  |  |
| FERD LENGTH / LAMBDA = | 14.15 |  |  |  |
| LAMBDA, cm | DIA,M | LEN,M | DIA, FT | LEN, FTT |
| 3 | 0.27 | 0.4 | 0.89 | 1.39 |
| 4 | 0.36 | 0.6 | 1.19 | 1.86 |
| 6 | 0.55 | 0.8 | 1.79 | 2.79 |
| 15 | 1.36 | 2.1 | 4.47 | 6.96 |
| 20 | 1.82 | 2.8 | 5.97 | 9.29 |
| 30 | 2.73 | 4.2 | 8.95 | 13.93 |
| Representative Feed Sizes, Narrow-band Corrugated Homs |  |  |  |  |
| Ke = | 4.00 |  |  |  |
| Plare Angle, THETA $f=$ | 0.16 | $=$ | 9.29 | DBGREES |
| FERD DIA / LAMBDA = | 4.92 |  |  |  |
| FEED LENGTH / LAMBDA $=$ | 15.25 |  |  |  |
| LAMBDA, cm | DIA,M | LEN, M | DIA, FT | LEN, FT |
| 3 | 0.15 | 0.5 | 0.48 | 1.50 |
| 4 | 0.20 | 0.6 | 0.65 | 2.00 |
| 6 | 0.30 | 0.9 | 0.97 | 3.00 |
| 15 | 0.74 | 2.3 | 2.42 | 7.50 |
| 20 | 0.98 | 3.0 | 3.23 | 10.00 |
| 30 | 1.48 | 4.6 | 4.85 | 15.01 |

FERD DRSIGN

INPUTS:
FERD HALF BEAM ANGLE, a
0.16 RADIANS

Representative Feed Sizes, Wide-band Corrugated Horns
Spherical Phase Rrror, DRL $=0.75$
THETA $* /$ THETA $f=0.80$
Flare Angle, THETA $f=0.20=11.44$ DRGRERS FBED DIA / LAMBDA $=14.98$
FRED LENGIH / LAMBDA $=37.77$

| LAMBDA, cm | DIA,M | LEN,M | DIA, FT | LEN, FT |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 0.15 | 0.4 | 0.49 | 1.24 |
| 2 | 0.30 | 0.8 | 0.98 | 2.48 |
| 3 | 0.45 | 1.1 | 1.47 | 3.72 |
| 4 | 0.60 | 1.5 | 1.97 | 4.96 |
| 6 | 0.90 | 2.3 | 2.95 | 7.43 |
| 9 | 1.35 | 3.4 | 4.42 | 11.15 |

Representative Feed Sizes, Narrow-band Corrugated Horns
Spherical Phase Error, DRL $=0.20$
$\mathrm{Ke}=4.00$
Flare Angle, THETA $f=0.10=5.72$ DRGRERS FRED DIA / LAMBDA $=8.01$ FEED LENGTH / LAMBDA $=40.17$

| LAMBDA, Cm | DIA,M | LEN,M | DIA, FT | LBN, FT |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 0.08 | 0.4 | 0.26 | 1.32 |
| 2 | 0.16 | 0.8 | 0.53 | 2.64 |
| 3 | 0.24 | 1.2 | 0.79 | 3.95 |
| 4 | 0.32 | 1.6 | 1.05 | 5.27 |
| 6 | 0.48 | 2.4 | 1.58 | 7.91 |
| 9 | 0.72 | 3.6 | 2.36 | 11.86 |

EEED DRSIGN

| INPUTS: <br> EEED HALP BEAM ANGLE, a | 8.00 | $=$ | 0.14 | RadIans |
| :---: | :---: | :---: | :---: | :---: |
| Representative Feed Sizes, Wide-band Corrugated HomsSpherical Phase Error, |  |  |  |  |
|  |  |  |  |  |
| THETA * THETA $\mathrm{f}=$ | 0.80 |  |  |  |
| Flare Angle, THETA $\mathrm{f}=$ | 0.17 | = | 10.00 | DEGRBES |
| FEED DIA / LAMBDA = | 17.15 |  |  |  |
| FEED LENGTH / LAMBDA $=$ | 49.37 |  |  |  |
| LAMBDA, cm | DIA,M | LEN,M | DIA, FT | LEN, FT |
| 1 | 0.17 | 0.5 | 0.56 | 1.62 |
| 2 | 0.34 | 1.0 | 1.13 | 3.24 |
| 3 | 0.51 | 1.5 | 1.69 | 4.86 |
| 4 | 0.69 | 2.0 | 2.25 | 6.48 |
| 6 | 1.03 | 3.0 | 3.38 | 9.72 |
| 9 | 1.54 | 4.4 | 5.06 | 14.58 |
| Representative Feed Sizes, Narrow-band Corrugated HornsSpherical Phase Rrror, DRL |  |  |  |  |
| Spherical Phase Error, DEL = | 0.20 |  |  |  |
| $\mathrm{Ke}=$ | 4.00 |  |  |  |
| Flare Angle, THETA $\mathrm{f}=$ | 0.09 | $=$ | 5.01 | DEGREES |
| FEED DIA / LAMBDA = | 9.15 |  |  |  |
| FEED LENGIH / LAMBDA = | 52.41 |  |  |  |
| LAMBDA, cm | DIA,M | LEN,M | DIA, FT | LEN, FT |
| 1 | 0.09 | 0.5 | 0.30 | 1.72 |
| 2 | 0.18 | 1.0 | 0.60 | 3.44 |
| 3 | 0.27 | 1.6 | 0.90 | 5.16 |
| 4 | 0.37 | 2.1 | 1.20 | 6.88 |
| 6 | 0.55 | 3.1 | 1.80 | 10.32 |
| 9 | 0.82 | 4.7 | 2.70 | 15.48 |

FEED DRSIGN

| INPUTS: <br> FRED HALP BRAM ANGLE, a | 7.16 | $=$ | 0.12 | RADIANS |
| :---: | :---: | :---: | :---: | :---: |
| Representative Feed Sizes, Wide-band Corrugated Horns |  |  |  |  |
| Spherical Phase Error, DRL = | 0.75 |  |  |  |
| THETA * / THETA $f=$ | 0.80 |  |  |  |
| Flare Angle, THETA $\mathrm{f}=$ | 0.16 | = | 8.95 | DRGREES |
| FRED DIA / LAMBDA = | 19.17 |  |  |  |
| FEED LENGIH / LAMBDA = | 61.60 |  |  |  |
| LAMBDA, cm | DIA,M | LEN,M | DIA, FT | LEN, FT |
| 1 | 0.19 | 0.6 | 0.63 | 2.02 |
| 2 | 0.38 | 1.2 | 1.26 | 4.04 |
| 3 | 0.57 | 1.8 | 1.89 | 6.06 |
| 4 | 0.77 | 2.5 | 2.52 | 8.08 |
| 6 | 1.15 | 3.7 | 3.77 | 12.13 |
| 9 | 1.72 | 5.5 | 5.66 | 18.19 |

Representative Feed Sizes, Narrow-band Corrugated Horns
Spherical Phase Error, DRL $=0.20$
$\mathrm{Ke}=\quad 4.00$
Flare Angle, THETA $f=0.08=4.48$ DEGRESS
FEED DIA / LAMBDA $=10.22$
FBED LENGIH / LAMBDA $=65.32$

| LAMBDA, cm | DIA,M | LEN,M | DIA, FT | LEN, FT |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.10 | 0.7 | 0.34 | 2.14 |
| 2 | 0.20 | 1.3 | 0.67 | 4.29 |
| 3 | 0.31 | 2.0 | 1.01 | 6.43 |
| 4 | 0.41 | 2.6 | 1.34 | 8.57 |
| 6 | 0.61 | 3.9 | 2.01 | 12.86 |
| 9 | 0.92 | 5.9 | 3.02 | 19.29 |



FIGURE 1. Current design, showing both subreflectors.


FIGURE 2. Geometry of main reflector.


FIGURE 3. Subreflector geometry.


FIGURE 4. Optical transfer of focal point.

