INTERFERENCE GROUP INTERNAL REPORT

# Measuring Polarization With The NRAO Three Element Interferometer

by

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## 1. INTRODUCTION

This memo describes two programs which calibrate polarization data taken on the NRAO three element interferometer. Both programs are now on disc. Program POLAR determines and analyses the instrumental polarization of the interferometer, and program POLCOR applies the appropriate corrections to the data. The theory of the instrumental polarization is discussed in detail, and some preliminary results for the new dual frequency system are given. The notation used is standard interferometer group usage, as much as possible.

## 2. DESCRIPTION OF THE PROGRAMS

When measuring linear polarization with an interferometer certain instrumental effects have to be removed from the data. These effects may be considerably larger than the true polarized signal, and arise basically in the following way. The feeds on the NRAO interferometer split the incoming radiation into two nominally orthogonal circularly polarized modes. These are the 'left' and 'right' hand channels coming from each of the three telescopes. The feeds are not perfect, however, and the signal in the LH channel contains a small percentage of the RH polarized signal, and vice versa. Thus we can describe the signals in the RH channel of telescope 1 and the LH channel of telescope 2 as

$$1R \propto R + \varepsilon_1 L$$

$$2L \propto L + \varepsilon_2 R \qquad (1)$$

where the $\varepsilon$ are smallish complex quantities, usually of the order of a few percent. The output of the correlator is

$$1R2L \propto RL^* + \varepsilon_1 LL^* + \varepsilon_2^* RR^* + \varepsilon_1\varepsilon_2^* LR^* \qquad (2)$$

In terms of Stokes' parameters, if circular polarization is absent, $RR^* = LL^* \propto I$, the unpolarized fringe amplitude. $RL^*$ is the signal we want and is proportional to $Q + jU$. If the source is unresolved, $RL^* \propto mIe^{2j\chi}$, where m is the percentage polarization, and $\chi$ is the position angle of the electric vector.

Thus, neglecting terms of order $\varepsilon^2$, we have

$$1R2L \propto (Q + jU) + (\varepsilon_1 + \varepsilon_2^*) I \qquad (3)$$

i.e. the unpolarized fringe is 'leaking' into the polarized signal, and this is the term we must remove from the data.

Terms like ($\varepsilon_1 + \varepsilon_2$*) are calculated for each correlator by POLAR. An input data set called CALTABLE is required, which contains the true values of the polarization of at least two unresolved calibration sources. The data is then fitted to an expression like

$$P_i = GG * F_i + DD \qquad i = 1, 2.... \qquad (4)$$

where $P_i$ is the assumed true polarization of the $i^{th}$ calibration source, $F_i$ is the polarized fringe amplitude (LR or RL) divided by the unpolarized fringe amplitude (LL). GG and DD are (complex) constants for any one correlator and are calculated for each of the 6 correlators (1R2L, 1L2R, 1R3L, 1L3R, 2R3L, 2L3R).

Comparing (3) and (4), you can see that the DD's are equal to terms like - ($\varepsilon_1 + \varepsilon_2$*) and the GG's give the gain and the phase center of the correlators. The GG and DD terms are punched out on cards, which are used as input to POLCOR. POLCOR corrects all the data, using (4), e.g.

$$\text{1R2L (corrected)} = \text{1R2L (observed)} * GG + LL * DD \qquad (5)$$

Equation (4) cannot be solved if you have only one calibration source. If you have two calibration sources, it is solved exactly. If you have more than two, GG and DD are calculated in the least squares sense:

$$\text{i.e.,} \quad \sum_i \omega_i \left| P_i - DD - GG * F_i \right|^2 = \text{minimum} \qquad (6)$$

Since the assumed values for the calibration sources are usually slightly wrong, it is wise to use as many calibration sources as possible. The lease squares solutions for GG and DD are given by

$$GG = \frac{\Sigma\, \omega_i\, P_i\, F_i{*} \cdot \Sigma\, \omega_i - \Sigma\, \omega_i\, F_i \cdot \Sigma\, \omega_i\, F_i{*}}{\Sigma\, \omega_i\, F_i\, F_i{*}\, \Sigma\, \omega_i \;-\; \Sigma\, \omega_i\, F_i \cdot \Sigma\, \omega_i\, F_i{*}} \qquad (7)$$

and

$$DD = \frac{\Sigma\, \omega_i P_i \cdot \Sigma\, \omega_i F_i F_i{*} - \Sigma\, \omega_i P_i F_i{*}\, \Sigma\omega_i F_i}{\Sigma\omega_i F_i F_i{*}\, \Sigma\omega_i \;-\; \Sigma\omega_i F_i \cdot \Sigma\omega_i F_i{*}} \qquad (8)$$

where the summation is over all the calibration sources.

The weights $\omega_i$ are taken as $\sqrt{n_i}$ where the $i^{th}$ source has $n_i$ data records. Also one solution is computed including weights $W_i$ for each source, assigned by the user in CALTABLE, i. e. $\omega_i = \sqrt{n_i W_i}$ . The $W_i$ can represent the relative reliability of the assumed calibration values. Other things being equal, it may be appropriate to make these the flux densities of the sources.

In the old 11 cm interferometer system, it was found that DD and GG changed as a function of delay. In particular, they jumped, when the delays were switched from one arm of the interferometer to the other. Hence in POLAR, the total delay range is divided into 10 equal steps, and DD and GG are calculated for each delay step, and POLCOR applies the corrections appropriately.

## 3. DESCRIPTION OF THE OUTPUT

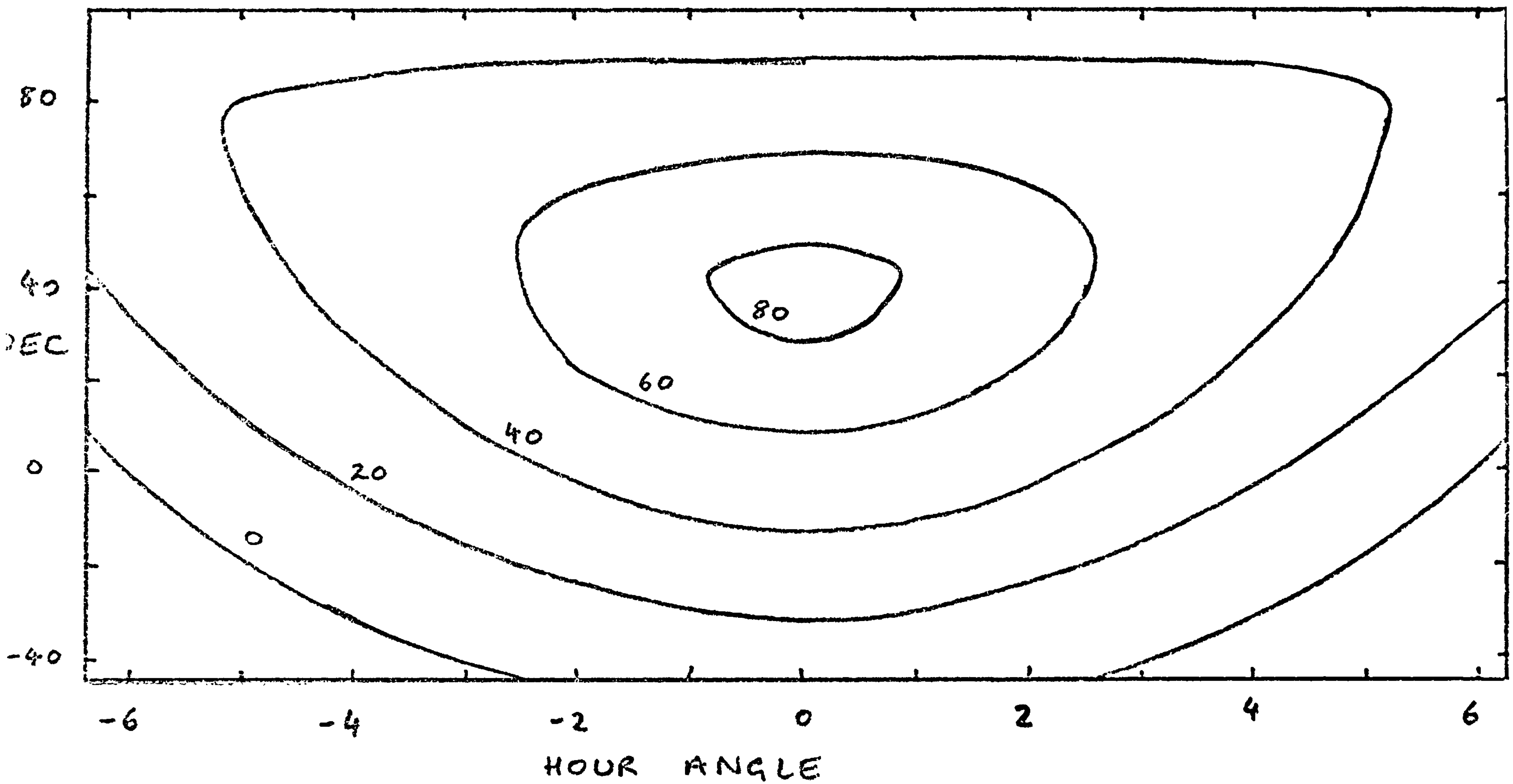
The first page states the wavelength of observation, the first and last scans read, and the assumed polarizations of the calibration sources. The fifth column, labelled FLUX, is the weight W described in the previous section.

The next few pages give the raw data for each source, read directly off the tape. The delay steps run from D = 1 to D = 10. Large numbers correspond to the source being in the East, and small numbers to it being in the West. The delays switch from one arm of the interferometer to the other between D = 5 and D = 6.

Next, the computed values of GG and DD are printed for each correlator and for each delay step for which there is data on at least two sources. The left hand side of the page gives the LR correlators and the right hand side gives the RL correlators. The column labelled GAIN gives the GG terms and the column labelled DEFLECTION gives the DD terms. The column labelled # gives the number of sources used in the solution, the column labelled ERROR gives the RMS error of the points about the mean. i.e. ERROR = $(\Sigma_i \omega_i |P_i - DD - GG * F_i|^2 / \Sigma\omega_i)^{1/2}$. Clearly, if # = 2, then ERROR = 0.0

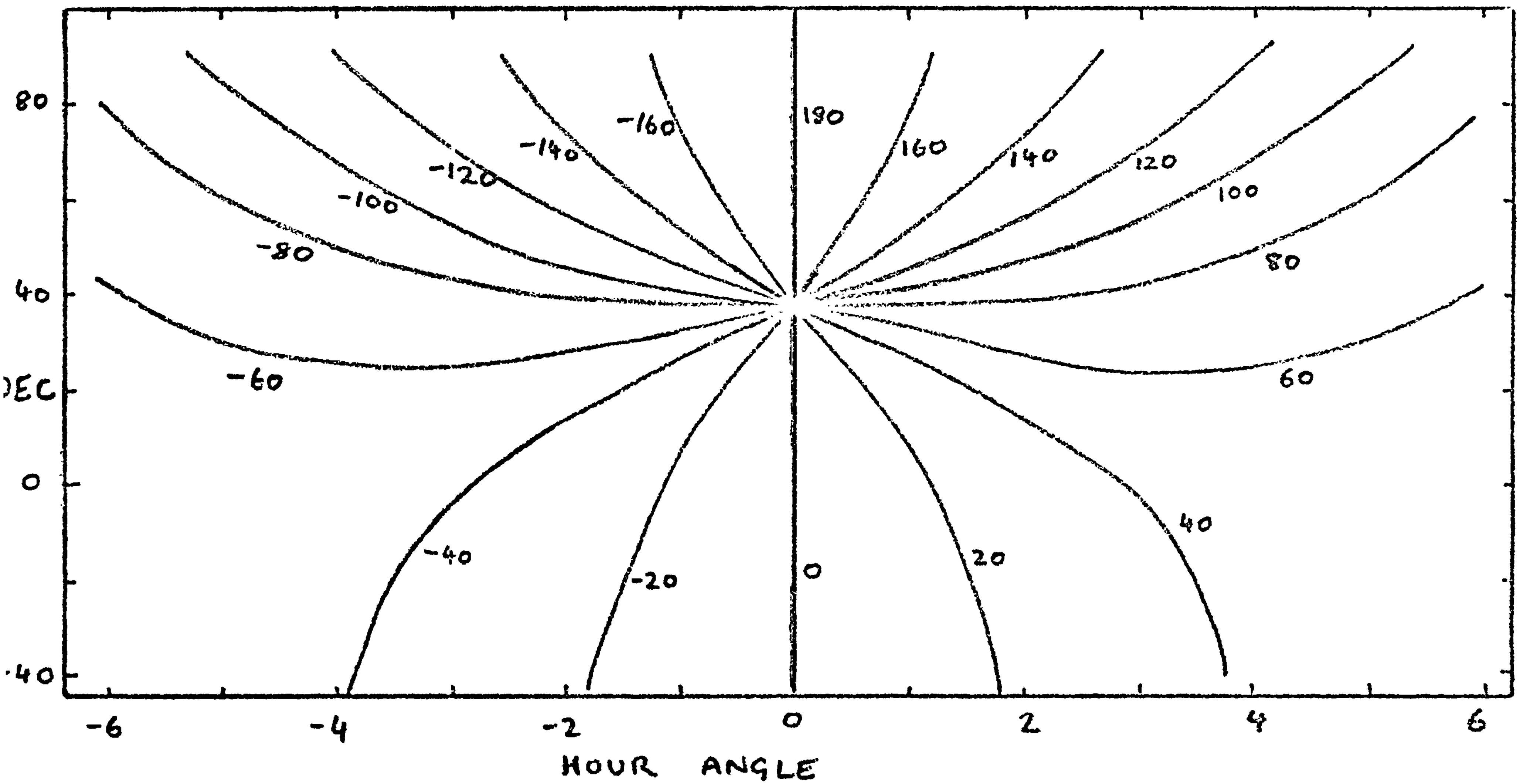
The next page gives the values of GG and DD for each correlator averaged over the delay steps, and then checks the 'closure' conditions on the solution. These are discussed fully in section 8 . The first three columns print out the left hand side of equation (18) for the three baselines, and the fourth column prints out the left hand side of equation (19). This information is useful in deciding on the validity of your solutions.

The next few pages give the data on the calibration sources, corrected with the values of DD and GG you have just calculated. The mean and standard deviation of the corrected data is computed and compared with the calibration value, at the bottom of each page.



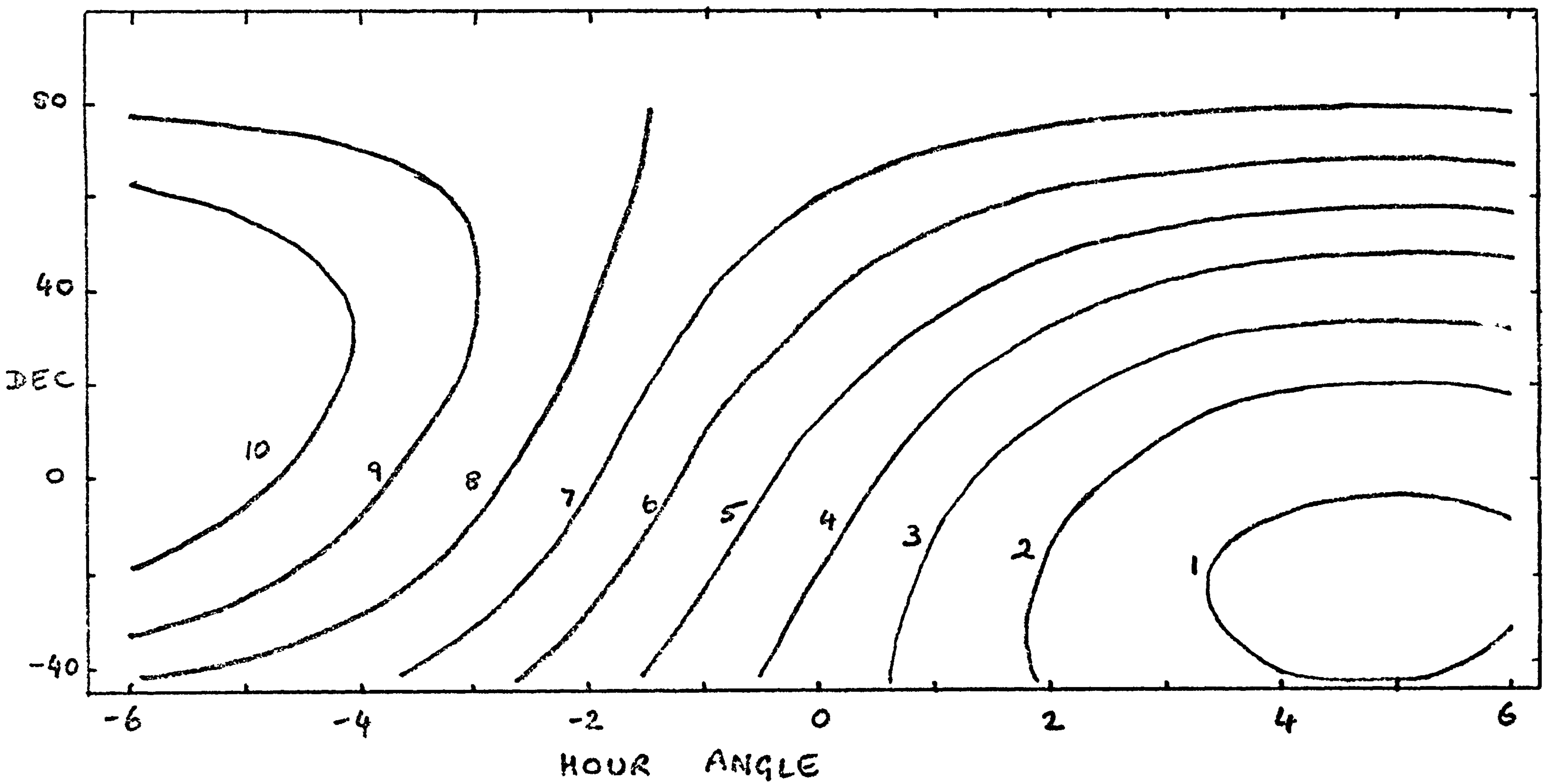

LINES OF CONSTANT ELEVATION

Fig 1




LINES OF CONSTANT PARALLACTIC ANGLE

Fig 2




LINES OF CONSTANT DELAY

Fig 3

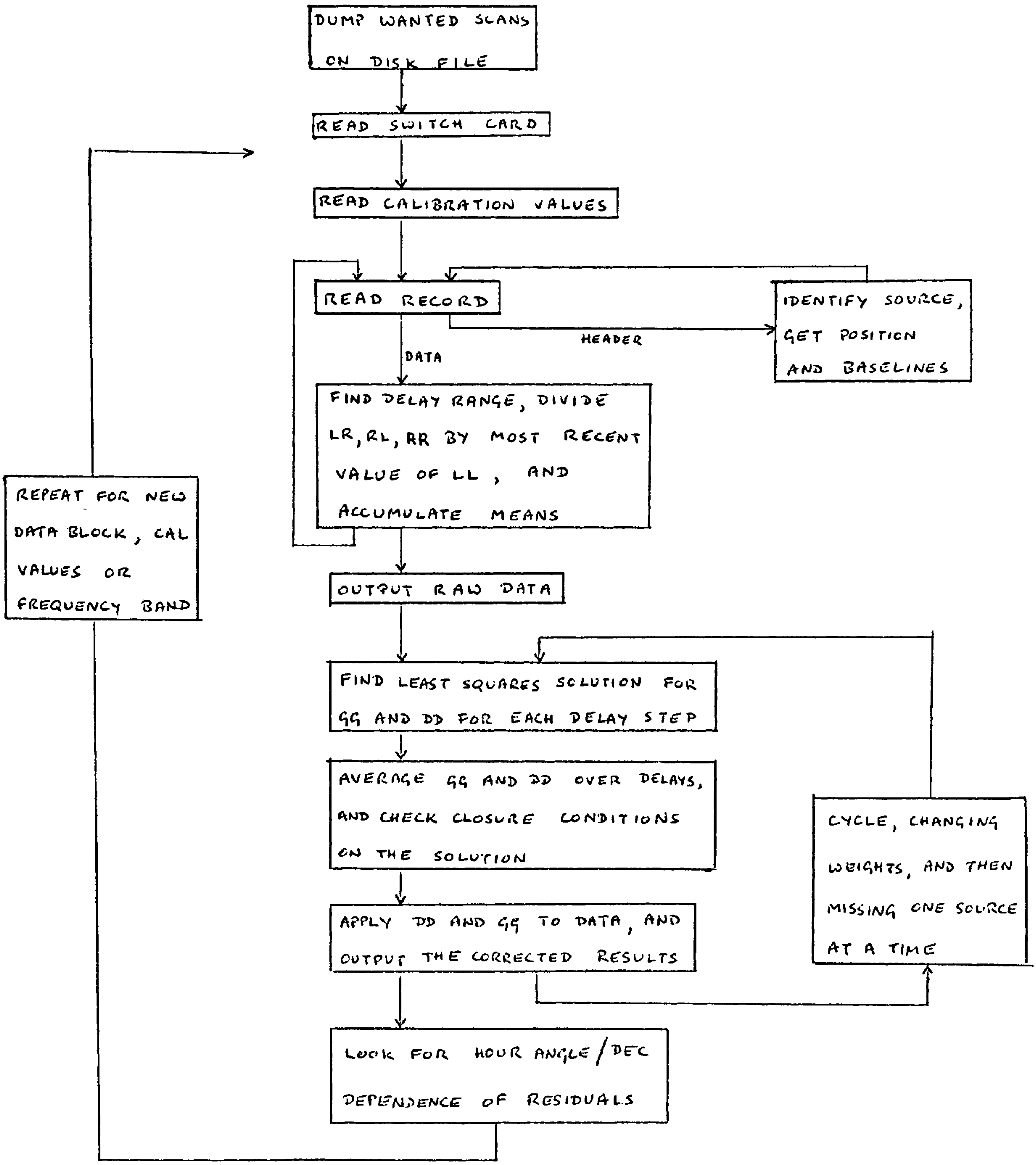

DUMP WANTED SCANS ON DISK FILE
READ SWITCH CARD
READ CALIBRATION VALUES
READ RECORD
HEADER
IDENTIFY SOURCE, GET POSITION AND BASELINES
DATA
FIND DELAY RANGE, DIVIDE LR, RL, RR BY MOST RECENT VALUE OF LL, AND ACCUMULATE MEANS
REPEAT FOR NEW DATA BLOCK, CAL VALUES OR FREQUENCY BAND
OUTPUT RAW DATA
FIND LEAST SQUARES SOLUTION FOR GG AND DD FOR EACH DELAY STEP
AVERAGE GG AND DD OVER DELAYS, AND CHECK CLOSURE CONDITIONS ON THE SOLUTION
CYCLE, CHANGING WEIGHTS, AND THEN MISSING ONE SOURCE AT A TIME
APPLY DD AND GG TO DATA, AND OUTPUT THE CORRECTED RESULTS
LOOK FOR HOUR ANGLE/DEC DEPENDENCE OF RESIDUALS

Fig 4

If you want to, the whole solution can be repeated skipping one calibration source from the solution at a time. If the assumed polarization of one source is badly wrong, it will stick out like a sore thumb.

The final six pages of output investigate the hour angle and declination dependence of the data. For each source, the polarized fringe near hour angle zero is subtracted from the rest of the data. The residuals are almost independent of the actual polarization of the source, and are displayed graphically for each correlator.

Figs. 1, 2 and 3 give lines of constant elevation, parallactic angle and delay as a function of hour angle and Declination. These are on the same scale as the line printer output and may be useful when inspecting the output.

The whole rigmarole can now be repeated for a different frequency band, a different block of data or different calibration sources or values. This is described in the next section.

## 4. HOW TO USE POLAR

Fig. 4 gives a flow diagram of POLAR, which may help a bit. The program consists of two job steps. The first step simply reads your input tape and dumps all the wanted data on a disk file. This is to save rewinding the tape many times in the second job step. The first step requires three JCL cards plus the usual INCLUDE EXCLUDE cards (see the 'Users Guide to the Interferometer Reduction System', by Bossermann and Rhudy). e.g.

```
//POLAR      JOB    (179,P,6,8,5),RAQUEL,MSGLEVEL=1,CLASS=C
//  EXEC POLAR,INTAPE=3824,INNAME=WELCH
//DUMP.SYSIN     DD    *
INCLUDE    27000     28000      3C286
etc.
```

It saves considerable time later on if your specify only the sources you want.

The next step requires two card data sets; POLAR.SYSIN specifies the data to be used for any particular run through the program, using INCLUDE EXCLUDE cards; CALTABLE gives the assumed values of the polarization of the calibration sources. There is one card for each source, giving the name, percentage polarization, position angle, and weight, in FORMAT (5A2, 3F10.2).

The first card in CALTABLE sets some program switches, and contains up to eleven numbers in FORMAT (11I5). These numbers set the following switches.

| | | |
|---|---|---|
| 1 | LAMDA | 1st Frequency |
| 2 | NUMBER | |
| 3 | CARDS | |
| 4 | CYCLE | |
| 5 | DELAY | |

| | | |
|---|---|---|
| 6 | LAMDA | 2nd Frequency |
| 7 | NUMBER | |
| 8 | CARDS | |
| 9 | CYCLE | |
| 10 | DELAY | |
| 11 | AGAIN | |

What they mean is:

LAMDA = 11, if the following cards give calibration values at 2695 MHz.

LAMDA = 4, if they give calibration values at 8085 MHz.

NUMBER = number of calibration sources for that frequency.

CARDS = $\leq$ -2 if you want no punched card output.

= -1 if you want card output giving GG and DD for the solution which included the weights you assigned to each source.

= 0 for the solution which ignored your weights.

CYCLE $\neq$ 0 if you want the solution to be repeated missing out one calibration source at a time.

= 0 if you don't want to do that.

if CYCLE $\neq$ 0, then you may set CARDS = n, and you will get punched card output for the solution which ignores the $n^{th}$ calibration source.

DELAY = 0 if you want a solution for each of the ten delay steps.

= 1 if you want all the delays averaged together.

= 2 if you want a solution for two delay steps: before and after crossover.

You can analyze both frequency bands at once, so normally switches 6 - 10 will specify similar information for the other band. Alternatively they could specify a different set of calibration sources on trial values for the same band. If you don't want to do any of this, leave switches 6 - 10 blank or zero.

AGAIN: If you analyzing more than one block of data (say, another configuration) and you want to use exactly the same calibration values, and set the same program switches, then set AGAIN equal to the number of blocks of data minus one. i.e., each set of calibration values is reused AGAIN times before the program reads the next set.

The different blocks of data to be analyzed are delineated by END REWIND cards in POLAR.SYSIN.

All this is getting ludicrously complicated, so lets have a few examples.

Example: You want to look at the S BAND data between scans 27000 and 28000. You have four calibration sources, you want a solution for all ten delay steps, and you want to cycle the solution missing out one source at a time, and you don't want punched card output.

```
//POLAR.SYSIN    DD    *
INCLUDE     27000    28000
//CALTABLE   DD   *
   11    4   -2
CTA102          5.05      20.0       4.0
3C286           9.55      30.0      10.3
3C48            1.93      62.20      9.0
3C147           1.20      87.0      10.0
```

Example: You have three separate configurations, and you want to look at both X BAND and S BAND data. You want ten delay steps, and punched card output for the solution which includes your assigned weights. You think you've already got the best values for the calibration sources, so you don't want to cycle the solution.

```
//POLAR.SYSIN   DD   *
INCLUDE    27000    27300
END    REWIND
INCLUDE    27301    27650
END REWIND
INCLUDE    27651    28000
//CALTABLE    DD    *
   11    5   -1    0    0    4    4   -1    0    0    2
CTA102
3C286
3C48                                   S BAND Values
3C309.1
3C147
CTA102
3C286
3C147                                  X BAND Values
3C48
```

Complete example: This time you have just one configuration, at X BAND, and you are going to play with it a little. First, you want a ten delay step solution using three calibrations whose values you think you know pretty well, (no CYCLE). Next you want to do it again including four other calibration sources, whose true polarizations are less certain (so this time you do want to cycle the solution, skipping one source at a time). Finally you want to divide the data into small chunks and look at the repeatability of the observations, and the time dependence of the solution. Ten delay steps would be stretching the data rather thin on the ground so this time you might use only two delay steps. The full program might look like this:

```
//POLAR     JOB  (203,P,6,8,5),HUMPTY,MSGLEVEL=1,CLASS=D
//  EXEC POLAR,INTAPE=3848,INNAME=DUMPTY
//DUMP.SYSIN   DD   *
INCLUDE    4000    5000    3C286
etc.
//POLAR.SYSIN   DD   *
INCLUDE    4000    5000
END   REWIND
INCLUDE    4000    5000
END   REWIND
INCLUDE    4000    4033
END   REWIND
INCLUDE    4034    4066
END REWIND
INCLUDE    4067    5000
//CALTABLE    DD    *
```

```
    4    3   -2
3C286
3C147           etc.
CTA102
    4    7   -2    1
3C286
3C147
CTA102
                etc.
3C209.1
3C287
3C48
CTA21
    4    3   -2    0    2                   2
3C286
3C147
CTA102
/*
```

The running time is very roughly 30 sec to 1 1/2 minutes per solution, depending on how much data you have, and whether you are cycling the solution. You would normally run as a CLASS C job.

## 5. HOW TO USE POLCOR

This program is very straight forward, and requires two data sets; SYSIN specifies the data to be corrected using INCLUDE EXCLUDE cards; and

INSTRPOL contains the cards punched out by POLAR, giving the GG and DD terms. These cards give the baseline ( 1 = 85-1/2, 2 = 85-1/3, 3 = 85-2/3), the correlator (2 = LR, 3 = RL), the delay step (1→10), the real and imaginary parts of GG, and the real and imaginary parts of DD. The FORMAT is (3I5, 4F10.2).

The first card in INSTRPOL sets program switches which specify the frequency band and the number of delay steps in the cards that follow. The FORMAT is (4I5). The switches are:

| | |
|---|---|
| 1 = LAMDA<br>2 = DELAY | $1^{st}$ frequency band |
| 3 = LAMDA<br>4 = DELAY | $2^{nd}$ frequency band |

LAMDA and DELAY are defined exactly as in POLAR.

Thus a typical switch card might be:

11 0 4 1

The program will then expect 66 cards to follow. The first 60 give the S BAND corrections (for 10 delay steps), and the next 6 give the X BAND corrections (for one delay step).

The whole program will now look like this:

```
//POLCOR    JOB    (179,P,6,8,5),POOH,MSGLEVEL=1,CLASS=C
//  EXEC POLCOR,INTAPE=1066,INNAME=PIGLET,
//  OUTTAPE=1984,OUTNAME=EYORE
//SYSIN   DD   *
INCLUDE    2400    2600
//INSTRPOL     DD     *
```

```
  11          4    1
   1    2     1         0.72    0.35    -0.37    1.60      60 cards at S BAND
           etc
           etc.                                             6 cards at X BAND
/*
```

Note: If the DELAY switch is set equal to zero, there must be 60 GG/DD cards to follow, or else the program goes haywire. POLAR usually does not punch out cards for the extreme delay steps, since you don't often calibrate in these regions. You must therefore decide what values of DD and GG you want for these delay steps (e.g. averaged or extrapolated values) and punch out appropriate cards. Similarly, if DELAY equals one or two, there must be six or twelve GG/DD cards respectively.

You can correct several different blocks of data, using different values of the corrections. The blocks are delineated by END cards in SYSIN. (You can REWIND if you have the blocks out of order, but this is very time consuming.) For each new block of data, there must be a new switch card and a new set of GG/DD cards. e.g.

```
//POLCOR   JOB   (179,P,6,8,5),BUSTER,MSGLEVEL=1,CLASS=C
//  EXEC POLCOR,INTAPE=1500,INNAME=KEATON
//  OUTTAPE=1450,OUTNAME=FIELDS
//SYSIN   DD   *
INCLUDE    2000    3000
END    3000
INCLUDE    3001    4000
//INSTRPOL   DD   *
```

```
   11    2
12  GG/DD  cards   for   S BAND
    4                 11
60 GG/DD cards   for   X BAND
60 GG/DD cards   for   S BAND
/*
```

The program prints out the values of GG and DD it has used, on the line printer. It will normally run as a CLASS C job.

## 6. CALIBRATION SOURCES

For a source to be a good calibrator, it must be strong, unresolved, unconfused, nonvariable, and you must know its true polarization accurately. At S BAND, this is not much of a problem. At X BAND we are not in very good shape yet; many of the compact sources are variable or suspected to be so, and there are not very many good determinations of the polarizations.

Table 1 gives some suggested values. The S BAND values by Ed Formalont were measured on the NRAO interferometer two years ago, and have excellent internal consistancy. The Australian results (Gardner; Morris, Whiteoak, 1969, Aust. J. Phys., 22, 79) are probably less reliable.

The best X BAND results are from Philipp Kronberg. These are an interpolation of recent measurements at Algonquin at wavelengths of 2.2 cms, 2.8 cms and 4.6 cms. Hopefully more sources will be added to this list quite soon. I have also added some estimates of my own, so as to extend the number of possible calibation sources. These are read from plots of m ($\lambda$) and $\chi(\lambda)$ made from data of many observers.

It is salutary to note the lack of agreement between different peoples' measurements.

TABLE 1

Possible Calibration Sources

| S BAND | FOMALONT (2695 MHz) | | | AUSTRALIAN (2650 MHz) | | |
|---|---|---|---|---|---|---|
| Source | m | χ | error | m | χ | error |
| 0023-26 | 0.5 | 39.5 | .5 | | | |
| 3C48 | 1.9 | 62.2 | .3 | | | |
| 0237-23 | 2.0 | 138.0 | .4 | 3.4 | 143 | .5 |
| CTA21 | 0.6 | 64.0 | .4 | 1.8 | 52 | .6 |
| 3C138 | 7.8 | 167.1 | .4 | 8.5 | 169 | .2 |
| 3C147 | 0.5 | 50.5 | .3 | | | |
| 1127-14 | 1.2 | 38.0 | .4 | 1.9 | 4 | .7 |
| 1151-34 | 0.5 | 34.0 | .5 | 1.6 | 165 | .8 |
| 3C287 | 3.3 | 104.4 | .4 | 3.4 | 110 | .4 |
| 3C286 | 9.5 | 30.0 | .3 | | | |
| 3C 309.1 | 1.6 | 83.0 | .4 | | | |
| 1827-36 | 0.4 | 58.5 | .5 | 0.5 | 88 | .5 |
| CTA102 | 5.0 | 20.0 | .3 | 5.3 | .6 | .6 |
| 3C119 | 0.6 | 50.0 | .3 | | | |

| X BAND | KRONBERG | | OTHER ESTIMATES | |
|---|---|---|---|---|
| Source | m | χ | m | χ |
| 3C48 | 6.03 | 113 | 6.8 | 78 |
| CTA21 | | | 5.0 | 70 |
| CTA26 | | | 1.0 | 55 |

| | | | | |
|---|---|---|---|---|
| 3C147 | 1.21 | 87 | 0.5 | 172 |
| 1055+01 | | | 6.6 | 140 |
| 1127-14 | | | 3.4 | 140 |
| 3C286 | 11.55 | 40 | 12.0 | 41 |
| 3C309.1 | | | 1.5 | 0 |
| 3C380 | | | 2.0 | 160 |
| CTA102 | 6.05 | 52 | 6.6 | 40 |

## 7. SOME RESULTS

Table 2 and Figs. 5-7 give some results for three configurations at both frequencies. The data were taken in the winter of 1970/71. the GG data will not be discussed here, since they depend on how much calibration and adjusting has been done to the LL data.

Table 2 shows that the mean values of DD (averaged over the delay steps) are roughly constant for each correlator, but do change slightly with configuration (presumably due to the different IF cables). In particular look at the two sets of values for the 1900 m baseline, when the IF cables were the same. The agreement at S BAND is almost too good to be true, while the agreement at X BAND is a factor of ten worse. This is due to the stronger delay dependence of DD at X BAND (see Fig. 5-7) and mainly to the less accurate calibration values used. However, the rough consistency between configurations gives us some faith that we are doing things more or less right.

Figs. 5-7 show for three configurations how DD changes as a function of delay step. It is not yet known how repeatable the apparent delay dependence of DD is. Certainly, some of the delay dependence is spurious, especially at X BAND, and is caused by inaccurate values for the calibration sources. If GG is assumed to be independent of delay, then the true delay dependence can be found simply by subtracting the data in, say, delay step 5 from the data at other delays. For this, you do not need the true polarization of any source. Effectively, this is done in the Hour Angle/Declination plot of POLAR, and you can interpret it using the overlay giving lines of constant delay, Fig. 3.

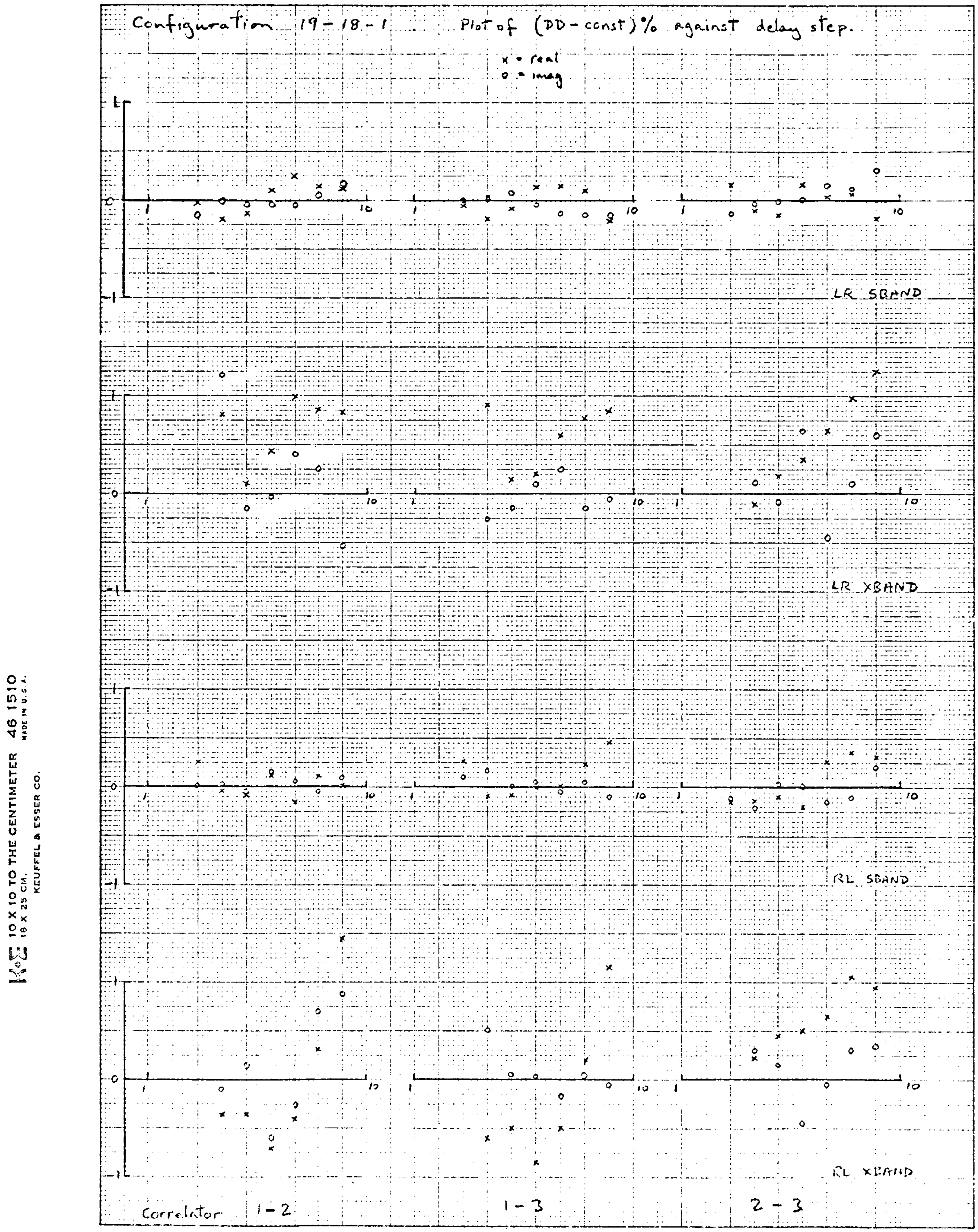

Configuration 19-18-1
Plot of (DD-const)% against delay step.
x = real
o = imag
LR SBAND
LR XBAND
RL SBAND
RL XBAND
Correlator 1-2
1-3
2-3
10 X 10 TO THE CENTIMETER 46 1510
18 X 25 CM.
MADE IN U.S.A.
KEUFFEL & ESSER CO.

Fig 5

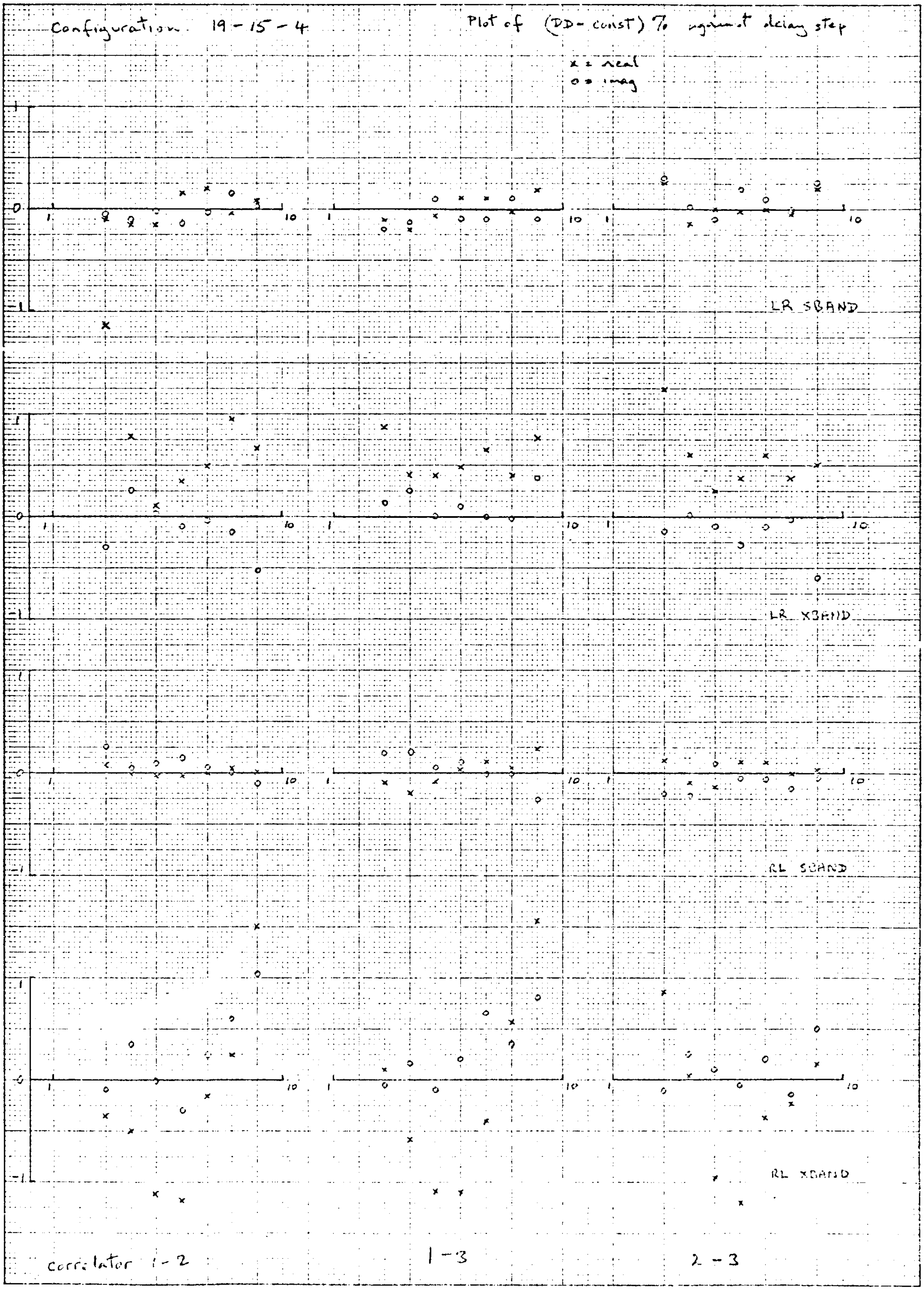

Configuration 19-15-4
Plot of (DD-const) % against delay step
x = real
o = imag
LR SBAND
LR XBAND
RL SBAND
RL XBAND
correlator 1-2
1-3
2-3

Fig 6

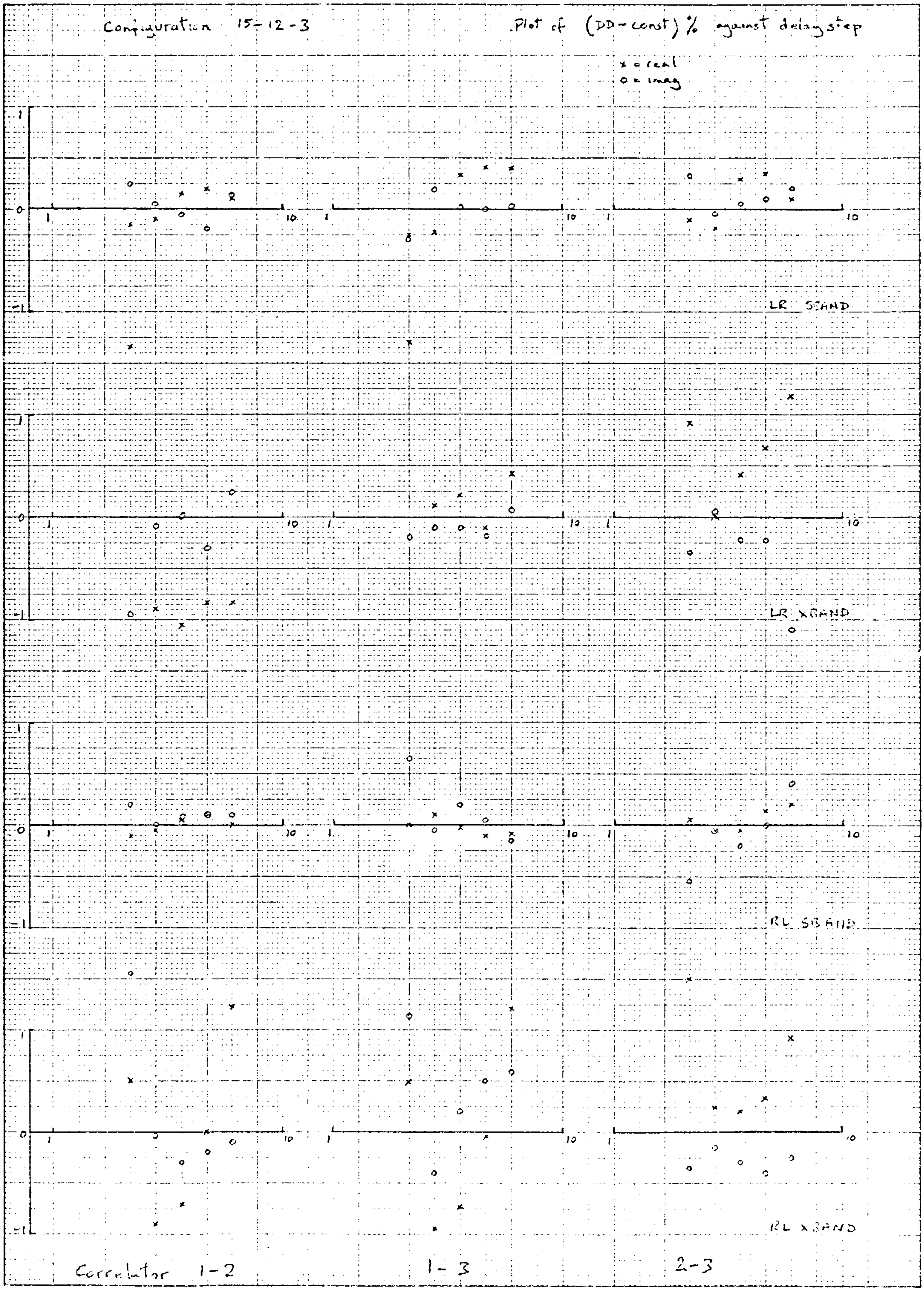

Configuration 15-12-3
Plot of (DD-const)% against delaystep
x = real
o = imag
LR SBAND
LR XBAND
RL SBAND
RL XBAND
Correlator 1-2
1-3
2-3

fig 7

TABLE 2

The Mean Values of DD for 3 Configurations

| Correlator | 1 - 18 - 19 | | 4 - 15 - 19 | | 3 - 12 - 15 | |
|---|---|---|---|---|---|---|
| S BAND | Real | Imag | Real | Imag | Real | Imag |
| 1L2R | 0.90 | -0.96 | 0.90 | -0.93 | 0.86 | -1.06 |
| 1R2L | -0.96 | 0.42 | -1.01 | 0.43 | -0.95 | 0.49 |
| 1L3R | 0.32 | -1.70 | 0.38 | -1.78 | 0.14 | -1.42 |
| 1R3L | -0.30 | -0.03 | -0.14 | -0.25 | -0.28 | -0.32 |
| 2L3R | -0.22 | 1.33 | -0.05 | 1.48 | -0.69 | 1.29 |
| 2R3L | 1.36 | 0.51 | 1.33 | -0.01 | 0.84 | 0.54 |
| X BAND | | | | | | |
| 1L2R | 1.31 | -0.36 | 1.83 | -0.07 | 2.23 | -0.28 |
| 1R2L | -4.77 | 0.75 | -5.13 | 1.23 | -5.14 | 0.96 |
| 1L3R | 0.74 | -0.01 | 1.30 | -0.21 | 1.65 | -0.34 |
| 1R3L | -4.15 | 1.67 | -5.00 | 1.62 | -4.50 | 1.73 |
| 2L3R | -2.89 | 0.35 | -2.03 | 0.08 | -2.06 | 0.32 |
| 2R3L | -1.44 | -1.59 | -1.55 | -1.00 | -1.58 | -1.31 |

## 8. THEORY OF THE INSTRUMENTAL POLARIZATION

In what follows, if one subscript is used, it refers to the telescope: if two are used, the first refers to the telescope, and the second is 1 for a right hand circularly polarized signal, and 2 for a left hand circularly polarized signal.

The signal voltage in the IR cahnnel can be written as

$$V_{11} \propto E_R + E_L \, \varepsilon_{11} \, e^{j\phi_{11}} \tag{8}$$

$E_R$ and $E_L$ represent the incoming radiation analyzed into perfectly circular modes. $\varepsilon_{11}$ is the fraction of left hand polarized radiaiton "leaking" into the right hand channel, and $\phi_{11}$ is the phase difference between the two signal components.

If the feed is rotated through an angle $\theta_1$ then at the correlators, the two signals from telescope 1 are

$$V_{11} = g_{11} \left\{ E_R \, e^{j\theta_1} + E_L \, \varepsilon_{11} \, e^{j(\phi_{11}-\theta_1)} \right\}$$

$$V_{12} = g_{12} \left\{ E_L \, e^{-j\theta_1} + E_R \, \varepsilon_{12} \, e^{j(\phi_{12}+\theta_1)} \right\} \tag{9}$$

The g terms are complex numbers representing the gains and phase shifts of all the amplifiers and delays in the signal path.

Write

$$G_{11} = g_{11} \, e^{j\theta_1}$$

$$G_{12} = g_{12} \, e^{-j\theta_1}$$

$$\delta_{11} = \varepsilon_{11} \, e^{j(\phi_{11}-2\theta_1)}$$

$$\delta_{12} = \varepsilon_{12} \, e^{j(\phi_{12}+2\theta_1)} \tag{10}$$

Then

$$V_{11} = G_{11} \left\{ E_R + \delta_{11} \cdot E_L \right\}$$

$$V_{12} = G_{12} \left\{ E_L + \delta_{12} \cdot E_R \right\} \tag{11}$$

and similarly for the other telescopes.

The outputs of the 12 correlators are

$$1R2R = V_{11} \cdot V_{21}^*$$

$$1L2L = V_{12} \cdot V_{22}^*$$

$$1L2R = V_{12} \cdot V_{21}^*$$

$$1R2L = V_{11} \cdot V_{22}^*$$

$$1R3L = V_{11} \cdot V_{31}^*$$

$$1L3L = V_{12} \cdot V_{32}^*$$

$$1L3R = V_{12} \cdot V_{31}^*$$

$$1R3L = V_{11} \cdot V_{32}^* \qquad (12)$$

$$2R3R = V_{21}^* \cdot V_{31}$$

$$2L3L = V_{22}^* \cdot V_{32}$$

$$2L3R = V_{22}^* \cdot V_{31}$$

$$2R3L = V_{21}^* \cdot V_{32} \qquad (13)$$

Note that the baseline is defined in the same <u>direction</u> for all three correlators.

i.e. $$1R2R = G_{11}\, G_{21}^*\ E_R\, E_R^* + O(\delta)$$

$$\sim G_{11}\, G_{21}^*\ I$$

$$1L2L \sim G_{12}\, G_{22}^*\ I$$

$$1L2R = G_{12}\, G_{21}^*\ [E_L E_R^* + \delta_{12} E_R E_R^* + \delta_{21}^*\ E_L E_L^* + O(\delta^2)]$$

$$\sim G_{12} G_{21}^* I [me^{-2j\chi} + \delta_{12} + \delta_{21}^*]$$

$$1R2L \sim G_{11} G_{22}^* I [me^{+2j\chi} + \delta_{11} + \delta_{22}^*]$$

$$1R3R \sim G_{11} G_{31}^* I$$

$$1L3L \sim G_{12} G_{32}^* I$$

$$1L3R \sim G_{12} G_{31}^* I [me^{-2j\chi} + \delta_{12} + \delta_{31}^*]$$

$$1R3L \sim G_{11} G_{32}^* I [me^{+2j\chi} + \delta_{11} + \delta_{32}^*] \qquad (13)$$

$$2R3R \sim G_{21}^* G_{31} I$$

$$2L3L \sim G_{22}^* G_{32} I$$

$$2L3R \sim G_{22}^* G_{31} I [me^{+2j\chi} + \delta_{22}^* + \delta_{31}]$$

$$2R3L \sim G_{21}^* G_{32} I [me^{-2j\chi} + \delta_{21}^* + \delta_{32}]$$

where I is the total intensity of the source, m is the percentage linear polarization, and $\chi$ is the position angle of the electric vector. The source is assumed to be unresolved and have negligible circular polarization.

In POLAR, routine CALC fits the observed fringes to expressions like

$$me^{-2j\chi} = GG(1, 2) \cdot 1L2R/1L2L + DD(1, 2)$$

$$= GG(2, 2) \cdot 1L3R/1L3L + DD(2, 2)$$

$$= GG(3, 3) \cdot 2R3L/2L3L + DD(3, 3) \qquad (14)$$

$$me^{+2j\chi} = GG(1, 3) \cdot 1R2L/1L2L + DD(1, 3)$$
$$= GG(2, 3) \cdot 1R3L/1L2L + DD(2, 3)$$
$$= GG(3, 2) \cdot 2L3R/2L3L + DD(3, 2)$$

The subscripts of GG and DD are those used in the program: the first subscript refers to the baseline (1 = 1-2, 2 = 1-3, 3 = 2-3), and the second refers to the polarization (2 = LR, 3 = RL).

From (6) and (7) we get

$$GG(1, 2) = G_{12}\ G_{22}^*/G_{12}\ G_{21}^*$$
$$GG(2, 2) = G_{12}\ G_{32}^*/G_{12}\ G_{31}^*$$
$$GG(3, 2) = G_{22}^*\ G_{32}/G_{22}^*\ G_{31}$$
$$GG(1, 3) = G_{12}\ G_{22}^*/G_{11}\ G_{22}^*$$
$$GG(2, 3) = G_{12}\ G_{32}^*/G_{11}\ G_{32}^*$$
$$GG(3, 3) = G_{22}^*\ G_{32}/G_{21}^*\ G_{32} \qquad (15)$$

and

$$DD(1, 2) = -[\delta_{12} + \delta_{21}^*]$$
$$DD(2, 2) = -[\delta_{12} + \delta_{31}^*]$$
$$DD(3, 2) = -[\delta_{22}^* + \delta_{31}]$$
$$DD(1, 3) = -[\delta_{11} + \delta_{22}^*]$$
$$DD(2, 3) = -[\delta_{11} + \delta_{32}^*]$$
$$DD(3, 3) = -[\delta_{21}^* + \delta_{32}] \qquad (16)$$

GG and DD are the quantities punched out on cards by POLAR, and are used as input to POLCOR to correct the data for instrumental polarization.

Both GG and DD can be checked to see that reasonable values have been derived. We know that

$$1R2R/1L2L = G_{11}\ G_{21}{*}/G_{12}\ G_{22}{*} \quad \text{etc.} \tag{17}$$

Hence

$$G(1,\ 2) \cdot G(1,\ 3) \cdot 1R2R/1L2L \equiv 1 \quad \text{(unit amplitude, zero phase)}$$

$$G(2,\ 2) \cdot G(2,\ 3) \cdot 1R3R/1L3L \equiv 1 \quad \text{"} \quad \text{"}$$

$$G(3,\ 2) \cdot G(3,\ 3) \cdot 2R3R/2L3L \equiv 1 \quad \text{"} \quad \text{"} \tag{18}$$

In (16) we have 6 equations for the 6 unknown $\delta$'s. However, the equations are not independent, so we have the 'closure' condition,

$$DD(1,2)-DD(2,2)-DD(3,3)+DD(3,2){*}+DD(2,3){*}-DD(1,3){*} \equiv 0 \tag{19}$$

In POLAR, the identities (18) and (19) are calculated and printed out for each solution. These help you recognize bad solutions.

## 9. THE EFFECT OF WRONG POLARIZATION VALUES ASSUMED FOR THE CALIBRATION SOURCES

The assumed values of the polarization of the calibrators, which are used as input data to POLAR, will usually be slightly in error, and sometimes grossly so. These lead to errors in the calculated values of DD and GG, and the identities (18) and (19) will no longer be satisfied. If only one calibrator is significantly wrong, then this will be obvious when the solutions are cycled, missing out one source at a time. If more than one calibrator is wrong, it gets a bit more tricky.

Assume that the true value the polarization of the $i^{th}$ source is $P_i = m_i\, e^{2j\chi_i}$, the assumed value is $P_i+\Delta P_i$, and the source enters the solution with weight $\omega_i$. Then the ratio of the calculated to the time value of GG for any correlator is

$$\frac{GG_{calc}}{GG_{true}} = 1 + \frac{\Sigma\ \omega_i\ \Sigma\ \omega_i P_i{}^*\ \Delta P_i\ -\ \Sigma\ \omega_i\ \Delta P_i\ \Sigma\ \omega_i P_i{}^*}{\Sigma\ \omega_i\ \Sigma\ \omega_i P_i{}^* P_i\ -\ \Sigma\ \omega_i P_i\ \Sigma\ \omega_i P_i{}^*} \tag{20}$$

If all the assumed values of the calibration sources are wrong by a constant (complex) factor,

i.e. $\Delta P_i = \alpha P_i$

then $\dfrac{GG_{calc}}{GG_{true}} = 1 + \alpha$ and the right hand sides of (18) are all

equal to $(1+\alpha)\ (1+\alpha^*)$

Also,

$$DD_{calc} - DD_{true} = \frac{\Sigma\omega_i \Delta P_i (\Sigma\omega_i P_i P_i{}^* - DD_{true}\ \Sigma\omega_i P_i{}^*) - \Sigma\omega_i P_i{}^* \Delta P_i\ (\Sigma\omega_i P_i - DD_{true} \Sigma\omega_i)}{\Sigma\ \omega_i\ \Sigma\ \omega_i\ P_i\ P_i{}^*\ -\ \Sigma\ \omega_i\ P_i\ \Sigma\ \omega_i\ P_i{}^*} \tag{21}$$

i.e. $$DD_{cale} = DD_{true} \cdot \frac{GG_{calc}}{GG_{true}} + \frac{\Sigma\omega_i \Delta P_i \Sigma\omega_i P_i P_i{}^* - \Sigma\omega_i P_i{}^* \Delta P_i \Sigma\omega_i P_i}{\Sigma\omega_i \Sigma\omega_i P_i P_i{}^* - \Sigma\omega_i P_i \Sigma\omega_i P_i{}^*} \tag{22}$$

If $\Delta P_i = \Delta P$ is the same for all sources, then $DD_{calc} - DD_{true} = \Delta P$.

If $\Delta P_i$ is random, and $DD_{true}$ is reasonably small, then

$$DD_{calc} - DD_{true} \simeq \frac{\Sigma\omega_i \Delta P_i}{\Sigma\omega_i} = \overline{\Delta P_i} \tag{23}$$

Under these conditions, the closure condition (12) still holds. Other instrumental effects prevent (12) from being satisfied: noise and instabilities in the amplifiers and delay lines, and non-identical band passes.

## 10. BANDWIDTHS, DISPERSION, DISH DEFORMATION

So far, the discussion has assumed that the interferometer is monochromatic, ie., has zero bandwidth. In fact it has a bandwidth of ~30 MHz, so all the quantities defined in section 9 should be replaced by integrals over the bandpass.

Several effects make $\varepsilon$ and $\phi$ in equation (1) vary accross the bandpass. The feeds contain orthogonal linear probes. The signals are transformed into orthogonal circular modes by hybrids. If the linear probes are not perfectly orthogonal, there will be a frequency independent contribution to the instrumental polarization. Missmatches and reflections in the feeds and hybrids introduce additional couplings between the circular modes, and these will vary as a function of frequency accross the bandpass.

Thus, if the normalized transfer function of channel 1R is $H_{11}(\omega)$, then equation (16), for example, should be written as

$$DD(1,\ 2) = -\int_{-\infty}^{\infty} [\delta_{12}(\omega) + \delta_{21}^{*}(\omega)]\ H_{12}(\omega)\ H_{21}{}^{*}(\omega)d\omega \qquad (13)$$

etc.

Equations (18) and (19) are now true only if $H(\omega)$ is the same for all six channels.

The coaxial cables carrying the signals from the telescopes to the control building, and the delay systems are all dispersive to some extent. This has the effect of changing $H(\omega)$, so we expect the instrumental polarization to change with delay and with the configuration of the telescopes.

The dispersion in the cables is caused mainly by the skin effect. This adds a series impedance of the form $Z_s \sim K\,(1 + j)\,\sqrt{\omega/2}$ ohms. Neglecting dielectric losses, we can estimate K from the attenuation of the cables. This is given by

$$\alpha \sim K\,\sqrt{\omega/2}\;/\;2Z_o \times 8.69 \text{ dB/meter}$$

$$\sim 9.2.10^{3} K \left(\frac{\nu}{1\text{MHz}}\right)^{1/2} \text{ dB/100 ft} \quad \text{for } 50\Omega \text{ cables.}$$

For the cables in use, $\alpha \sim .44$ db/100 ft at 35 MHz, and $\sim .26$ dB/100 ft at 5 MHz. The propogation coefficient is now

$$\beta \sim j\,\omega\,\sqrt{LC} + j\,K\,\sqrt{\omega/2}\;/\;2\,Z_o$$

This increases the phase delay in the cable by a factor

$$1 + 750\;K\left(\frac{\nu}{1\text{MHz}}\right)^{-1/2}$$

In the worst case, this can give phase shifts of over 90° at the high end of the band pass. To first order this phase shift is taken out in euqalizing networks inserted in the cables at 300 meter intervals; however, these are adjusted to flatten the amplitude rather than the delay characteristic. Missmatches in cable connectors will also produce configuration dependent changes in $H(\omega)$.

The variable delay lines are specified to have a better than 10° rms phase ripple between 10 MHz and 30 MHz. At the extremes of the bandpass, the phase errors may be considerably larger. Missmatches at the switches will also produce delay dependent effects which may increase with aging. The largest phase errors probably occur at the transducers in front of the larger delays. These have a very low input impedance, and it is difficult to make a satisfactory match over such a wide band pass.

As a dish moves, the surface deforms under gravitational stresses. Any ellipticity in the dish will introduce coupling between the left and right hand circularly polarized modes, and will contribute to the instrumental polarization. These effects should be larger at 8085 MHz and should be apparent on the plot of residuals as a function of hour angle and declination. Since an equatorally mounted dish rotates with the parallactic angle, we would expect dish effects to depend on both elevation and parallactic angle. The overlays in Figs. 1 and 2 should help determine this.

## 11. FEED ROTATION

Equations (2) and (3) show that the instrumental terms depend on orientation of the feeds. For any one correlator, DD is minimized when the major axes of the polarization ellipses of the two feeds are orthogonal. If you determine all the DD terms, and then rotate, one feed and determine them again, you can find out the individual ε's and ϕ's for each feed, using equations (3) and (16). It is then easy to calculate the optimum orientation of each feed so as to minimize the DD terms.

## 12. CONFUSION

At the shortest spacings, estimates of the instrumental polarization may be affected by confusion. There are two contributions to the confusing fringe: 1) Other polarized sources in the primary beam of the telescopes, and 2) sources (not necessairly polarized) in the cross polarized side lobes.

The cross polarized sidelobes have not yet been measured, but the amplitude of the first maximum (located) near the first null in the unpolarized power polar diagram) is expected to be about 2 to 3% of the forward gain. This is slightly larger than usual because the feeds somewhat over illuminate the dishes. The median polarization of extra-galactic sources is about 3.2% at both frequencies, so the two contributions are roughly equally important.

Of course a confusing source will usually have a very different fringe rate then the main source. However, observations over a short period of time, or near cross-over may be appreciably affected.

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