

SHIFT OF PRIME FOCUS LOCATION BY SMALL SURFACE ADJUSTMENTS

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1. Problem

In GBT Memo 78 of May 20, 1972, S. Srikanth drew attention to the fact that gravitational deformations will change the locations of both the prime focus and the secondary feed, and he investigated the best movements of the secondary mirror to make up for it, maximizing the gain of the system.

The relevant deformations have five parameters: shifts (in y and z) of the prime focus, plus shifts (y, z) of the feed, plus rotation (about x) of the feed axis. Whereas the movements of the secondary have only three degrees of freedom: shifts (y, z) plus rotation. This leaves two deformations uncorrected. (Here, as in Memo 78, x is parallel to the elevation axis, z is parallel to the primary axis, and the y,z-plane is the plane of symmetry.) Regarding these uncorrected items: first, we cannot change the eccentricity of the elliptic secondary, its two foci have a given distance from each other and keep it. Which means we cannot correct for a changing distance between the prime focus and the phase center of the feed. Second, if the three degrees of freedom of the secondary are used up for maximizing the gain, then the feed axis may not point to the (projected) center of the secondary, which will cause some spillover beyond its rim.

If we would readjust the active surface such that the prime focus moves in y and z by wanted amounts, then we had two more degrees of freedom, and all five deformation parameters could exactly be corrected for. Since, in principle, this can be done, the actual question is only:

*Can the wanted focal shift be achieved with sufficiently **small** adjustments? -*

Table 1 of Memo 78 gives the changes of y and z, for horizon and zenith pointing (rigging pointing at 30° elevation), of prime focus and feed. Taking the differences, δ = focus - feed, and calling $R = \sqrt{(\delta y)^2 + (\delta z)^2}$ the length of the wanted change, we find $R(\text{horizon}) = 6.59 \text{ inch} = 16.74 \text{ cm}$, and $R(\text{zenith}) = 11.97 \text{ inch} = 30.41 \text{ cm}$. Since observations close to the zenith can be avoided mostly, we shall investigate (as a large example) a wanted change of length R with

$$R = 25 \text{ cm.} \quad (1)$$

2. Method

Fig.1 shows the geometry of the GBT primary. With special readjustments normal to the surface, $\epsilon(Y)$, we can achieve all four possible changes of a parabola in the plane of symmetry: two translations ΔY and ΔZ , a change of focal length ΔF , and one rotation $\Delta \phi$. They shall be chosen such that the primary focus moves by dPY and dPZ , and that **rms(ϵ) = min.**

Since the maximum and minimum values of ϵ will occur at $X = 0$, we will treat only the plane of symmetry. Then $Z = Y^2/4F$, and the slope α is given by $\tan \alpha = Y/2F$. The four changes of the parabola are considered small, to be superimposed. It follows that the four cases of needed adjustments $\epsilon(Y)$, normal to the surface), then are:

$$\begin{aligned} \epsilon_1(Y) &= -\sin \alpha \Delta Y, & \epsilon_2(Y) &= +\cos \alpha \Delta Z, \\ \epsilon_3(Y) &= \sin \alpha \tan \alpha \Delta F & \epsilon_4(Y) &= (Y \cos \alpha + Z \sin \alpha) \Delta \phi \end{aligned} \quad (2)$$

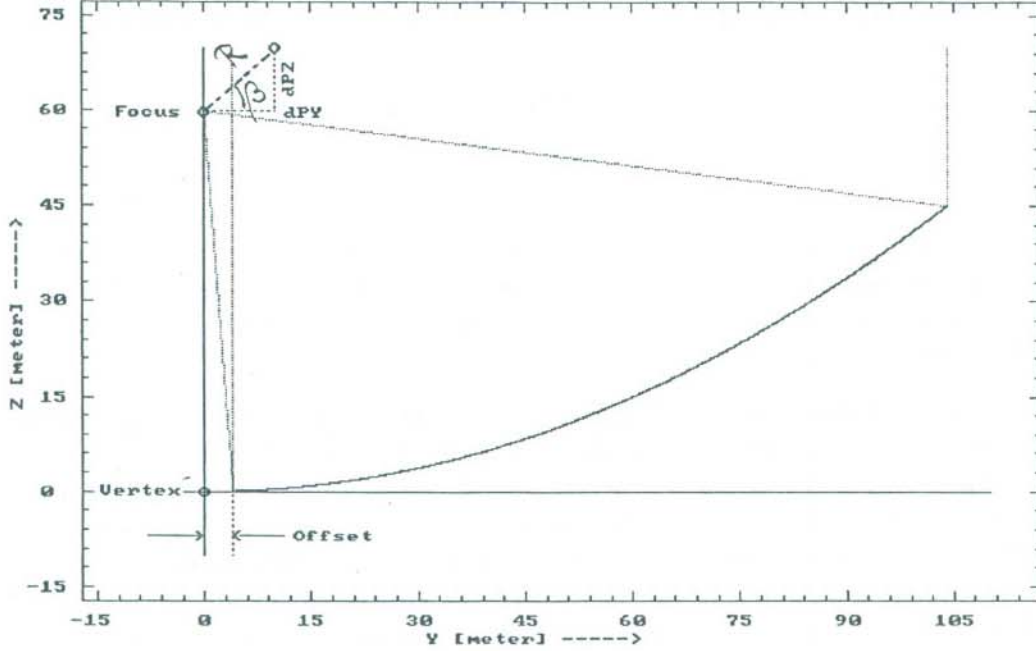


Fig. 1. GBT, geometry. dPY, dPZ = wanted prime focus shift.

and the four changes Δ must yield the two wanted shifts of the primary focus, dPY and dPZ:

$$\Delta Y - F \Delta \phi = dPY \quad \Delta Z + \Delta F = dPZ \quad (3)$$

which are used to eliminate ΔF and $\Delta \phi$ in equations (2), leaving ΔY and ΔZ for minimizing $\text{rms}(\epsilon)$. We call

$$\begin{aligned} f_1(Y) &= f_3 - \sin \alpha & f_3(Y) &= (Y \cos \alpha + Z \sin \alpha)/F \\ f_2(Y) &= f_4 + \cos \alpha & f_4(Y) &= \sin \alpha \tan \alpha \end{aligned} \quad (4)$$

the sum

$$\epsilon(Y) = f_1 \Delta Y + f_2 \Delta Z + f_3 dPY + f_4 dPZ \quad (5)$$

and mean square

$$\rho(\Delta Y, \Delta Z) = (1/D) \int_s^{D+s} \epsilon^2 dY \quad (6)$$

where $s = 4$ meter is the offset (vertex to rim). For minimizing $\text{rms}(\epsilon)$, we let $\partial \rho / \partial \Delta Y = 0$, and $\partial \rho / \partial \Delta Z = 0$, yielding two equations for the two unknowns ΔY and ΔZ , which are solved as

$$\Delta Y = (M_{22} V_1 - M_{12} V_2) / \text{Det} \quad (7)$$

$$\Delta Z = (M_{11} V_2 - M_{21} V_1) / \text{Det} \quad (8)$$

with

$$M_{ij} = (1/D) \int_s^{D+s} f_i f_j dY \quad (9)$$

and

$$V_1 = M_{13} dPY + M_{14} dPZ$$

$$V_2 = M_{23} dPY + M_{24} dPZ$$

$$\text{Det} = M_{11} M_{22} - M_{12} M_{21}$$

finally After having solved for ΔY and ΔZ , we calculate $\epsilon(Y)$ from (5), the integral ρ from (6), and $\text{rms}(\epsilon) = \sqrt{\rho}$. (10)

3. Results

We call β the direction of the focus shift ($\tan \beta = dPZ/dPY$). From Memo 78 (if I get the signs right), the wanted focal shifts should go in direction $\beta(\text{horizon}) = 60^\circ$, and $\beta(\text{zenith}) = 208^\circ$. In any case, and to learn the dependence on the angle β , we have calculated the full range, $0 < \beta < 360^\circ$, see Table 1 and Fig. 2. In addition to the $\text{rms}(\epsilon)$, the table shows also the extremes (max, min) of ϵ , and the location Y/D of their occurrence. The larger extreme is always at the primary rim (Y/D = 0.04 and 1.04 because of the offset). The best angle is at $\beta = 67^\circ$ (with $\epsilon \text{ max} = +0.47 \text{ cm}$, $\text{rms} = 0.18 \text{ cm}$), and the worst is at $\beta = 147^\circ$ ($\epsilon \text{ min} = -2.37 \text{ cm}$, $\text{rms} = 0.99 \text{ cm}$). All these ϵ values hold for a focal change of length $R = 25 \text{ cm}$.

If we use for the wanted focus shift the angles and lengths as derived from Memo 78, we get the following results for the needed surface readjustments ϵ :

	angle β	length R	extreme ϵ	$\text{rms}(\epsilon)$
horizon	60°	16.74 cm	+ 0.48 cm	0.15 cm
zenith	208°	30.41 cm	- 2.02 cm	0.81 cm

(11)

So far, it seems that the needed readjustments are within the range of the activators, but this needs some remarks. First, the deformations of focus and feed as used in Memo 78 were not yet the final ones. Second, the actuators have first to correct for the deformed surface, and then it is the question of how much range is still left (on the side of our readjustment sign). Third, the extremes as calculated here along $X = 0$ should be correct, but the $\text{rms}(\epsilon)$ would still be much smaller if the lateral dimension were included, too, in the calculation. Fourth, observation close to the zenith can mostly be avoided. In summary, we hope that this method will allow the correction of all five deformation parameters mentioned in the beginning; either completely or at least with some good compromise.

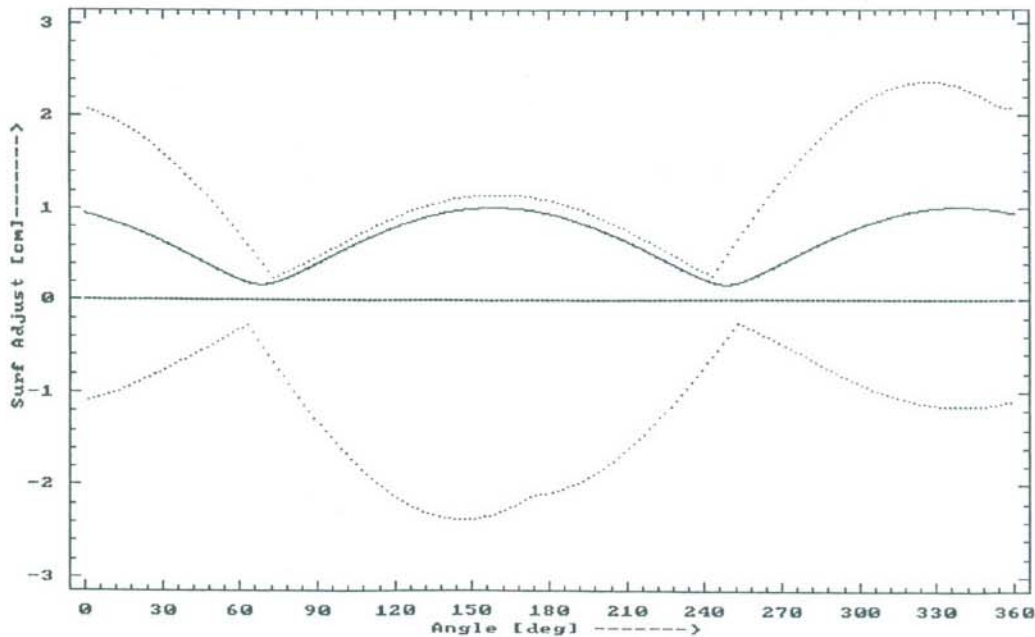


Fig. 2. Surface adjustments, for focus shift of 25 cm at various angles.
Full = rms, dotted = max and min (extremes always at the rim).

TABLE 1. Wanted shift of prime focus, of length $R = 25$ cm at various angles, by small adjustments of prime surface, $\epsilon(Y)$. Aperture diameter $D = 10000$, focal length $F = 6000$. Offset (vertex to rim) = 400. All lengths in cm.

dPY, dPZ = shift of prime focus coordinates;
dY, dZ = resulting vertex shift, for $\text{rms}(\epsilon) = \text{min}$;
 ϵ_{max} , ϵ_{min} = max and min of ϵ , at diameter fraction Y_{max} , Y_{min} ;
 R/rms , R/max = focal shift / surface change.

angle	dPY	dPZ	dY	dZ	ϵ_{max}	Y_{max}/D	ϵ_{min}	Y_{min}/D	rms	R/rms	R/max
0	25.00	0.00	38.37	2.36	2.08	1.04	-1.09	0.57	0.94	26.54	12.04
10	24.62	4.34	40.22	2.02	1.98	1.04	-1.01	0.58	0.87	28.87	12.62
20	23.49	8.55	40.86	1.62	1.82	1.04	-0.90	0.59	0.77	32.67	13.70
30	21.65	12.50	40.26	1.17	1.61	1.04	-0.77	0.60	0.64	38.86	15.50
40	19.15	16.07	38.43	0.69	1.35	1.04	-0.62	0.62	0.51	49.43	18.49
50	16.07	19.15	35.43	0.18	1.05	1.04	-0.46	0.65	0.36	69.27	23.80
60	12.50	21.65	31.36	-0.33	0.72	1.04	-0.31	0.69	0.23	109.15	34.88
67	9.77	23.01	27.93	-0.68	0.47	1.04	-0.43	0.04	0.18	140.48	53.23
70	8.55	23.49	26.34	-0.83	0.36	1.04	-0.55	0.04	0.18	140.33	45.40
80	4.34	24.62	20.51	-1.31	0.33	0.39	-0.94	0.04	0.27	94.08	26.54
90	-0.00	25.00	14.06	-1.74	0.48	0.44	-1.30	0.04	0.41	61.66	19.17
100	-4.34	24.62	7.19	-2.13	0.63	0.47	-1.63	0.04	0.55	45.51	15.36
110	-8.55	23.49	0.09	-2.45	0.78	0.49	-1.90	0.04	0.68	36.62	13.15
120	-12.50	21.65	-7.00	-2.69	0.90	0.51	-2.12	0.04	0.80	31.31	11.81
130	-16.07	19.15	-13.89	-2.85	1.01	0.52	-2.27	0.04	0.89	28.02	11.02
140	-19.15	16.07	-20.35	-2.93	1.09	0.53	-2.35	0.04	0.96	26.05	10.64
147	-20.97	13.62	-24.52	-2.93	1.13	0.54	-2.37	0.04	0.99	25.23	10.57
150	-21.65	12.50	-26.19	-2.92	1.14	0.54	-2.36	0.04	1.00	25.01	10.59
160	-23.49	8.55	-31.24	-2.81	1.15	0.55	-2.30	0.04	1.01	24.76	10.87
170	-24.62	4.34	-35.34	-2.63	1.14	0.56	-2.17	0.04	0.99	25.24	11.52
180	-25.00	-0.00	-38.37	-2.36	1.09	0.57	-2.08	1.04	0.94	26.54	12.04
190	-24.62	-4.34	-40.22	-2.02	1.01	0.58	-1.98	1.04	0.87	28.87	12.62
200	-23.49	-8.55	-40.86	-1.62	0.90	0.59	-1.82	1.04	0.77	32.67	13.70
208	-22.07	-11.74	-40.48	-1.26	0.80	0.60	-1.66	1.04	0.67	37.36	15.07
210	-21.65	-12.50	-40.26	-1.17	0.77	0.60	-1.61	1.04	0.64	38.86	15.50
220	-19.15	-16.07	-38.43	-0.69	0.62	0.62	-1.35	1.04	0.51	49.43	18.49
230	-16.07	-19.15	-35.43	-0.18	0.46	0.65	-1.05	1.04	0.36	69.27	23.80
240	-12.50	-21.65	-31.36	0.33	0.31	0.69	-0.72	1.04	0.23	109.15	34.88
250	-8.55	-23.49	-26.34	0.83	0.55	0.04	-0.36	1.04	0.18	140.33	45.40
260	-4.34	-24.62	-20.51	1.31	0.94	0.04	-0.33	0.39	0.27	94.08	26.54
270	0.00	-25.00	-14.06	1.74	1.30	0.04	-0.48	0.44	0.41	61.66	19.17
280	4.34	-24.62	-7.19	2.13	1.63	0.04	-0.63	0.47	0.55	45.51	15.36
290	8.55	-23.49	-0.09	2.45	1.90	0.04	-0.78	0.49	0.68	36.62	13.15
300	12.50	-21.65	7.00	2.69	2.12	0.04	-0.90	0.51	0.80	31.31	11.81
310	16.07	-19.15	13.89	2.85	2.27	0.04	-1.01	0.52	0.89	28.02	11.02
320	19.15	-16.07	20.35	2.93	2.35	0.04	-1.09	0.53	0.96	26.05	10.64
330	21.65	-12.50	26.19	2.92	2.36	0.04	-1.14	0.54	1.00	25.01	10.59
340	23.49	-8.55	31.24	2.81	2.30	0.04	-1.15	0.55	1.01	24.76	10.87
350	24.62	-4.34	35.34	2.63	2.17	0.04	-1.14	0.56	0.99	25.24	11.52
360	25.00	0.00	38.37	2.36	2.08	1.04	-1.09	0.57	0.94	26.54	12.04