GBT Memo No. 56

REMARKS ABOUT DYNAMICS, ATMOSPHERE AND SUMMARIES

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The following treats some of the items which we discussed in Charlottesville on May 24-25, 1990.

1. Dynamical Estimate

When designing a structure, deformations and stresses under all loads are immediately analyzed, checking stability. The lowest dynamical frequency (complicated and time-consuming) is calculated only at a more final state. But the dead load deformations (sag) give already an estimate of the lowest frequency in vertical direction. And since the direction of gravity is easily changed in the computer, all lowest modes of translation (but not rotation) can already be estimated during stability checks.

Fig.1a has a mass M on a spring of stiffness K=Force/Displacement=F/X. For an oscillation, X=A sin wt. the acceleration then is $X'' = -Aw^2 \sin wt = -w^2X$. Regarding the spring , the force is F = -KX and the acceleration is F/M = -KX/M. Thus: $w^2 = K/M$. The frequency is $f = w/2\pi$ or

$$f = (1/2\pi) \quad \sqrt{K/M} \tag{1}$$

The static sag is S=F/K where F=M*Gravity, and G = 981 cm/sec². Thus S = (M/K) *G, or K/M = (981/S) cm/sec². And with (1) we have the frequency of oscillation, as measured by the sag.

$$f = \frac{4.99\ \text{Hz}}{\sqrt{S/\text{cm}}} = \frac{3.13\ \text{Hz}}{\sqrt{S/\text{inch}}} \tag{2}$$

The same holds for the mass at the end of a beam, Fig. 1b, which is the worst case. More realistic would be an equal-distribution, which we replace by all mass at the center, Fig. 1c, but measuring the sag still at the end of the beam. For equal sag, the oscillation is now faster by $\sqrt{5/2}$, and instead of (2) we have

$$f = \frac{7.88\ \text{Hz}}{\sqrt{S/\text{cm}}} = \frac{4.95\ \text{Hz}}{\sqrt{S/\text{inch}}} \tag{3}$$

Sometimes two separately measured dynamical frequencies are given. For example from our NRAO 25-m design I find only dynamics for the dish on stiff elevation bearings, and for the supporting towers with the dish mass lumped at the bearings. Regarding the (worst) case of complete coupling, Fig.1d estimates how to combine two frequencies. Since the combined sag is $S = S_1 + S_2$, and with the same reasoning as before, the frequency f of the combined system is estimated from (4) which is always on the safe side:

$$1/f^2 = 1/f_1^2 + 1/f_2^2 \tag{4}$$

2. Scaling

We want to compare the dynamics of telescopes with different diameters D. The axial deformation of a bar is (Force*Length)/(Modulus of Elasticity * Bar Area) = FL/EA. For a simple scaling, $L \approx D$, and $A \approx D^2$. And for dead loads the force of the weight is $F \approx LA$ or $F \approx D^3$. Thus the telescope deformation or sag, $S \approx FL/A$, scales as

$$S \approx D^2 \tag{5}$$

Inserting this in (2) or (3), we find that the lowest dynamical frequency f will scale with the telescope diameter D simply as

$$f \approx 1/D \tag{6}$$

Table 1 gives the lowest frequency f of several telescopes. And f(100 m), scaled with (6), is to be expected for a telescope with 100 m diameter and of comparable design.

Table 1. Lowest frequency f, scaled to 100 m diameter.

Telescope	D [m]	f [Hz]	f (100 m)
SMT, Mt Graham (1994)	10	7.0	0.70
25-m design, 1975	25	5.81	1.45
VLA and VLBA, NRAO	25	4.0	1.00
Pico Veleta, 1985	30	3.9	1.17
65-m design, 1972	65	1.52	0.99
Deep Space Network	70	1.3	0.91
300-ft design, 1969	91	1.20	1.09
Effelsberg, 1971	100	1.3	1.30
GBT prelim. design, 1990	100	0.5	0.50

Table 1 shows first, that the scaling of equation (6) does make sense: the frequencies cover a much smaller range when scaled to the same diameter. Second, for a size of 100 m we have an average of $f(100) = 1.08 \pm 0.08$ Hz. Third, our present GBT design is still down a factor of two and wants to be improved. A possible improvement is the original tripod-like arm.

3. Atmosphere and Laser Ranging

We plan to measure and to correct the fine pointing, and the surface shape, by a closed-loop laser ranging. The atmosphere will give two kinds of errors. First, the fast turbulence of smaller eddies gives a "jitter" instead of a constant output, thus adding different random noise to all distances. This effect can be corrected, if needed, using two different colours for correction.

Second, Memo 45 mentioned a constant term of about 60 mm, for a 200-m round-trip, by which the distance is measured longer than it actually is (and would be, measured in vacuum). And for a correct subtraction, we would have to know temperature and pressure very accurately, or to correct again with two colours.

If we measure all distances (also those between the lasers) too large by the same factor, then all angles are still correct. Thus the pointing, being an angle, is measured correct. In a radome we might have small errors from stratification, but not in the open.

Regarding the surface shape, we find the telescope a bit larger but all in proportion (with a "homologous error" as it were). Our best-fit paraboloid then seems a bit larger, and to this one will the surface be adjusted. We measure also the distance to the focal equipment a bit larger by the same factor, which fits perfectly the larger best-fit paraboloid. Thus all is OK. Let us keep in mind that the incoming radio wave also travel a bit slower and with somewhat shorter wavelength in air than in vacuum, which does not change the focussing of a mirror (as opposed to a lens). Thus we may have problems with the jitter, but none with the bias.

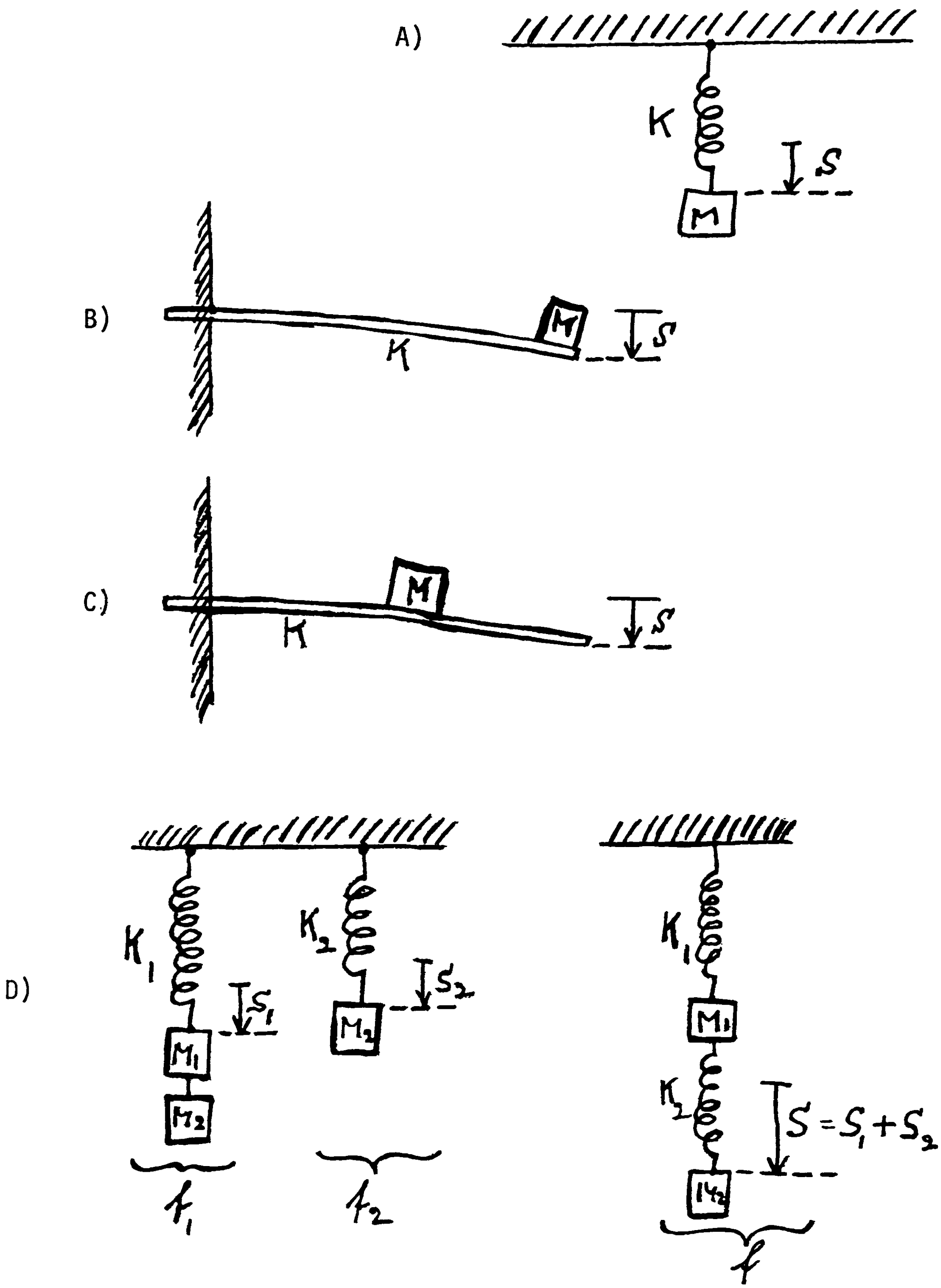

Fig. 1. Gravitational sag, for load of mass M and stiffness K.
a) Mass under spring; b) End of beam; c) Center of beam.
d) Combination of two separately measured frequencies.