GBT MEMO 211

Calibration Of The GBT Triplet Retroreflector Units.

M.A. Goldman

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Abstract

Triplet Retroreflector Units are assemblies of three rangefinder targets, to be mounted at the rim of the Green Bank Telescope's main reflector surface. They provide metrology references to tie the locations of feed arm rangefinders to the ground survey control net of the telescope. Laboratory optical calibration of these units is described in this report.

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See L0632 — "GBT Triplet – Calibration, Locations, Work Breakdown, Drawings, and Software" for additional materials.

1. Introduction.

1.1. The Use Of Triplet Target Units In Telescope Metrology.

The surface and pointing of the Green Bank Telescope are measured using a laser metrology system. Its optical components are: an array of ground-based rangefinders spaced about the telescope's base, rangefinders on the vertical feed arm, and retroreflecting targets on the main dish surface, subreflector surface, feed arm and tipping structure, and alidade and ground benchmark targets. In 1992, Don Wells and John Payne proposed the use of clusters of retroreflecting targets at the rim of the main dish of the telescope, to tie the locations of the feed arm rangefinders to a ground reference frame survey-control-network [1, 2]. Each cluster would serve as a Janus-faced target which could be viewed and located from both the feed arm rangefinders and ground-based rangefinders. These units were subsequently designed and fabricated as triples of targets rigidly placed relative to one another. Two of the targets are solid glass cube corner prisms which are designed to face towards the feed arm rangefinders, the third target, called a "ball" or cat's-eye target, is a specially fabricated wide-angle target which is viewed by ground-based rangefinders. The three targets are packaged together as a "triplet target unit."

For metrology purposes, the location of a rangefinder is specified by the position of its scan reference point, which is the unique point on its scan mirror's surface at which the two rangefinder scan drive axes intersect. The effective location of a retroreflecting target is defined by its "fiducial reference point" which is a unique point which can be found from the known optical geometry and refractivity of the target. The cat's-eye target can be considered geometrically to consist of two half-balls of different radius, which are concentric and meet at a common boundary plane.

Both are types of reflector are made of BK7 glass. The first is a commercially available cube corner reflector prism. The second type is a ball and ball-cap cat's-eye retroreflector, manufactured for NRAO by the University of Arizona Optical Sciences Center [3, 4].

The prism targets and the "ball" target are mounted on a flat-ground rigid plate (the "top plate" of the unit). The prism targets are mounted to look normally outward from the "upper surface" of the top plate, while the cat's-eye reflector faces oppositely, through a hole in the plate [Fig.'s 1A,2]. The fiducial reference points of the prism and ball targets are designed to lie in the plane of the top plate's upper surface and to be co-linear and, furthermore, the ball target's reference point is designed to coincide with the midpoint of the line segment joining the prism reference points. This effectively provides a two-faced metrology target, with both faces at the same location but looking in opposite directions. The fiducial reference point of the cat's-eye target is defined to be the center point, BC, of the small ball.

Using ten of these triplet units mounted at the edge of the GBT main reflector dish it is possible to make the following metrology measurements:

1. By making trilateration range measurements from three or four ground-based rangefinders to a visible triplet ball, one can determine the position of that ball reflector's reference point with respect to the ground-coordinate reference frame of the telescope.

2. Feed arm rangefinders can range to one another and to all triplet prism targets to generate a local survey control reference frame and local coordinates (in that frame) of the feed arm rangefinders' scan reference points and the triplet target prism reference points. One gets, thereby, a set of adjusted ranges and local coordinates for those rangefinders' scan points and all of the triplet prism fiducial points. By simple computations, the local coordinates of all of the triplet ball target reference points are then calculated, together with the ranges of the ball target reference points to the feed arm ranger scan points.

3. Ground-reference-frame coordinates of the feed arm rangers' scan reference points are then calculated from the already-determined ball target ground frame coordinates and their range distances to the feed arm ranger scan points.

1.2. Calibration Of The Target Units.

The target units, as fabricated, depart from their ideal design geometry. The ball target center point is not perfectly co-linear with the prism reference points and does not lie exactly at equal distances from them. The three target reference points determine a plane which is not exactly that of the upper surface of the top plate. Such deviations from the ideal design introduce small corrections in the computation of ball target reference point ranges to rangefinder scan points, as derived from adjusted ranges between triplet prism targets and these scan points. By calibrating each triplet unit in the laboratory, a correction constant will be found which will largely compensate for deviations of the as-built units from their ideal geometry.

The calibration procedure consists of three types of measurement:

1. Mechanical measurements are made to locate the position of the three target reference points with respect to a synthetic reference plane which is locally very close to and effectively (for metrology purposes) parallel to the upper top plate surface. Physically, this plane is a plate surface of the triplet calibration fixture. (This will be explained in the discussion to follow). The mechanical measurements are made using a granite surface plate, electronic displacement indicators and gauge blocks. The positions of the ball target center and the prism reference point midpoint are determined with respect to this synthetic plane.

2. Measurements of range distance are made from a rangefinder to the triplet prisms. During these measurements the triplet unit is positioned by angle block limit stops on the rear and side (Fig 1A) so that the synthetic reference plane is vertical, the rangefinder illuminating beam is horizontal, and the line joining the prism fiducial reference points is horizontal. The latter positioning is accomplished by levelling the upper long side plate of the triplet unit. The triplet is positioned so that its top plate is perpendicular to the rangefinder illuminating beam; this is accomplished by clamping a plane mirror to the upper surface of the top plate and rotating the triplet about a vertical axis until the rangefinder beam is in autocollimation with the mirror and reflects back on itself. The limit stop blocks are then locked in position to provide a kinematic mount for the triplet unit (Fig. 1A).

3. Range measurements are made from the rangefinder to the ball target of the triplet unit. During the latter measurements the triplet unit will have been rotated about a horizontal axis (the common line of target centers) so the reference plane is flipped 180° but remains coincident with its former position during the prism measurements, and the ball target has not moved laterally or vertically upwards. The triplet unit was designed so that the line of target centers lies mid-

way between the long side-plates of the unit, so that by flipping the unit 180° and resting it on its lower long side plate and pushing it against its limit stops the plate will reverse its orientation with respect to the rangefinder, but will leave the transverse position of the ball target reference point fixed. (That is, the ball target's reference point might move towards or away from the scan point when the unit is flipped, but it's displacement perpendicular to the rangefinder beam will be small and cause negligible error in range).

4. Range measurements are repeated several times, and the unit is flipped repeatedly, to obtain statistics for the calibration ranging precision and standard errors on the measurements.

5. Gauge block spacers, or other precision spacers of known thickness are placed in front of the rear limit stops to decrease the distance of the triplet unit from the rangefinder by a known increment. The ranges are remeasured. This will give independent redeterminations of the calibration constant.

6. The range measurements are analyzed using the analysis provided here. The best distance, H_{corr}, from the midpoint MP of the line segment joining the prism target reference points to the ball target center BC (as determined by reducing and adjusting the range calibration measurements) is taken to be the calibration constant of the triplet unit. For practical purposes, the vector from MP to BC is effectively perpendicular to the synthetic reference plane, to the upper surface plane of the triplet top plate, and also to the line of centers of the prism reference points, for the purpose of making range corrections to the triplet metrology range measurements on the GBT telescope.

The distance from MP to BC is also measured mechanically by the surface plate measurements. The mechanically determined value, which we denote by L_{MPBC}, provides a check on the accuracy of the rangefinder calibration measurements. The calibration is considered successful, and the triplet unit is considered to have met its mechanical fabrication specifications only if $|H_{corr} - L_{MPBC}| \leq 0.15\,\text{mm}$.

1.3. Description Of The Triplet Geometry.

The triplet, as designed ideally, consists of the following elements:

1. A flat (3/4" thick) top surface plate.

2. Two cups, each holding a cube corner prism retroreflector, of BK7 optical glass. The cups are recessed into the top plate and are pinned, so that the prism ray entry face is parallel to the top plate and the fiducial reference point of each prism lies in the plane of the top plate. The prisms both face outwards from the top plate's upper surface. The cube corner diagonals of the prism are parallel. The prism refernce points are 25.000" (635.00 mm) apart. (Fig. 2). The prisms are designated as "left" and "right." The left prism has an odd serial number and mounts on the GBT to the left when viewed from the GBT dish center; the right prism has an even serial number and mounts to the right when viewed from the dish center. The prisms are Edmund Scientific trihedral prisms, stock no.43299: annealed uncoated BK7 glass, dia. 50.80 (+0 / -0.1) mm, height 38.10 ± 0.25 mm, deviation $180° \pm 3$ arc-second.

3. A cat's-eye retroreflector. Optically it may be considered to consist of two concentric half-balls meeting at their boundary disks. The hemisphere surface of the smaller half-ball is the reflector's ray entry surface. The hemisphere surface of the larger half-ball is metal coated and is reflecting. This reflector is positioned to lie on a spacing ring so that the common half-ball meeting plane is parallel to the plane of the top plate. The small half-ball faces downwards through the triplet box, through an aperture in the top plate. The spacing ring is trimmed in thickness for each triplet unit, so that the small half-ball's center point, BC, lies in the plane of the top plate, and is centered to lie on the line segment between the two prism fiducial reference points, and lie at the midpoint, MP, of this line segment. (Fig. 2).

4. A box structure bolted to the top plate, consisting of side and end plates, and split bottom plates, assembled to form a rectangular box. The side and end plates stiffen the top plate. The planes of the long side plates are designed to lie parallel to the line of optical reference points, which lies midway between them (Fig. 2). Steel trunnions are set into the end plates to provide a rotary attachment to a triplet unit mount support assembly.

5. A mount assembly to join the triplet reflector unit to the telescope. The mount assembly provides the triplet unit with two degrees of freedom in rotation, to allow the triplet to be mounted on the telescope and be oriented so that the outward normal to the top plate's plane points approximately to the centroid of the set of feed arm rangefinder scan points. A fixture holding a rifle optical sight is available to aim and orient the triplet unit on the GBT.

6. Two half-fixtures, which can bolt onto the triplet unit, are used to position the triplet during laboratory-calibration measurements. (Fig's. 5, 6). Each half consists of a flat 0.500"-thick plate, two 0.500" thick cylindrical spacers, and four 3.000" long posts. The plate, spacers and posts bolt together so that a pair consisting of a spacer and a post extends from one side of the plate, while a pair of posts extends from the other side of the plate. This half-fixture assembly has the property that when it rests on a granite surface plate, the fixture's plane surface adjacent to the cylindrical spacers will lie 3.500" from the surface plate, regardless of whether the half-fixture faces up or faces down. The corresponding plane surfaces of the two half-fixtures (adjacent to the cylindrical spacers) form a virtual reference plane when they rest on a flat plane surface, for example - a granite surface plate or limit stop blocks arranged to have coincident stop planes. The synthetic reference surface lies 3.500" from the physical surface upon which the half-fixtures rest. The half-fixture bolts to the triplet unit so that this plate face is against the upper surface of the unit's top plate. The fixture geometry is shown in Fig.'s 2, 5 and 6.

The triplet's upper top plate surface lies close to the half-fixture's synthetic reference plane but does not coincide with it because the top surface is painted (to cut down heating and thermal expansion due to solar irradiation when on the GBT). The metrology calibration measurements are independent of the thickness of paint on the triplet, because all distances are referred to the fixture's synthetic reference plane and not to the painted surface of the triplet unit.

2. Calibration Procedure.

2.1. Required Equipment.

01. Large granite surface plate.
02. Electronic surface height gauge.
03. Gauge blocks.
04. Small flat mirror for autocollimation.
05. Laser rangefinder.
06. Three limit stops and a flat plate.
07. Pair of triplet unit calibration half-fixtures.
08. Rangefinder 18 meter test range.
09. Calibration Spreadsheet File.
10. Data sheet containing geometric information for triplet unit targets.
11. Calibration range, with length $\geq$ 18 meter.
12. Infrared laser beam viewer.
13. Data sheet containing serial numbers and target geometry data for the triplet unit.

2.2. Surface Plate Mechanical Measurements.

Calibration of the triplet units is done in the laboratory prior to installation on the GBT. The calibration half-fixtures are bolted to the triplet unit as in Fig. 2, and the fixtured unit is placed on the granite surface plate. The prisms face upwards, away from the surface plate, and the ball target faces downwards towards the surface plate. Using electronic dial indicators (Mitutoyo) and gauge blocks, the distances of the prism entry faces and the ball vertex from the surface plate are measured. (Optionally, the distance of the painted upper top plate surface to the surface plate may be sampled at several locations, to gain information about the paint layer thickness on the unit).

The distances L_1, L_2, and L_{BT} of the prism pole points and ball target vertex from the surface plate are measured.

A spreadsheet template file for the triplet unit under test is generated. The triplet unit's data-sheet information and the above mechanically-measured distances are entered into the spreadsheet. Prism heights $H_{rp}(T_1)$ and $H_{rp}(T_2)$ and glass correction constants $Pc(T_1)$ and $Pc(T_2)$, are given in the triplet unit data-sheet. The spreadsheet will compute L_{MPBC}.

2.3. Triplet Calibration Rangefinder Measurements.

The three limit-stop blocks are set in position to satisfy the following criteria. The two back-stop blocks are set with their surfaces co-planar and vertical, using a flat plate as a fixture. The rangefinder is turned on and the rangefinder beam is set so it is horizontal and points along the range axis. Using the small mirror flat in autocollimation against the flat plate, the plane of the back stops is set $\perp$ to the incoming rangefinder beam so the beam returns along itself and is $\perp$ to the plate and the plane of the back stops.

The side stop is positioned so its plane is vertical and $\perp$ to the plane of the back stops. The perpendicular distance from the scan point of the rangefinder *to the plane of the backstops* is measured (either mechanically or optically, using the rangefinder together with a hollow cube corner retroreflector). *This distance, minus the calibration fixture half-height* (which is $L_{post} = 88.90$ mm) is entered into the spreadsheet as the distance D_{TOPPL}.

Remove the handles from the triplet unit and push the unit against the back stops, with prisms facing towards the rangefinder. Level the unit so the long uppermost side plate is levelled and the line of target centers is horizontal. Using the infrared laser beam viewer, observe the landing height of the rangefinder beam on the triplet unit, and mechanically measure the beam landing height vertically above the horizontal line of target centers. Enter this height as h_S into the spreadsheet.

Measure the range from the rangefinder to each of the triplet target prisms and to the rangefinder reference prism "*ZRG*." Each measurement gives three range phases (in radians) as data: Φ_1, Φ_2, Φ_{ZRG}. Air temperature, pressure and % relative humidity are recorded, and the rangefinder's index of refraction program is used to compute the group index of air η_A and the modulation air-half-wavelength $HWLA$ for the ambient measurement conditions. This data is entered in the spreadsheet.

As part of the rangefinder instrument calibration an instrument offset-phase reading to adjust to zero-range distance, Φ_{Cal} , has been previously measured and recorded in the instrument's database. This instrument calibration offset-phase is also entered in the spreadsheet.

The triplet unit is then flipped so the ball target faces the rangefinder. The measured range phase Φ_{BCfp} is entered in the spreadsheet, together with ambient temperature, pressure, relative humidity, and computed air modulation-half wavelength, $HWLA$.

3. Data Analysis.

3.1. Analysis Of Surface Plate Measurements.

The first objective in reducing the calibration measurements is to find the distance, L_{MPBC}, of the ball center BC above the prism fiducials' midpoint MP, measured perpendicular to the synthetic reference plane. This is the geometrically-determined correction distance. This distance is obtained directly from the gauge block and electronic-distance-gauge measurements when the fixtured triplet unit is on the granite surface plate. The ball center BC is the fiducial reference point of the cat's-eye retroreflector.

Call the measured distances of prism pole points PP_1 and PP_2 above the surface plate L_1 and L_2, respectively (Fig. 4). The heights, $L(T_1)$ and $L(T_2)$, of prism fiducial reference points T_1 and T_2 above the surface plate are, respectively, $L_1 - H_{rp}(T_1)$ and $L_2 - H_{rp}(T_2)$. Prism constants $H_{rp}(T_1)$ and $H_{rp}(T_2)$ are supplied in the triplet unit's data-sheet. The distance L_{MP} of the prism fiducials' centroid MP is the average of $L(T_1)$ and $L(T_2)$.

At the same time, the distance from the ball vertex point from the surface plate is measured, L_{BT} , using gauge blocks and height indicator. The ball center's distance from the surface plate, L_{BC} , is then $L_{BC} = L_{BT} + R_1$, where the radius R_1 of the ball is provided in the triplet unit's data-sheet. The mechanically determined distance of ball center point BC above midpoint MP is then

(3.1) $\quad L_{MPBC} = L_{BC} - L_{MP}$.

During these measurements the synthetic reference plane is parallel to that of the surface plate, and lies at the distance $L_{post} = 88.900\,\mathrm{mm}$ above the surface plate. The prism fiducial points T_1 and T_2 are designed to lie at the upper plane surface of the triplet's top plate. The spacing between the synthetic reference plane and the upper top plate surface is generated by the paint layer, of thickness t_{paint} on the top plate. This thickness is estimated to be in the range $0.05\,\mathrm{mm} < t_{paint} < 0.15\,\mathrm{mm}$. The distance of T_1 to the top plate's upper surface is $L(T_1) - L_{post} + t_{paint}$. The distance of T_2 to this surface is $L(T_2) - L_{post} + t_{paint}$. The distance of BC to this surface is $L_{BC} - L_{post} + t_{paint}$. These distances give bounds for the departure of the as-built triplet unit from the ideal design.

3.2. Analysis Of Prism Range Measurements.

The rangefinder instrument was designed so that the instrument's output phase reading is modular, modulo the vacuum wavelength $\frac{c}{f_{\mathrm{mod}}} = 2 * HWLV$ in the round-trip group optical path when ranging to a target. The instrument design is such that the output range phase reading decreases as target distance increases (when the target distance increase is sufficiently small that no 2π phase jump occurs). The round-trip optical path length is the sum of the round trip optical path length from the rangefinder scan point S to the target and back to S and common-mode one-way propagation paths. The common-mode paths are: an outgoing path from the rangefinder laser diode face and through the steering mirrors and nose prism to the scan point, and a return path from the scan point S through the large focusing lens to the detector diode, and inaccessible electronic delay paths within the rangefinder's circuitry. The common-mode paths within the circuitry and between the laser diode and scan point and between the detector diode and scan point may be removed, except for a fixed offset phase, by ranging to a comparison reference target prism (ZRG) and using the difference of the target phase and the sum of, the ZRG-reference-phase plus the offset-phase, Φ_{Cal} , as the round-trip propagation phase.

When one measures the range completely in air (of group index η_A and corresponding modulation wavelength λ_A), from the rangefinder scan point S, to the corner point of a *hollow* cube corner retroreflector prism whose corner point is at a geometric distance D from S ,one has the following basic instrumental relation:

$$(3.2) \quad D = (N) \cdot \left(\frac{\lambda_A}{2}\right) - \left(\frac{\Phi_{\text{hollow\_target}} - \Phi_{ZRG} - \Phi_{Cal}}{2\pi}\right)\left(\frac{\lambda_A}{2}\right).$$

We can rewrite this as

$$(3.3) \qquad \frac{D}{\left(\frac{\lambda_A}{2}\right)} - N + \left(\frac{\Phi_{\text{hollow\_target}} - \Phi_{ZRG}}{2\pi}\right) = \Phi_{Cal} \ ,$$

subject to the constraints that:

$$(3.4a) \qquad 0 < \Phi_{\text{hollow\_target}} \ , \ \Phi_{ZRG} \ , \ \Phi_{Cal} \ < 2\pi \, .$$

$$(3.4b) \qquad 0 < \Phi_{ZRG} \ + \Phi_{Cal} \ < 2\pi \, .$$

The first constraint comes from the fact that the instrument readout phases are always non-negative. The second is an empirical observation. They lead to the inequality

$$(3.5) \qquad -1 \ < \ \left(\frac{\Phi_{\text{hollow\_target}} - \Phi_{ZRG} - \Phi_{Cal}}{2\pi}\right) \ < \ 1 \, .$$

By measuring the hollow cube reflector's phase versus its corner-point distance from S, and also measuring the comparison prism phase,the offset-phase Φ_{Cal} (which is a constant of the rangefinder instrument) may be determined. The meaning of the integer N is seen by rewriting (3.2) as

$$(3.6) \qquad \frac{D}{\left(\frac{\lambda_A}{2}\right)} = N + \left(\frac{\Phi_{ZRG} + \Phi_{Cal} - \Phi_{\text{hollow\_target}}}{2\pi}\right) \, .$$

Taking the integer part of both sides of the equation, we get

$$(3.7) \qquad INT\left\{\frac{D}{\left(\frac{\lambda_A}{2}\right)}\right\} = INT\left\{N + \left(\frac{\Phi_{ZRG} + \Phi_{Cal} - \Phi_{\text{hollow\_target}}}{2\pi}\right)\right\} \, .$$

We now use the following feature of modular counting. If N is a positive integer and r is a real number in the range $-1 < r < 1$ it is true that

$$\begin{aligned} INT(N+r) &= N \quad (0 \leq r < 1), \\ INT(N+r) &= N-1 \quad (-1 < r < 0). \end{aligned} \tag{3.8}$$

We then have, for the case of a hollow cube corner retroreflector,

$$N = INT\left\{\frac{D}{\left(\frac{\lambda_A}{2}\right)}\right\}, \qquad (\Phi_{ZRG} + \Phi_{Cal} - \Phi_{\text{hollow\_target}} \geq 0), \tag{3.7a}$$

$$N = 1 + INT\left\{\frac{D}{\left(\frac{\lambda_A}{2}\right)}\right\}, \qquad (\Phi_{ZRG} + \Phi_{Cal} - \Phi_{\text{hollow\_target}} < 0). \tag{3.7b}$$

The sum over phases on the right side is directly available from the measured phases. We note that, as functions of D: $\Phi_{\text{hollow\_target}}(D)$ is a sawtooth function (negative slope serrodyne), while $N(D)$ is an ascending staircase.

Now assume that a cube corner retroreflector prism of BK7-glass, of height D_1, is substituted for the hollow retroreflector so it is at normal incidence and its fiducial reference point T_1 is set at the same distance, D, from scan point S. The one-way optical group path length in the prism itself is less than $\frac{\lambda_A}{2}$. The instrumental rangefinder phase output for the glass prism will be either: that of the hollow target decremented by the additional one-way optical path length introduced by the prism substitution multiplied by $2\pi \cdot \left(\frac{\lambda_A}{2}\right)^{-1}$, or will be this quantity plus an integer multiple of 2π. The prism reference point T_1 lies at a distance $H_{rp}(T_1)$ from the pole point of the prism, and distance $(D_1 - H_{rp}(T_1)$ from the corner-point of the prism. Substitution of the hollow corner by the solid prism increases the optical path by

$$\Delta(\text{Optical Path}) = [(\eta_{gtoup})_{\text{BK7\_glass}} - \eta_A]\cdot D_1 + (D_1 - H_{rp}(T_1)) \cdot \eta_A. \tag{3.8}$$

The terms on the right side are respectively the optical path increase caused by replacing air by optically denser glass over the length D_1, and also increasing the air path by moving back the retroreflector corner. If the prism reference point is chosen to be the optical nodal point of the prism, measured target range will be minimally sensitive to small departures of the prism orientation from normal incidence; this is done by choosing

$$(3.9) \qquad H_{rp}(T_1) = \left(\frac{\eta_A}{(\eta_{gtoup})_{\mathrm{BK7\_glass}}} \right) \cdot \mathrm{D}_1 \; = \frac{\mathbf{D}_1}{n} \; .$$

With this choice of H_{rp} the optical path increment (3.8) reduces to

$$(3.10) \quad \Delta(\text{Optical Path }) \; = \; \left(n - \frac{1}{n} \right) \cdot \mathrm{D}_1 \cdot \eta_A \; = \; (-Pc(T_1)\,).\eta_A$$

The rangefinder distance equation then becomes, for the case of a cube corner prism at normal incidence, whose fiducial point is at distance D from the rangefinder scan point S,

$$(3.11) \quad D + \left(n - \frac{1}{n} \right) \cdot \mathbf{D}_1 = (N_1) \cdot \left(\frac{\lambda_A}{2} \right) - \left(\frac{\Phi_1 - \Phi_{ZRG} - \Phi_{Cal}}{2\pi} \right) \cdot \left(\frac{\lambda_A}{2} \right) .$$

For reasons similar to our earlier case,

$$(3.12) \qquad -1 \; < \; \Phi_{ZRG} + \Phi_{Cal} - \Phi_{\mathrm{hollow\_target}} \; < \; 1 \; .$$

This gives, as before:

$$(3.13a) \qquad N_1 = INT \left\{ \frac{D - Pc(T_1)}{\left(\frac{\lambda_A}{2} \right)} \right\} \; , \qquad (\Phi_{ZRG} + \Phi_{Cal} - \Phi_1 \geq 0) \, ,$$

$$(3.13b) \quad N_1 = 1 + INT \left\{ \frac{D - Pc(T_1)}{\left(\frac{\lambda_A}{2} \right)} \right\} \; , \qquad (\Phi_{ZRG} + \Phi_{Cal} - \Phi_1 < 0)..$$

A corresponding result holds for prism 2; for our calibration geometry we find $N_1 = N_2$.

Under the conditions of the calibration the rangefinder beam is near $\perp$ to the top plate's upper plane surface (while ranging to each prism) and the prism fiducial points are very close to this plane. For this reason the $\perp$ distance from S to this plane, D_{TOPPL}, may be used in (3.13) to compute N_1 and N_2, in place of distances D_1 and D_2.

For calibration measurements on the left prism (prism 1) we have $D = D_1$. For computations it is more transparent to rewrite (3.11) as

$$(3.11a) \quad D_1 = \left[INT \left\{ \frac{D_{TOPPL} - Pc(T_1)}{\left(\frac{\lambda_A}{2}\right)} \right\} + \frac{1 - sgn(\Phi_{ZRG} + \Phi_{Cal} - \Phi_1)}{2} \right] \cdot \left(\frac{\lambda_A}{2}\right)$$

$$- \left(\frac{\Phi_1 - \Phi_{ZRG} - \Phi_{Cal}}{2\pi}\right) \cdot \left(\frac{\lambda_A}{2}\right) + Pc(T_1) \, .$$

Corresponding results hold for the ball target. The glass correction term for this target, whose fiducial reference point is the small ball's center point, is

$$(3.14) \quad Pc_B = (-1) \cdot (n \cdot (R_1 + R_2) - R_1) \, .$$

The range equation for the ball target, with the triplet unit flipped is

$$(3.15) \quad D_{BCfp} = (N_{BC}) \cdot \left(\frac{\lambda_A}{2}\right) - \left(\frac{\Phi_{BCfp} - \Phi_{ZRG} - \Phi_{Cal}}{2\pi}\right) \left(\frac{\lambda_A}{2}\right) + Pc_B \, , \quad \text{where}$$

$$(3.16a) \qquad N_{BC} = INT \left\{ \frac{D_{TOPPL} - Pc_B}{\left(\frac{\lambda_A}{2}\right)} \right\} \, , \quad (\Phi_{ZRG} + \Phi_{Cal} - \Phi_{BCfp} \geq 0) \, ,$$

$$(3.16b) \qquad N_{BC} = 1 + INT \left\{ \frac{D_{TOPPL} - Pc_B}{\left(\frac{\lambda_A}{2}\right)} \right\} \, , \quad (\Phi_{ZRG} + \Phi_{Cal} - \Phi_{BCfp} < 0).$$

Here D_{BCfp} is the distance from rangefinder scan point S to the ball center.

For transparency, we may rewrite (3.15) in the form

(3.15b)

$$D_{BCfp} = \left[INT \left\{ \frac{D_{TOPPL} - Pc_B}{\left(\frac{\lambda_A}{2}\right)} \right\} + \frac{1 - sgn(\Phi_{ZRG} + \Phi_{Cal} - \Phi_{BCfp})}{2} \right] \cdot \left(\frac{\lambda_A}{2}\right)$$

$$- \left(\frac{\Phi_{BCfp} - \Phi_{ZRG} - \Phi_{Cal}}{2\pi}\right) \cdot \left(\frac{\lambda_A}{2}\right) + Pc_B \,.$$

Range equation (3.11a) is not fully general. If the prism is not at normal incidence there is a small differential range correction $\delta Pc(\mathfrak{I}_1)$which is a function of the illumination incidence angle $\mathfrak{I}_1$. The correction is discussed in Appendix B. For calibration, that additional correction may be neglected. However, for completeness the range equation becomes, taking into account the correction for non-normal incidence,

(3.16)

$$D_1 = \left[INT \left\{ \frac{D_{TOPPL} - Pc(T_1) - \delta Pc(\mathfrak{I}_1)}{\left(\frac{\lambda_A}{2}\right)} \right\} + \frac{1 - sgn(\Phi_{ZRG} + \Phi_{Cal} - \Phi_1)}{2} \right] \cdot \left(\frac{\lambda_A}{2}\right)$$

$$- \left(\frac{\Phi_1 - \Phi_{ZRG} - \Phi_{Cal}}{2\pi}\right) \cdot \left(\frac{\lambda_A}{2}\right) + Pc(T_1) + \delta Pc(\mathfrak{I}_1) \,.$$

We next calculate the distance D_{MP} from scan point S to the midpoint MP of the prism fiducials, using the formula for the length of a triangle median in terms of its side lengths (Fig. 7).

$$(3.17) \qquad D_{MP} = \sqrt{\frac{D_1^2 + D_2^2}{2} - \left(\frac{D_{12}}{2}\right)^2} \,.$$

Define

$$(3.18) \quad \Delta = N_1 \left(\frac{\lambda_A}{2}\right) , \quad \delta_1 = D_1 - \Delta \,, \quad \delta_2 = D_2 - \Delta \,, \quad \delta_{MP} = D_{MP} - \Delta \,.$$

Substituting (3.18) into 3.17) gives

$$(3.19)\qquad \delta_{MP} = -\Delta + \Delta \cdot \sqrt{1 + \left(\frac{\delta_1 + \delta_2}{2}\right) + \left(\frac{2\delta_1^2 + 2\delta_2^2 - D_{12}^2}{4\Delta^2}\right)} .$$

Using the series expansion

$$(3.20)\qquad \sqrt{1+\epsilon} = 1 + \left(\frac{\epsilon}{2}\right) - \left(\frac{\epsilon^2}{8}\right) + O(\epsilon^3) \qquad (0 < \varepsilon << 1) ,$$

and noting that δ_1 , δ_2 are $\leq \frac{1}{180}$ we get, to μm computational accuracy,

$$(3.19a)\qquad \delta_{MP} = \left(\frac{\delta_1 + \delta_2}{2}\right) + \frac{(\delta_1 - \delta_2)^2}{8\Delta} - \frac{D_{12}^2}{8\Delta} , \quad \text{and}$$

$$(3.19b)\qquad D_{MP} = N_1 \cdot \left(\frac{\lambda_A}{2}\right) + \left(\frac{\delta_1 + \delta_2}{2}\right) + \frac{(\delta_1 - \delta_2)^2}{8\Delta} - \frac{D_{12}^2}{8\Delta} .$$

Rangefinder scan point S is offset laterally and vertically from MP. The vertical offset, h_S, was measured mechanically; (only moderate accuracy is needed). Length D_{TOPPL} of the $\perp$ from S to the top plate's upper plane was also measured. The $\perp$ distance- D_0 from S to line segment T_1T_2 is

$$(3.20)\qquad D_0 = \sqrt{D_{TOPPL}^2 + h_S^2} .$$

The incidence angles $\Im_1$ and $\Im_2$ of the rangefinder beam with the prism entry faces is calculated as follows. Assume that the $\perp$ from S to T_1T_2 divides T_1T_2 into pieces of length $\alpha \cdot D_{12}$ and $(1-\alpha) \cdot D_{12}$ respectively (Fig. 9). The Pythagorean theorem gives

$$(3.21)\qquad D_1^2 = D_0^2 + (\alpha \cdot D_{12})^2 , \quad D_2^2 = D_0^2 + (1-\alpha)^2 \cdot (D_{12})^2 , \quad \text{giving}$$

$$(3.22)\qquad \alpha = 0.5 + \frac{D_1^2 - {D_2}^2}{2 \cdot D_{12}^2} \simeq 0.5 + \frac{D_0 \cdot (\delta_1 - \delta_2)}{D_{12}^2} .$$

The angles of incidence are then

$$(3.23)\qquad \Im_1 = \arccos\left(\frac{D_0}{D_1}\right) \quad \text{and} \quad \Im_2 = \arccos\left(\frac{D_0}{D_2}\right) .$$

The correction constant to be found by ranging is

$$(3.24) \qquad H_{corr} = \text{Distance}(MP_{fp}\,,\, BC_{fp}).$$

It is found as follows. Let Ψ be the angle between the line segments S-MP and MP-BC_{fp} (Fig. 8). It is easily found that

$$(3.25) \qquad \Psi = \arcsin\left[\frac{(0.5-\alpha)\cdot D_{12}}{D_0}\right].$$

The distance between the position of the fiducial point T_1 during the prism range measurements, and its position $(T_1)_{fp}$ during the ball target range measurement is easily seen to be (Fig. 8):

$$(3.26) \qquad \text{Distance}(T_1\,,\,(T_1)_{fp}) = 2\cdot L_{post} - 2\cdot L(T_1)\,. \qquad \text{Similarly,}$$

$$(3.26b) \qquad \text{Distance}(T_2\,,\,(T_2)_{fp}) = 2\cdot L_{post} - 2\cdot L(T_2)\,. \qquad \text{giving,}$$

$$(3.26c) \qquad \text{Distance}(MP\,,\,MP_{fp}) = 2\cdot L_{post} - 2\cdot L_{MP}\,.$$

Call δ_{dsp} the distance from BC_{fp} to MP, as expected from the surface plate measurements. This distance is

$$(3.27) \qquad \delta_{dsp} = 2\cdot L_{post} - 2\cdot L_{MP} - L_{MPBC}\,.$$

This distance is appreciably less than 1 mm and the angle between the rays from S to MP and to BC_{fp} is small, and the value of the distance from BC_{fp} to MP, as derived from the laser ranging measurements is then

$$(3.28) \qquad \text{Distance}\ (MP\,,\, Bc_{fp})_{\text{ranging}} = (\delta_{MP} - \delta_{BCfp})\cdot \sec\Psi \equiv \delta_{range}\,.$$

The distance from MP_{fp} *to* BC_{fp} obtained by laser ranging is then

$$(3.29) \qquad H_{corr} \; = \; 2\cdot L_{post} - 2\cdot L_{MP} - \delta_{range}\,.$$

The sign of H_{corr} is the same as the sign of the right hand side of (3.29)

This completes the computation of the range correction constant. It is expected that the value H_{corr} obtained from the ranging measurements will be close to the value of L_{MPBC} obtained from surface plate measurements. It is desirable that the ranging calibration measurements be repeated for several spacings between the triplet unit and the rangefinder, to check the accuracy, self-consistency and thereby the validity of the calibration procedure.

3.3. Calibration Spreadsheet And Data Reduction.

A computational spreadsheet template TRPCAL_n.WQ2 was generated in Quattro Pro to simplify data entry and reduction for triplet unit calibrations. An example of the spreadsheet output is included with this memo. Calibration computations are performed in this spreadsheet. The values of H_{corr} and L_{MPBC} are reported in the spreadsheet printout as part of the calibration report, which includes printout of intermediate variables involved in the computations.

3.4. Notation: List of Symbols.

Symbol	Description	Design Value
T_1	Fiducial reference point of left prism.	
T_2	Fiducial reference point of right prism.	
MP	Midpoint of line segment T_1-T_2.	
BC	Center point of cat's-eye target's small ball.	
PP_1 , PP_2	Pole point of prism. (Foot of $\perp$ from corner point to opposite face).	
$\mathbf{D_1}$	Height of left prism. (Corner to pole point).	($\simeq$ 38.4 mm)
$\mathbf{D_2}$	Height of right prism.	($\simeq$ 38.4 mm)
R_1	Radius of small ball of cat's-eye reflector.	1.969"
R_2	Radius of large ball-cap of cat's-eye reflector.	3.799"
D_{12}	Distance from T_1 to T_2.	635.00 mm
Pc_B	Range correction constant for glass path in cat's-eye (ball) target.	($\simeq$ −173.6 mm)
$Pc(T_1)$, $Pc(T_2)$	Prism range correction constant for glass path in prism, at normal incidence.	($\simeq$ −33.5 mm)
$\mathfrak{I}_1$, $\mathfrak{I}_2$	Incidence angle of illumination onto prism.	
$\delta Pc(\mathfrak{I}_1)$ $\delta Pc(\mathfrak{I}_2)$	Differential range correction for non-normal illumination on prism target.	small

Surface Plate Measurements - Measured Parameters

Parameter	Description	Design Value
$L(T_1)$	Distance of T_1 to granite surface plate when prisms face away from plate.	
$L(T_2)$	Distance of T_2 to granite surface plate when prisms face away from plate.	
L_{MP}	$\frac{L(T_1) + L(T_2)}{2}$	
L_{BT}	Closest distance of small half-ball to granite surface plate, when facing plate.	
L_{BC}	Distance of BC to granite surface plate when prisms face away from plate.	
L_{MPBC}	$L_{BC} - L_{MP}$ Distance from ball center to MP.	($\simeq 0.0\,\text{mm}$)
L_{post}	Half-length of calibration half-fixture.	88.900 mm
D_{TOPPL}	Horizontal component of ($\perp$) distance from S to top plate of triplet during calibration, prisms towards rangefinder.	
δ_{dsp}	$2 \cdot L_{post} - 2 \cdot L_{MP} - L_{MPBC}$	

Symbol	Description	Design Value
$HWLV$ ($= \lambda_0/2$)	Rangefinder vacuum modulation half-wavelength ($= c/2 * f_{\mathbf{mod}}$)	99.930819 mm
T	Air temperature during calibration.	$20^\circ\, C$
P	Air pressure during calibration.	$933\, mb$
$\%\, h$	Relative humidity during calibration.	$50\, \%$
$\eta_A(T, P, \%\, h)$ $= \eta_A$	Group refractive index of air at 0.780 μm wavelength.	$(\eta_A - 1) =$ 2.5324×10^{-4}
η_G	Group refractive index of BK7 glass at 0.780 μm wavelength	1.527463
n	$n = \eta_G \,/\, \eta_A$	1.527077
$HWLA$ ($= \lambda_A/2$)	Rangefinder modulation half-wavelength in air ($= HWLV \,/\, \eta_A$)	
$H_{rp}(T_1) \equiv \dfrac{\mathbf{D_1}}{n}$	Distance from T_1 to PP_1.	($\simeq 25.2\,$mm)
$H_{rp}(T_2) \equiv \dfrac{\mathbf{D_2}}{n}$	Distance from T_2 to PP_2.	($\simeq 25.2\,$mm)

Symbols Related To Range Measurements:

Symbol	Description
S	Rangefinder scan reference point.
D_1	Actual distance from S to T_1 during prism range calibration measurement.
D_2	Actual distance from S to T_2 during prism range calibration measurement.
fp	Subscript denoting flipped position of triplet unit during range measurement (ball target towards rangefinder).
BC_{fp}	Small ball's center point position during ball range calibration measurement.
D_{BCfp}	Actual distance from S to BC_{fp} during ball calibration range measurement.
h_S	Vertical height of scan point S above line T_1-T_2 during calibration range measurements.

Symbol	Description	Design Value
Φ_1	Range measurement phase angle, left prism.	
Φ_2	Range measurement phase angle, right prism.	
Φ_{ZRG}	Range measurement phase angle, rangefinder reference prism.	
Φ_{Cal}	Measured rangefinder zero-distance calibration phase angle.	
Φ_{BCfp}	Range measurement phase angle, ball target.	
N_1	$\mathrm{INT}[\frac{D_{TOPPL} - Pc(T_1)}{HWLA}]$ Integer modulation air half-wavelengths from S to T_1 during calibration.	182
N_2	$\mathrm{INT}[\frac{D_{TOPPL} - Pc(T_2)}{HWLA}]$	182
N_{BC}	$\mathrm{INT}[\frac{D_{TOPPL} - Pc_B}{HWLA}]$ Integer modulation air half-wavelengths from S to BC_{fp} during calibration.	184
Δ_1	$HWLA \cdot N_1$	
Δ_2	$HWLA \cdot N_2$	
Δ_{BC}	$HWLA \cdot N_{BC}$	
Δ	$HWLA \cdot N_1$	

Symbol	Description
D_0	$\sqrt{h_S^2 + D_{TOPPL}^2}$ Length of the $\perp$ from S to line T_1-T_2.
δ_1	$D_1 - \Delta$
δ_2	$D_2 - \Delta$
δ_{MP}	$D_{MP} - \Delta$
δ_{BCfp}	$D_{BCfp} - \Delta$
α	$0.5 + \frac{D_1^2 - D_2^2}{2D_{12}^2}$
$\Im_1$	$\arccos(\frac{D_0}{D_1})$
$\Im_2$	$\arccos(\frac{D_0}{D_2})$
Ψ	$\arcsin\left[\frac{(0.5-\alpha)\cdot D_{12}}{D_0}\right]$
δ_{range}	$(D_{MP} - D_{BCfp}) \cdot \sec\Psi$ $= (\delta_{MP} - \delta_{BCfp}) \cdot \sec\Psi$
H_{corr}	Distance(MP_{fp}, BC_{fp}) = Distance(MP, BC)

It is expected that the value of H_{corr} found from reduction of laser rangefinder measurements will agree with the value of $L_{MPBC} = L_{BC} - L_{MP}$ measured by gauge measurements of the triplet unit while on the granite surface plate.

4. Summary.

A procedure has been presented to obtain a correction constant H_{corr} for each triplet target unit, which is used to reduce measured range from the scan point of a feed arm rangefinder to the ball reflector's center point (which is its fiducial reference point). The equations for making range reductions on the GBT are shown in Fig. 10, and are discussed in Appendix C.

The need for this correction constant is to compensate measured range for the effect of a departure of the ball center point from the midpoint of the prism reference points (which is its design location) in a direction perpendicular to the surface plane of the triplet unit's top plate.

The correction constant is measured in two independent ways. It is obtained by a purely geometric method, by using distance gauges to measure heights of triplet unit features above a granite surface plate. It is also obtained by analysis of three laser range measurements: to the triplet unit prisms, and to the ball reflector after flipping the unit. Self-consistency and accuracy of the calibration is found by making the calibration for several distances between the triplet unit and the rangefinder.
In the appendices to this memo we carry out calculations indicating the effects of departures of an as-built triplet unit from its ideal design geometry. We expect that if the units are built within their assigned fabrication tolerances, the accuracy of the calibration should be within 150 μm.

A spreadsheet was constructed to do computer analysis of the calibration measurements; a sample is included with this memo.

References.

1. D. Wells, *The GBT Precision Pointing System.* Unpublished draft memo. August 5,1992.

2. J. Payne, GBT Memo 84, *Pointing The GBT.* September 16, 1992.

3. NRAO GBT Drawing D35420M072, Feb. 17, 1995.

4. NRAO GBT Drawings D35420A003, D35420A004, Feb. 17,1995.

5. M.A. Goldman, GBT Memo 148. *Ball Retroreflector Optics*, March 1996.

6. M.A. Goldman, GBT Memo 154. *GBT Dish Laser Range Measurement Corrections*, June 1996, pg. 13.

Appendix A. Fabrication Error-Bound Estimates.

In this appendix we study the effects of fabrication tolerances and errors on measured ranges. We introduce synthetic triplet unit geometry errors into the range reduction analysis and give bounds for the distance errors which they produce.

We first find the range change caused by an increment δD_{12} in the spacing of the prism centers (given fixed measured ranges D_1 , D_2 to the scan point). From the triangle median formula:

$$\text{(A.1)} \qquad D_{MP}^2 = \frac{D_1^2}{2} + \frac{D_2^2}{2} - \frac{D_{12}^2}{4}\,.$$

The change in D_{MP} caused by this prism spacing increment is then

$$\text{(A.2)} \qquad \delta D_{MP} = -\frac{D_{12}\cdot\delta D_{12}}{4\,D_{MP}}\,.$$

Construction tolerances give $|\delta D_{12}| < 0.15\,\mathrm{mm}$, which implies, $|\delta D_{MP}| < 1.4\,\mu\mathrm{m}$ for $D_{12} = 635\,\mathrm{mm}$ and $D_{MP} > 18{,}000\,\mathrm{mm}$.

We next consider the range error $D_{BC} - D_{MP}$ when the ball center lies on line T_1-T_2 but is displaced from the midpoint MP. The geometry is shown in Fig. 11. Assume BC to be displaced by distance x from MP, towards T_2. Stewart's Theorem (Fig. 11) gives the length of a line segment drawn from a vertex of a triangle to the opposite side. For vertices S, T_1, T_2 and a line segment drawn from S to point BC on side $\overline{T_1T_2}$ which divides that side in the proportion $\dfrac{\overline{T_1BC}}{\overline{BCT_2}} = \dfrac{p}{1-p}$, Stewart's Theorem gives:

$$\text{(A.3)} \qquad D_{BC} = \sqrt{(1-p)\,D_1^2 + (p)\,D_2^2 - p(1-p)\,D_{12}^2} \qquad \text{and}$$

$$\text{(A.4)} \qquad D_{BC}^2 = \left(\frac{1}{2} - \frac{x}{D_{12}}\right) D_1^2 + \left(\frac{1}{2} + \frac{x}{D_{12}}\right) D_2^2 - \left(\frac{1}{4} - \frac{x^2}{D_{12}^2}\right) D_{12}^2\,.$$

Using (A.1) we get

$$\text{(A.5)} \qquad D_{BC}^2 - D_{MP}^2 = x\left(\frac{D_2^2 - D_1^2}{D_{12}}\right) + x^2 \qquad \text{or}$$

(A.6) $$(D_{BC} - D_{MP}) = \frac{x \cdot (D_2 - D_1) \cdot (D_2 + D_1)}{D_{12} \cdot (D_{BC} + D_{MP})} + \frac{x^2}{(D_{BC} + D_{MP})} \; .$$

Since $(D_2 + D_1) \simeq (D_{BC} + D_{MP})$ and $|x| << (D_{BC} + D_{MP})$ this gives

(A.7) $$D_{BC} - D_{MP} \simeq \frac{x \cdot (D_2 - D_1)}{D_{12}} .$$

During calibration, when the top plate is normal to the illuminating beam, the inequality holds that $|D_2 - D_1| < \frac{D_{12}^2}{2D_0}$. This gives the inequality

(A.8) $$|D_{BC} - D_{MP}| < x \cdot \left(\frac{D_{12}}{2D_0}\right) .$$

For $D_0 > 18,000\,\mathrm{mm}$ and $|x| < 0.15\,\mathrm{mm}$ this gives $|D_{BC} - D_{MP}| < 3\,\mu\mathrm{m}$

We next consider the case that the triplet unit was fabricated so that ball center BC lies in the top plate's upper plane surface but is displaced from MP by a distance x perpendicular to line segment $\overline{T_1T_2}$. Applying Pythagoras' theorem to right triangle S-MP-BC gives the same error bound as the previous case.

The arguments above indicate that the maximum fabrication errors given above in ball center location relative to the prism fiducial midpoint lead to negligible errors in calibration ranging.

Appendix B. Differential Range Correction For Non-zero Angle Of Incidence Onto A Prism Retroreflector.

From reference [6], the differential range correction to account for non-zero incidence angle of illumination onto a cube corner prism is:

(B.1) $$\delta Pc(\Im) = n\,\mathbf{D}\left(1 - \sqrt{1 - (\frac{\sin \Im}{n})^2}\right) - \frac{\mathbf{D}}{n}\,(1 - \cos \Im) \le 0 \; , \quad \text{where}$$

$n = 1.527077$ and $\mathbf{D}$ is the prism reflector's height and $\Im$ is the angle of incidence.

Under calibration conditions, the top plate upper surface is nearly normal to the incidence beam and

$$(\text{B.2})\qquad \tan\mathfrak{I}_1 = \frac{\alpha\, D_{12}}{D_0} \quad , \quad \tan\mathfrak{I}_2 = \frac{(1-\alpha)\, D_{12}}{D_0} \quad \leq \frac{D_{12}}{D_0} \simeq 0.0353\,.$$

Using $\mathbf{D} = 38.1$ mm and $0 < \mathfrak{I}_1, \mathfrak{I}_2 < 0.0353$ radian we find from (B.1) that $\delta Pc(\mathfrak{I}_1)$, $\delta Pc(\mathfrak{I}_2) < 0.1\ \mu$m.

We then see that, for an 18 meter calibration range length, the differential glass correction for non-normal illumination is negligible.

Appendix C. GBT Range Measurements Using Triplet Units.

The method of using the triplet units at the rim of the GBT main dish is illustrated in Fig. 10.

Each unit on the dish rim is aimed towards a common point on the feed arm, A, located approximately at the centroid of the scan points of the feed arm rangefinders. (For this purpose a bulls'-eye or other visible survey target can be situated on the feed arm). After each triplet mount assembly is bolted in place on the telescope rim, but before the unit itself is installed, a rifle scope fixture is substituted for the triplet unit and is aimed to point A, to orient the triplet mount. The mount is then locked in orientation. When the triplet unit is installed into its mount assembly, the prism target axis and normal to the top plate will then point towards A.

Given approximate main-reflector-system coordinates of aim-point A, the ball center BC_j , the prism reference points T_{1j} and T_{2j} of triplet unit j, and the scan point S_k of feed arm rangefinder k, one can compute the angles $\angle A\text{-}T_{1j}\text{-}S_k$, $\angle A\text{-}T_{2j}\text{-}S_k$, and $\angle\ A\text{-}BC_j\text{-}S_k$. (Only low accuracy is needed for the values of these angles, to make the corrections for incidence angle and obliquity of the plate normal to the lines of sight).

Prism target range distances $(D_1)_{j,k}$ and $(D_2)_{j,k}$ for each feed-arm-rangefinder triplet-unit pair (j,k) can be corrected to account for oblique incidence using these angles. The distance $D_{MP_j\,S_k}$ from the rangefinder scan point S_k to the prism fiducial midpoint MP_j is then computed using the formula for the length of the median to a side of a triangle, or its series expansion. The distance $D_{BC_j\,S_k}$ from

S_k to BC_j is then gotten by subtracting the correction term $(\hat{e}_{BC_j S_k} \cdot \hat{e}_{TRj}) \cdot H_{corr}$ from $D_{MP_j S_k}$, where $\hat{e}_{BC_j S_k}$ is the unit vector from BC_j to S_k and $\hat{e}_{TRj}$ is the unit vector from BC_j to A.

Appendix D. Effects Of Refractivity Shift On Range Calibration.

In the range reduction computations, the group index ratio

$$\text{(D.1)} \qquad n \equiv \frac{(n_{group})_{BK7\_glass}}{(n_{group})_{Air}}$$

(for illumination corresponding to 0.780 μm vacuum wavelength) was assumed to have numerical value $n = 1.527077$. This comes from group indices

$$\text{(D.2)} \quad (n_{group})_{BK7\_glass} = 1.527463 \qquad \text{and}$$
$$\text{(D.3)} \quad (n_{group})_{Air} = 1.00025324\,, \quad \text{at } T_0 = 20.5^\circ\ C,\ P_0 = 933\,\text{mb},\ \%h = 50\%\ .$$

The partial derivatives of air's group refractivity are

$$\text{(D.4)} \qquad \frac{\partial \eta_A}{\partial T} = -0.985 \times 10^{-6}\,(^\circ C)^{-1},\ \frac{\partial \eta_A}{\partial P} = 0.272 \times 10^{-6}\,(\text{mb})^{-1},$$
$$\frac{\partial \eta_A}{\partial \%h} = -0.0096 \times 10^{-6}\,(\%h)^{-1}\,.$$

For moderate changes of atmospheric variables the group refractivity change of air is

$$\text{(D.5)} \qquad \Delta \eta_A = \frac{\partial \eta_A}{\partial T}\Delta T + \frac{\partial \eta_A}{\partial P}\Delta P + \frac{\partial \eta_A}{\partial \%h}\Delta \%h\,.$$

When reducing measured range phases the values of HWLA used should correspond to the actual atmospheric conditions during calibration. For the prism glass correction term Pc, the tabulated value can be used, because the change in optical path due to change in n is insignificant for a glass path of a few cm length.

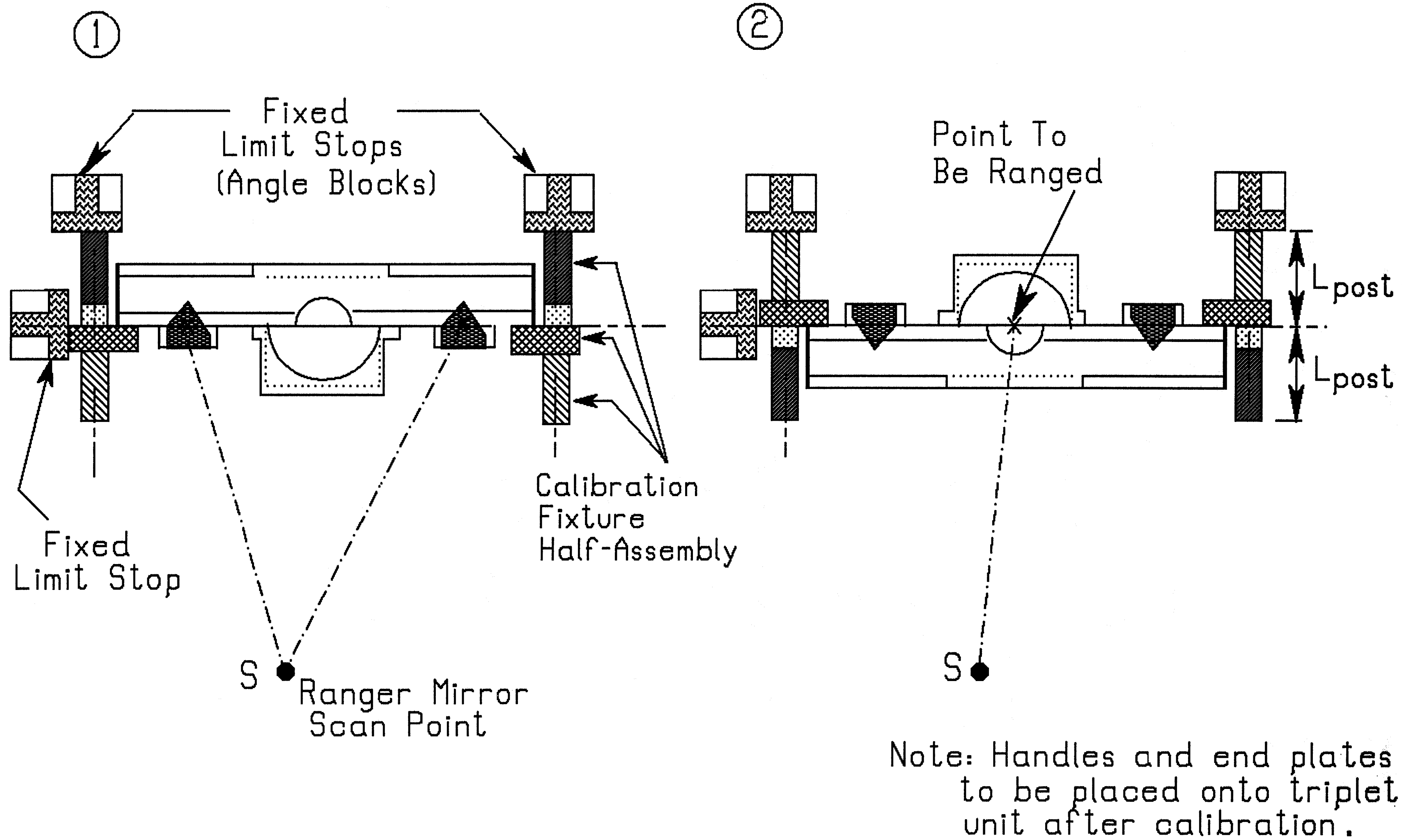


Figure 1A. Retroreflector Triplet Assembly Calibration.

1. Assemble triplet unit onto triplet fixture halves. Place fixtured triplet onto granite surface plate. with prisms facing up & ball facing down to plate. Measure heights of ball vertex and prism faces to granite plate using height gauge & gauge blocks.

2. Place fixtured unit hard against rear and side stops. Triplet top plate and prisms are to face rangefinder and lie perpendicular to horizontal ranging beam. Measure ranger scan point $\perp$ distance to top plate and vertical distance h above line of prism and ball reference points.

3. Measure range from ranger scan point S to prisms. Measure range to rangefinder reference prism. Record three range measurement phases and the rangefinder's calibration (zero distance) phase. Record temperature, pressure, relative humidity.

4. Flip the triplet unit 180 degrees about a horizontal axis and replace it against the limit stops. Measure range from the scan point S to the ball. Record the range measurement phase.

5. Compute the ball center distance from the centroid of the prism reference points, using the triplet calibration spreadsheet program.

6. Repeat range measurements with the triplet unit located closer to the rangefinder.

Figure 1B. Retroreflector Triplet Assembly Calibration.

Retroreflector Triplet Assembly Calibration

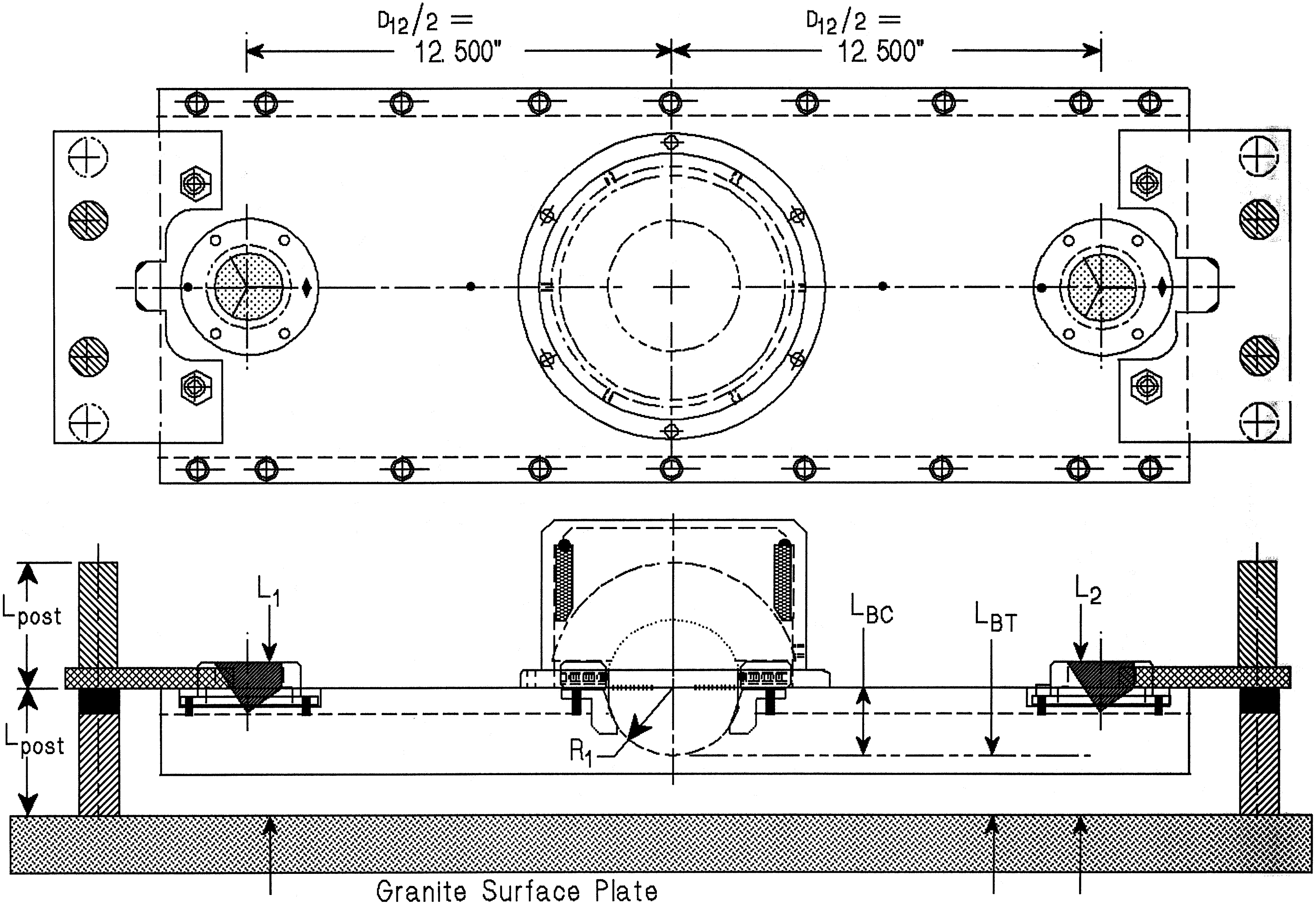
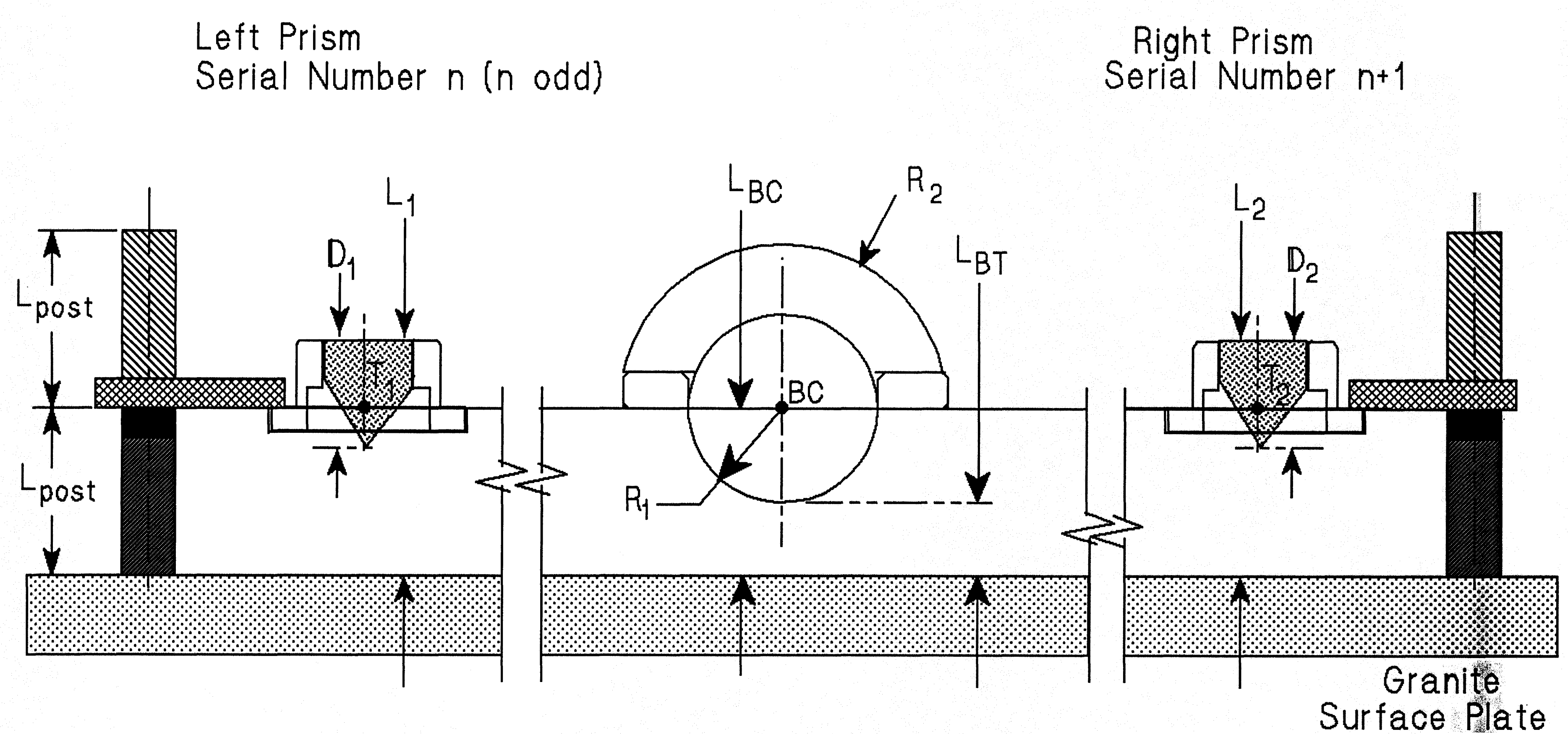


Figure 2.

Retroreflector Triplet Assembly Calibration



BC is the center point of the small ball

T_1 and T_2 are the prism fiducial reference points

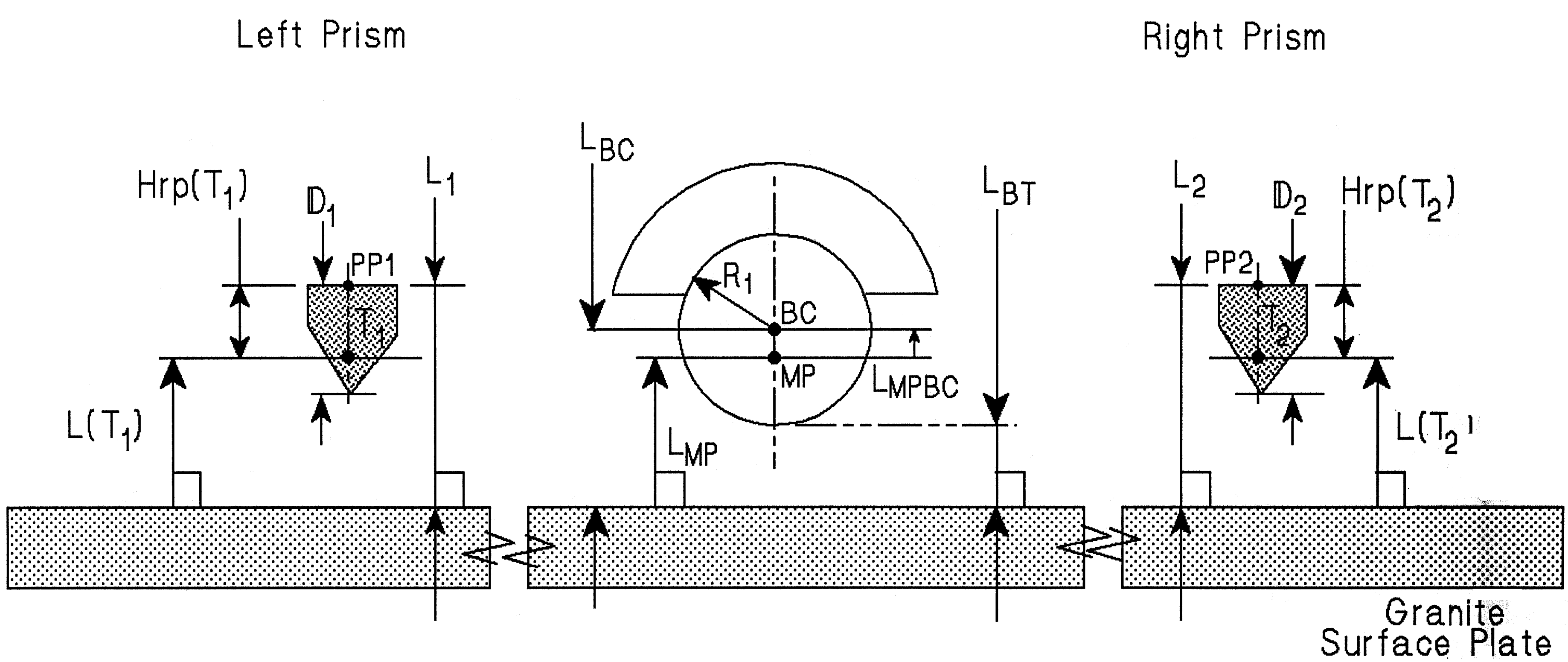
$L_{post} = 3.500"$

Measure L_1, L_2, L_{BT}

Prism Heights D_1 and D_2 are in prism data base

Figure 3.

Retroreflector Triplet Assembly Calibration



MP is defined to be the midpoint of line segment $T_1 T_2$

$L_{BC} = L_{BT} + R_1$

$L_{MP} = (1/2) * (L(T_1) + L(T_2))$

$L_{MPBC} = L_{BC} - L_{MP}$

$L(T_1) = L_1 - Hrp(T_1)$

$L(T_2) = L_2 - Hrp(T_2)$

Figure 4.

Half-Fixture For Retroreflector Triplet Assembly Calibration

Make Two Units

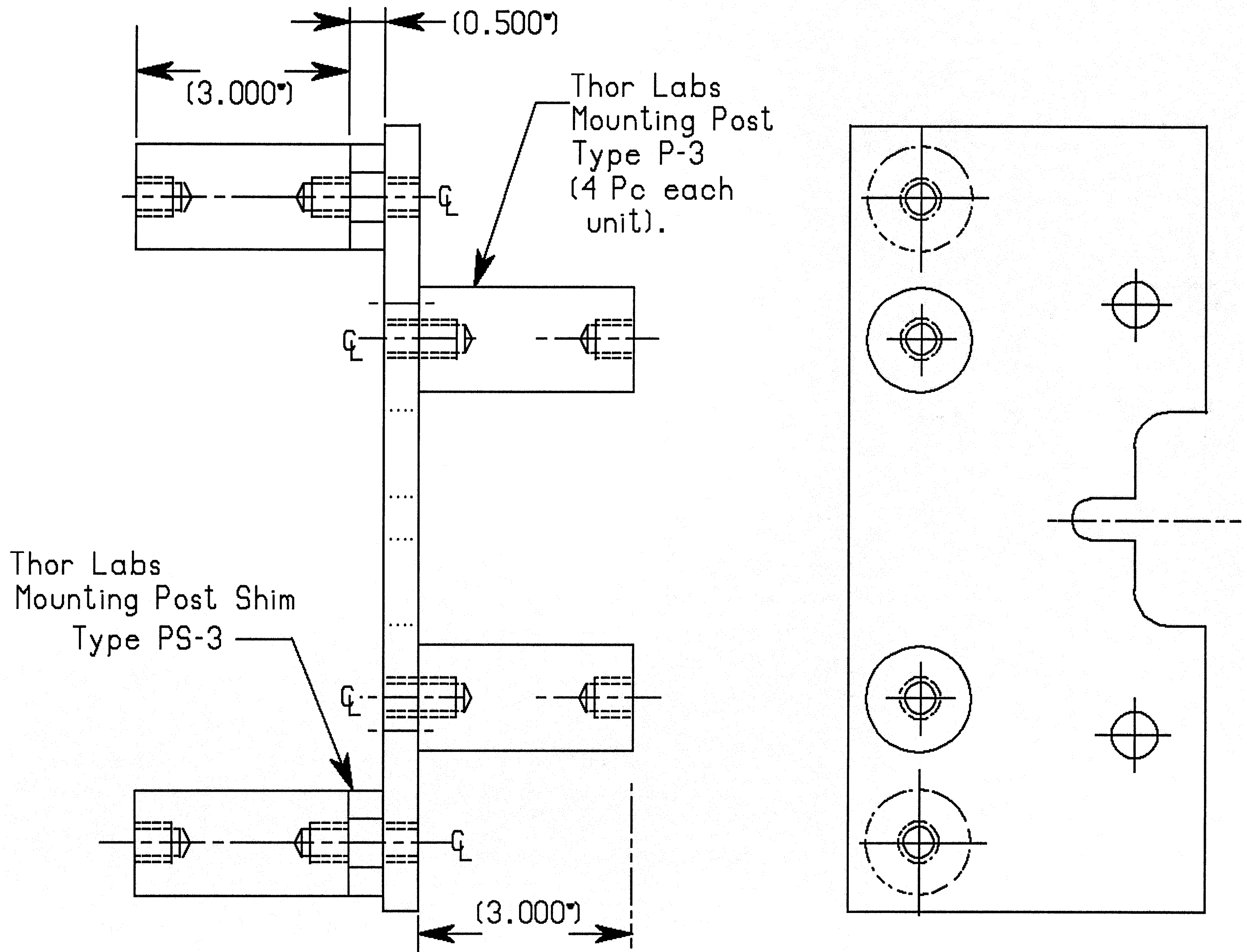
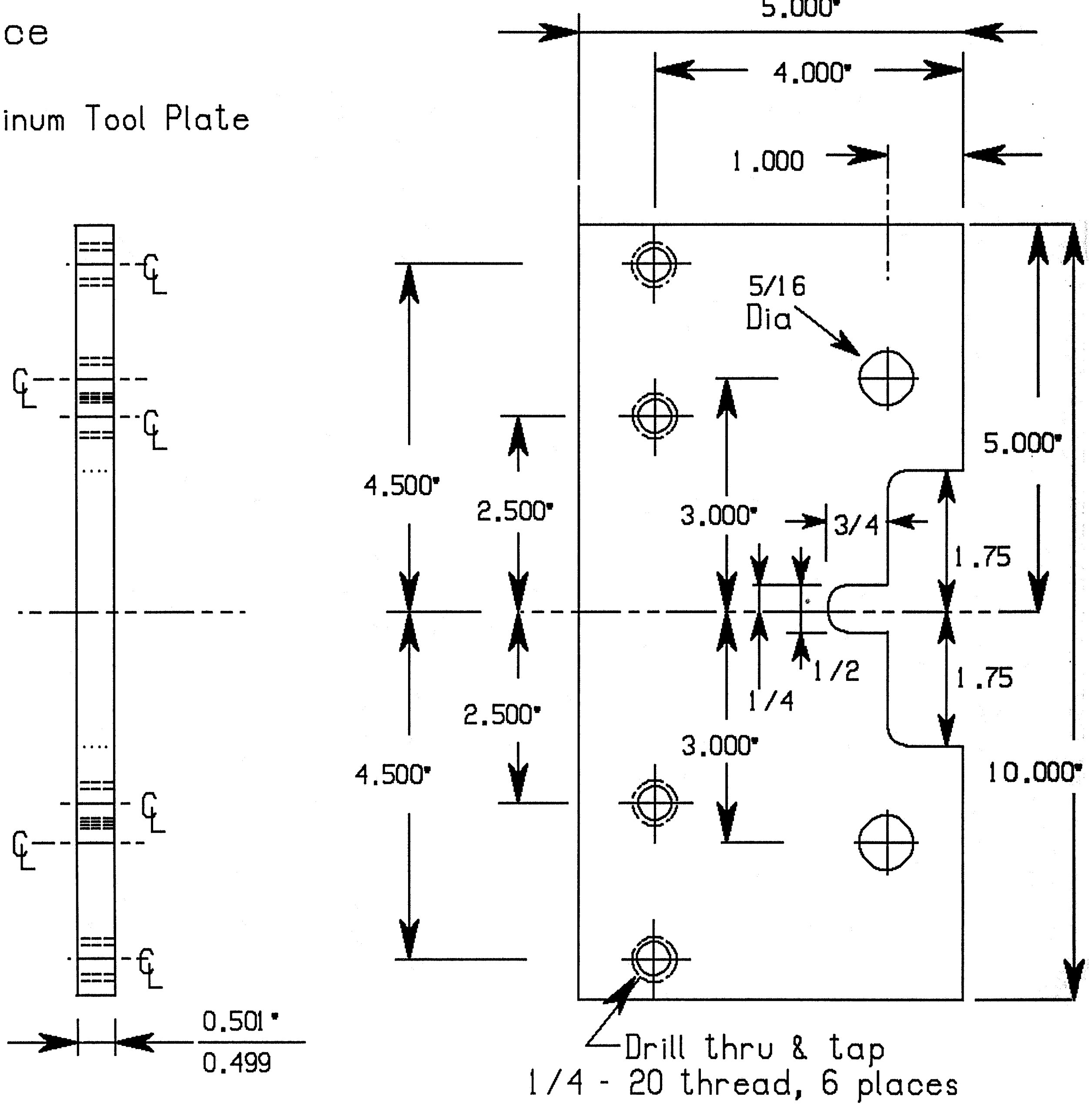


Figure 5.

Retroreflector Triplet Assembly Calibration
Half-Fixture Plate Piece

Make 2 units of 0.500" Aluminum Tool Plate



Tolerance ±0.003" on 3-decimal dimensions unless otherwise specified.

Figure 6.

Retroreflector Triplet Assembly Calibration

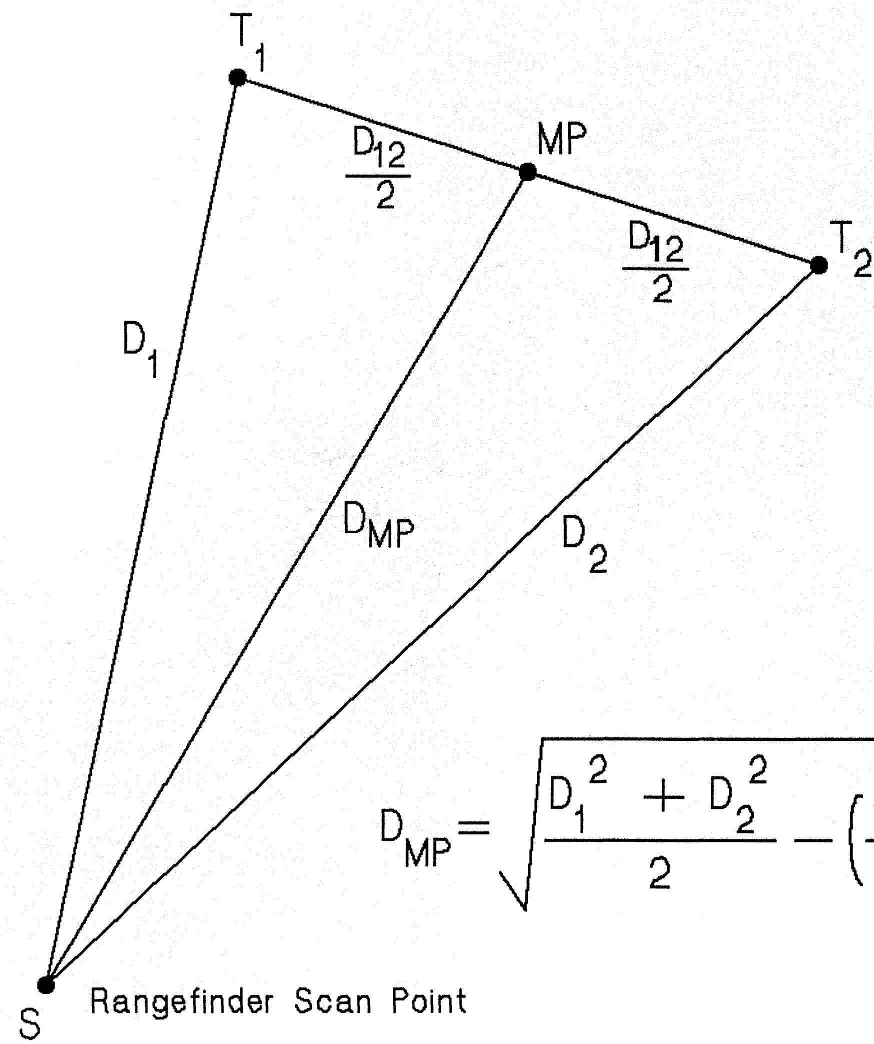
Computation Of Range From Scan Point S To Midpoint MP Of Segment T_1T_2.

$$\Delta = N_1\left(\frac{\lambda_A}{2}\right)$$

$$D_1 = \Delta + \delta_1$$

$$D_2 = \Delta + \delta_2$$

$$D_{MP} = \Delta + \delta_{MP}$$

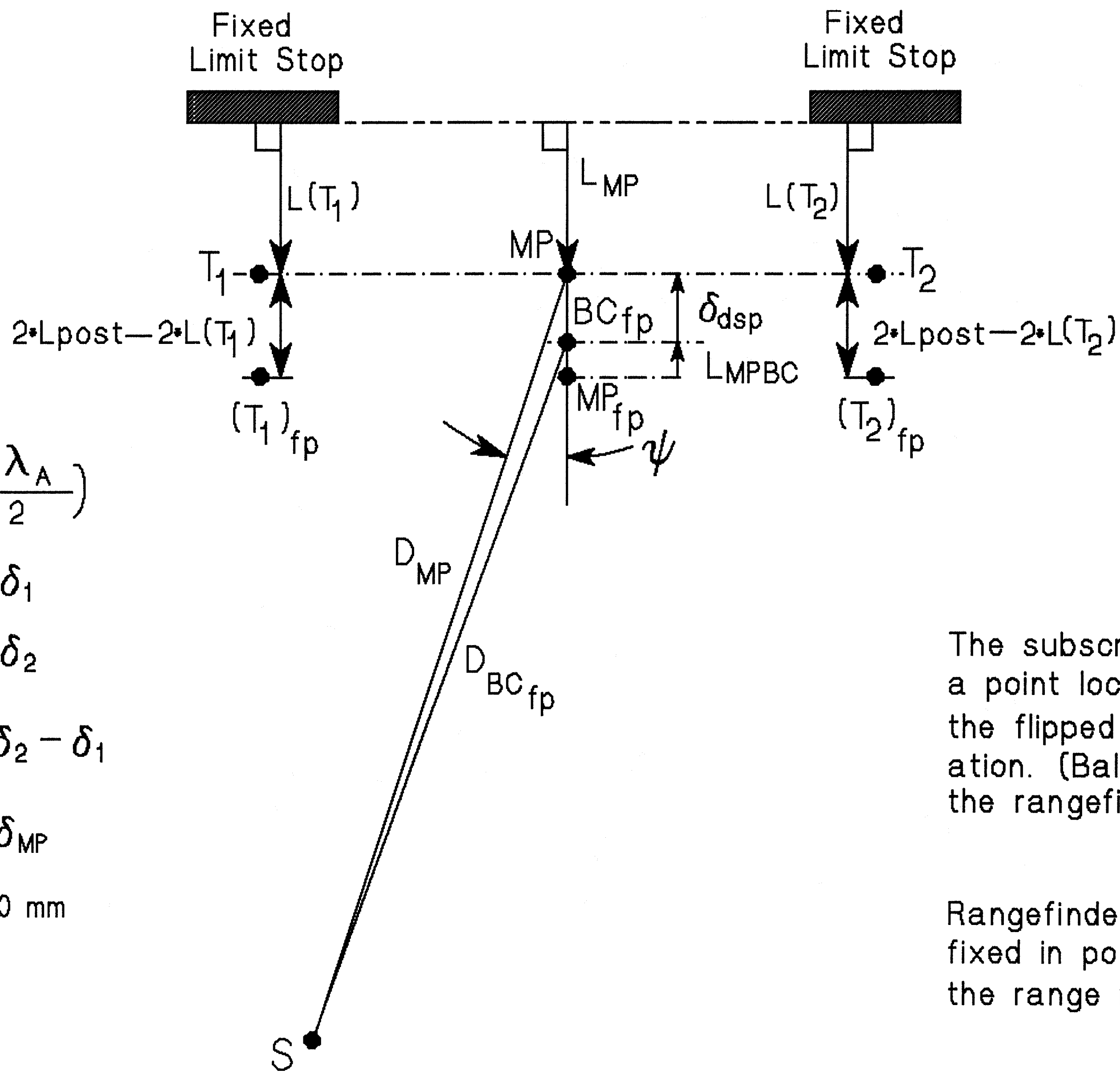


$$D_{MP} = \sqrt{\frac{D_1^2 + D_2^2}{2} - \left(\frac{D_{12}}{2}\right)^2}$$

Figure 7.

Retroreflector Triplet Assembly Calibration

Range Determination Of L_{MPBC}



$$\Delta = N_1\left(\frac{\lambda_A}{2}\right)$$

$$D_1 = \Delta + \delta_1$$

$$D_2 = \Delta + \delta_2$$

$$D_2 - D_1 = \delta_2 - \delta_1$$

$$D_{MP} = \Delta + \delta_{MP}$$

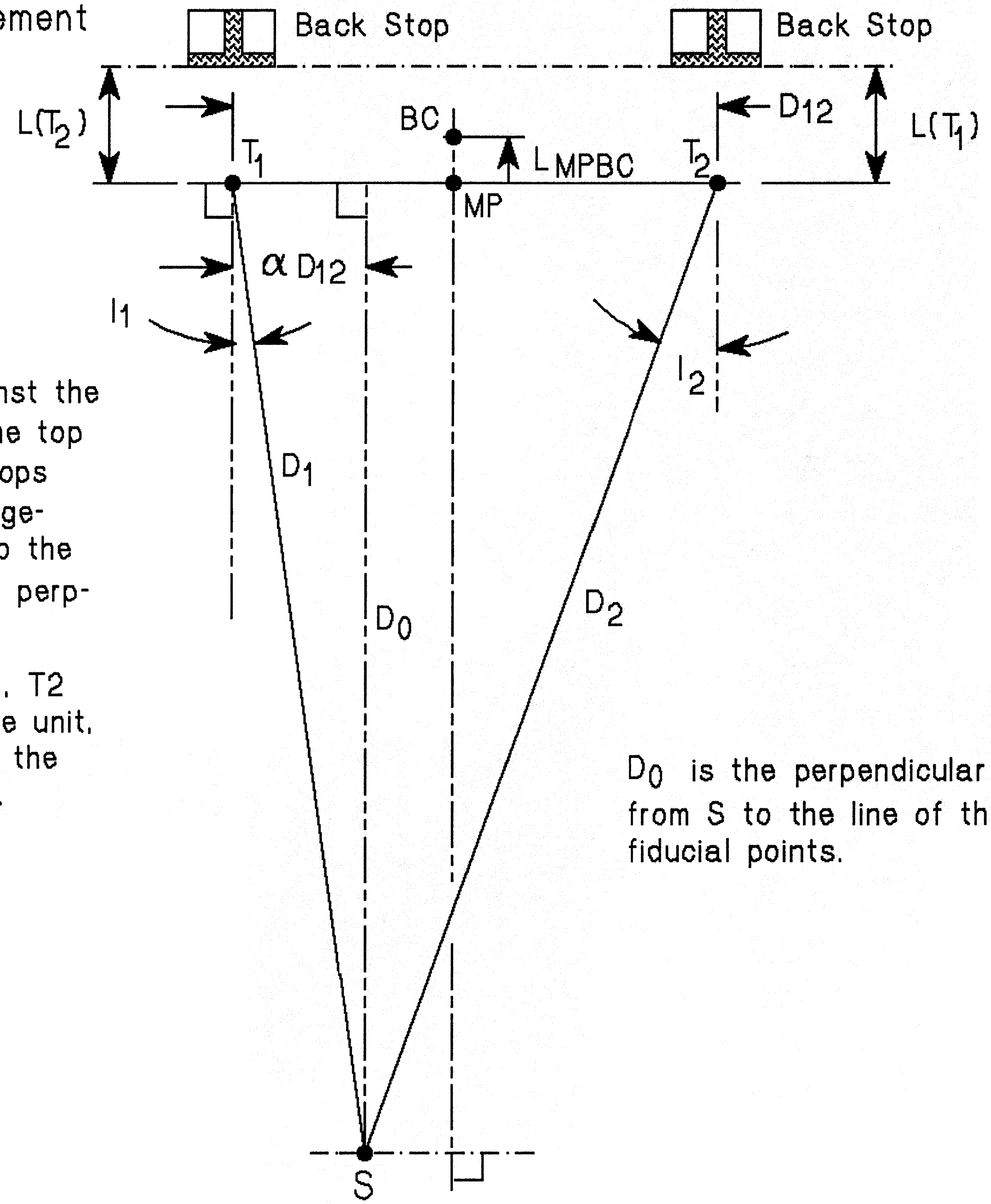
$$D_{12} = 635.00 \text{ mm}$$

The subscript "fp" denotes a point location referred to the flipped assembly configur-ation. (Ball reflector towards the rangefinder).

Rangefinder Scan Point S is fixed in position during all of the range measurements

Figure 8.

Prism Target Range Measurement

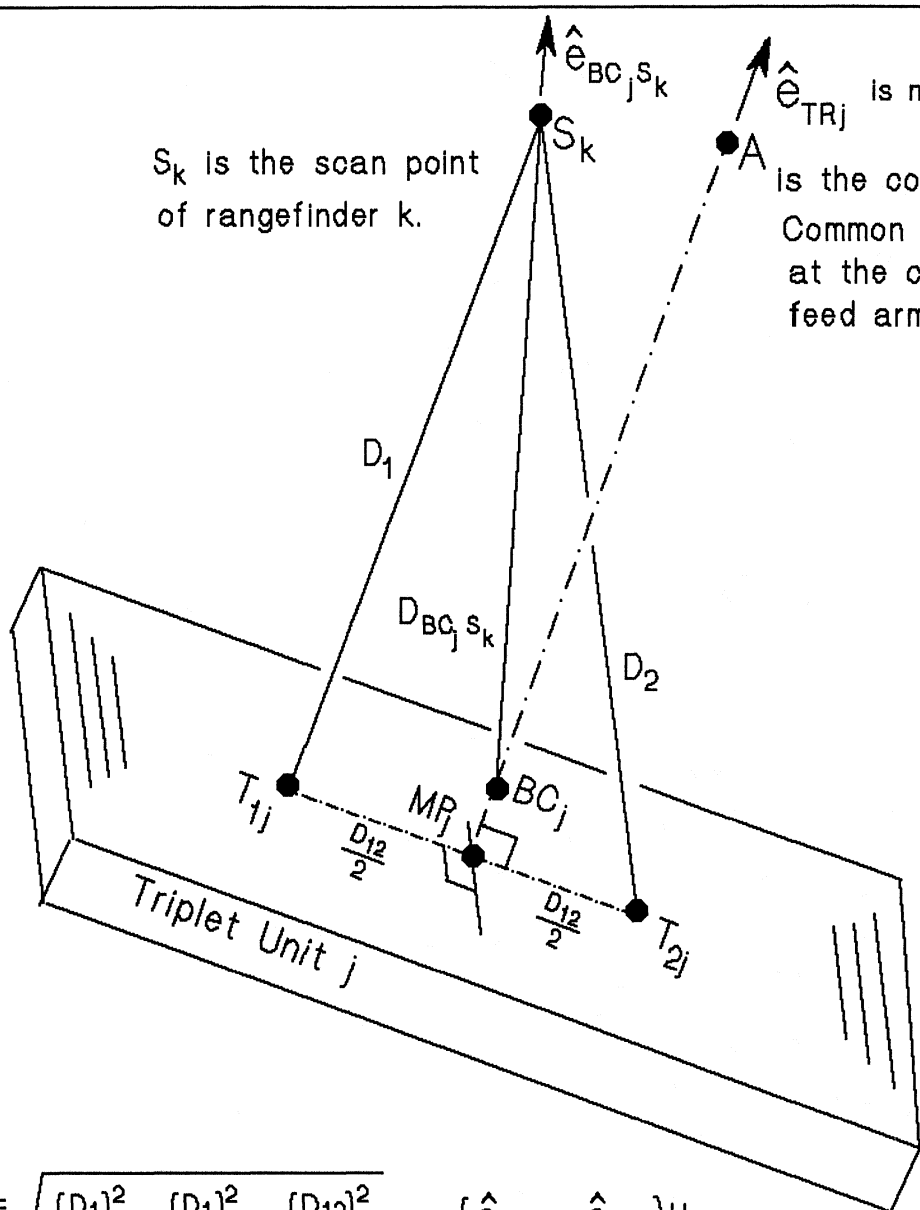


Using a plane mirror placed against the top plate of the triplet unit, set the top plate and the plane of the backstops perpendicular to the horizontal rangefinder beam (by autocollimating), so the planes are accurately vertical and perpendicular to the rangefinder beam.

Level the line of prism fiducials T1, T2 by levelling the topside face of the unit, while preserving autocollimation of the top plate to the rangefinder beam.

D_0 is the perpendicular distance from S to the line of the prism fiducial points.

Figure 9.



$\hat{e}_{TRj}$ is normal to the top plate of triplet unit j.

S_k is the scan point of rangefinder k.

A is the common aiming point for the triplet units. Common aiming point A lies approximately at the centroid of the scan points of the feed arm rangefinders.

T_{1j} and T_{2j} are prism fiducial reference points for triplet unit j.

MP_j is the midpoint of line segment T_{1j} - T_{2j}

BC_j is the ball reflector's reference point. It is displaced by $H_{corr} \cdot \hat{e}_{TRj}$ from the midpoint MP_j

$$D_{MP_j s_k} \cong \frac{(D_1 + D_2)}{2} - \frac{(D_{12})^2}{8\Delta} + \frac{(D_1 - D_2)^2}{8\Delta}$$

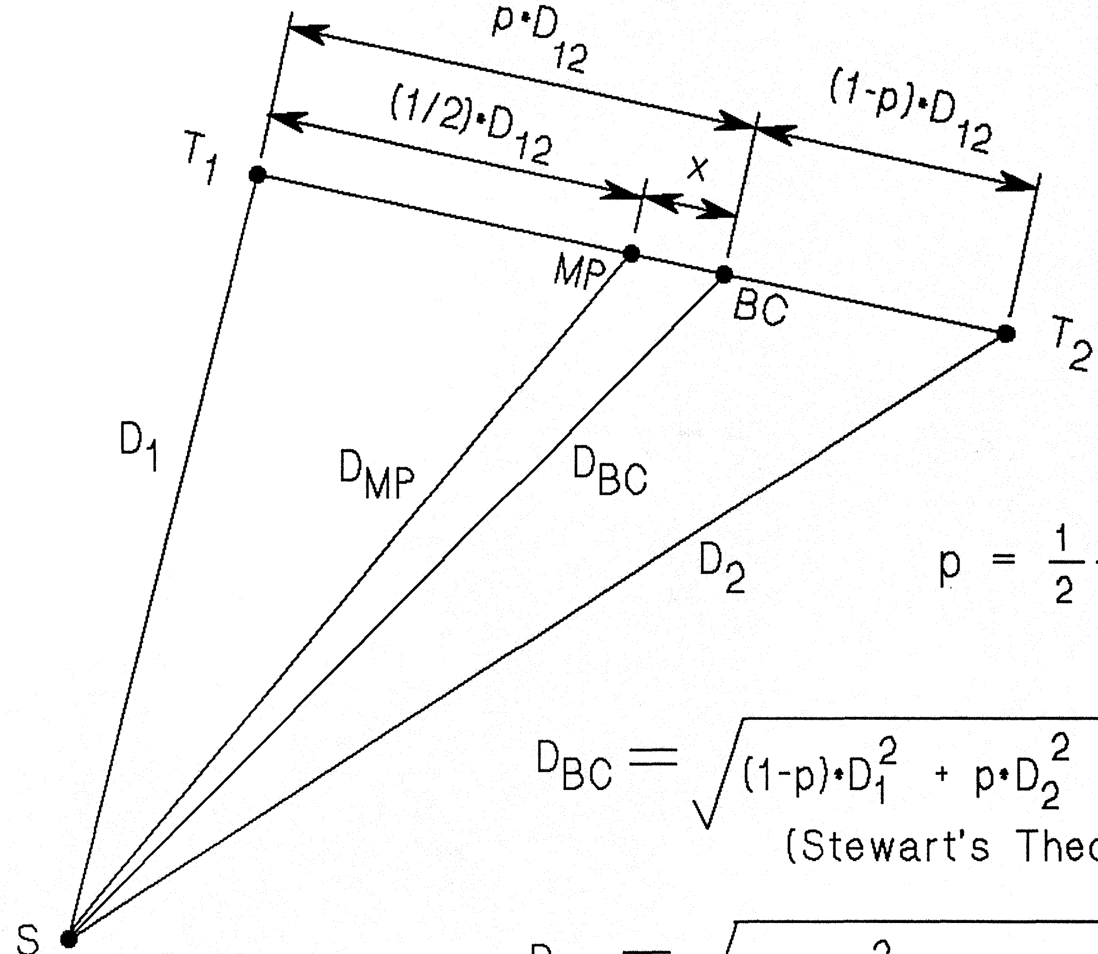
$$\Delta = \frac{c \cdot N_1}{2 \cdot f_{mod} \cdot n_A}$$

H_{corr} is measured during lab calibration of the triplet unit.

$$D_{BC_j s_k} \cong D_{MP_j s_k} - (\hat{e}_{BC_j s_k} \cdot \hat{e}_{TRj}) H_{corr}$$

$$D_{BC_j s_k} = \sqrt{\frac{(D_1)^2}{2} + \frac{(D_1)^2}{2} - \frac{(D_{12})^2}{4}} - (\hat{e}_{BC_j s_k} \cdot \hat{e}_{TRj}) H_{corr}$$

Figure 10. Triplet Range Computations.



$$p = \frac{1}{2} + \frac{x}{D_{12}}$$

$$D_{BC} = \sqrt{(1-p)\cdot D_1^2 + p\cdot D_2^2 - p(1-p)\cdot D_{12}^2}$$

(Stewart's Theorem)

$$D_{MP} = \sqrt{(1/2)\cdot D_1^2 + (1/2)\cdot D_2^2 - (1/4)\cdot D_{12}^2}$$

Figure 11. Effect Of A Lateral Shift Of Ball Center Relative To The Prism Fiducials" Midpoint.