1. Damping the Oscillation

This is in response to a letter from Paul Vanden Bout of May 1, containing a Memo from Bob Hall of April 20, and Memo 127 from John Payne of April 17. The problem is: fast ON-OFF changes in azimuth, of 1 degree (as simulated with the future pointing system) will cause large oscillations with small damping.

From Figs. 9a and 9b of the Memo I read, roughly: it takes a time \( T_e = 8 \) sec to reach the goal, causing an oscillation of \( f_r = 0.75 \) Hz with an amplitude of 1% of the move, and a damping decrement of 5% between successive maxima. It would take more than 30 seconds to reduce the oscillation error below 10 arcsec.

Regarding the damping, I suggest to investigate aerodynamic damping which might help. And regarding additional damping devices (like lead balls in a container, mentioned long ago): if they are strong enough for effective damping, they may also cause hysteresis. The only case without hysteresis may be a heavy viscous fluid. But a large mass will be needed.

A good proposal is from Lee King: increasing the arm stiffness by connecting the base of the feed arm to the backup structure of the dish, which could reduce the arm vibration by at least a factor two. It would increase the gravitational surface deformation, but this would then be corrected in Phase 3 by the laser system. (Would the needed activator movements then still be within their physical limits, including wind and thermal corrections?)

I would suggest a modification: let the arm and the backup be connected by good shock absorbers only, with a short time constant of very few seconds. This may give a good damping, to both arm and dish, without any remaining surface deformations to be corrected. It may also help to dampen wind gusts too fast for the pointing system.

2. Avoiding the Oscillation?

My main concern is not the damping of the oscillations, but their avoidance by simple means, if possible. The short ON-OFF move consists of two opposing parts: its acceleration, and its deceleration. It should be possible to let the second part counteract not only the average velocity of the first part, but as well its oscillation, too. Since both parts have opposite actions, we may need to adjust only one free parameter: proper timing of the switch between both parts, which means proper choice of the applied force. For simplicity, we adopt constant and equal forces for both parts. A strong damping will finally have damped the oscillation of the first part more than that of the second part, leaving a small residual oscillation, to be investigated numerically.

The case of a STOP after a long fast movement (slew, scan) will also be treated in a similar way. We do not have two opposing parts here; thus we try to imitate this, by introducing a short pause, without any force, after half the deceleration. This will need two free parameters to adjust: strength of the force, and duration of the pause.

I will treat only the simplest case of a forced harmonic oscillator: a Driver \( Y(t) \), with steady acceleration \( Y'' = \frac{d^2Y}{dt^2} = +A, \) 0, or \(-A, \) and \( A = \text{const} \leq 0.2 \) deg/sec\(^2\); acting on a Spring of stiffness \( K \), acting on a Mass \( M \) with Friction \( B \), which then moves as \( X(t) \), which should not exceed its velocity limit of \( X' = V(t) \leq 0.67 \) deg/sec. Both limits as given in the Memos. We call the Deviation, or Error, \( \text{Dev}(t) = X(t) - Y(t) \); and the task is to reach the goal, after a short duration \( T_e \), with both \( \text{Dev} = 0 \) and \( V = 0 \), or both at least as small as possible, to avoid or to minimize any residual oscillations.
3. Fast Move of 1 degree

We call $T_a = \frac{1}{2}T_e$. The differential equation to be integrated is

$$X'' = -(B/M) X' - (K/M)(X - Y(t))$$

with $Y'' = +A$ for $0 < t < T_a$, and $Y'' = -A$ for $T_a < t < 2T_a = T_e$.  

At $t=0$ let $Y = Y' = X = X' = 0$. Since $0.5\text{ deg} = \frac{1}{2}A T_a^2$, then $T_a = 1/\sqrt{A}$ and the full duration is $T_e = 2T_a$. The fastest move, with $A = 0.2$, then is $T_e = 4.472$ sec.

Regarding the oscillation frequency, we have $Fr = (1/2\pi) \sqrt{(K/M)} = 0.75$ Hz; and the damping decrement between maxima we call $1 - Qd$, with $Qd = \pi B/\sqrt{(MK)} = 0.05$. These are only two equations for three unknowns: $M$, $K$, $B$. But Equation (1) contains only $B/M = 2*Qd*Fr$, and $K/M = (2\pi*Fr)^2$; which both do not depend on $M$. We thus set $M = 1$.

For a start we integrate the fastest move ($A=0.2$) and obtain Fig.1, with a maximum error of $Devmax = 84.6$ arcsec. The upper curve is the movement, the lower curve the deviation. $De$ and $Ve$ are deviation and velocity at the end time $T_e$. $Te/W$ is the end time in terms of the wavelength $W=1/\Fr$ of the oscillation. This fastest case would be a bad choice.

![Graph](image)

**Fig.1.** The fastest move, with $A = 0.2$ deg/sec².

Some examples have shown that the two parts will exactly counter act only if $Te/W$ is an even number, the smallest being $Te/W = 4$ for $A < 0.2$. This best case is shown in Fig.2. We see that the residual oscillation is practically zero. And this best move is also quite fast, with $T_e = 5.33$ sec, only less than a second longer than the fastest move.

Fig.3 shows the same case, plotting the velocity $V(t)$, which stays well below its limit of 0.67 deg/sec. We see from these two figures, that at time $T_e$ the deviation $De$, and the velocity $Ve$, are both practically zero, as we demanded for vanishing oscillations.

Stronger damping will not yield much larger oscillations. For decrements $Qd < 0.20$ we still get $Dmax < 2.5$ arcsec. For timing errors, $Dmax$ increases quadratically. To keep $Dmax < 3$ arcsec (for $Fr = 0.75$ and $Qd = 0.10$) we need $5.2 \leq T_e \leq 5.4$ sec. And for $Dmax < 7$ arcsec, we would need $5.1 \leq T_e \leq 5.6$ sec. As the timing seems not too critical.
4. Stop after a Slew

The task is to get a fast stop with no residual oscillations. I found no simple rule for best adjustments of deceleration time $T_e$, and pause $T_p$ (without force) at half the time, $T_a = \frac{1}{2} T_e$. For a slew rate of $S = 0.6 \text{ deg/sec}$, oscillation $F_r = 0.75 \text{ Hz}$, decrement $Q_d = 0.05$, and by trial and error, the best solution obtained is shown in Fig.4, with a pause of $T_p = 0.37 \text{ sec}$, after a deceleration of $T_a = 1.53 \text{ sec}$ with $A = 0.196 \text{ deg/sec}^2$, and a total duration of only $T_e = 2T_a + T_p = 3.43 \text{ sec}$. Leaving a negligible oscillation of only $0.6 \text{ arcsec}$ amplitude.
The method suggested has the same basic idea as the homologous deformation had, where I did not try to avoid the gravitational sag but to shape it into a harmless form. The present method does not try to avoid the initial oscillation, but to shape it such that the pointing deviation and its velocity are both zero at the same time, using a simple drive function. The form of this smooth drive functions $Y(t)$ can be judged on Figs.2 and 4, where the deviations $Dev(t)$ are small, such that $Y(t)$ is about equal to the plotted actual movement $AZ(t)$.

The essential point is that I have adopted a Driver, $Y(t)$, executing a given simple program and nothing else. This would mean: The drive motors of the telescope are given commands according to $Y(t)$, without the use of any sensors or other feedback from the telescope, which would interfere with the proper execution of the simple and smooth $Y(t)$. Thus, the sensors should be used only to avoid any dangers, but not for the usual guidance.

Without this guidance and because of all tolerances, the telescope will not exactly hit the goal. Thus I assume that right after $Te$, the normal telescope drive system is again switched on, with its usual functions, to correct the residual deviation from the exact position of the goal. Which, for small deviations, should neither take long, nor cause much oscillation.

So far I have dealt with only one dynamic mode, the lowest. This mostly is the worst one, higher modes being generally of lower amplitude and faster damping, easier to be tolerated. Figures 9b and 10b of the Memo (April 17) for example show a beat (four waves long), indicating a second mode of 0.94 Hz, with less than half the amplitude, and fast damping. In this case, and if not tolerable, one could adjust the one-degree move for a length of $Te = 10.66$ sec, which then is $Te/W = 8$ for the first mode, and $Te/W = 10$ for the second one. If two low modes are closer together, one could adjust for their average.

The first thing should be a direct measurement of the actual dynamics. With the normal system, drive fast toward a strong point source, make a fast stop at half the beam before the source. Do not move now, just register the oscillating receiver output. Then repeat it, but with a smooth stop long before the source. Subtracting both outputs, and dividing by the slope of the second (the beam), yields the telescope oscillation: frequencies, amplitudes and damping.