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Phase Drifts in Interferometer Observations of Point Sources

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I. Introduction

The true phase angle of the complex visibility function of a point source is zero at all baseline lengths and orientations. When we derive from observation a phase angle other than zero for such a source, it implies that at least one of the constants used in the solution is incorrect.

The fringe reduction procedure determines the displacement between the fringes actually observed and those we would expect to see for a point source at the same position as the centroid of the source in question. The phase is said to be positive if the fringes come later in time than those expected for the point source. Thus if the observed fringes come one quarter of a fringe period late, the phase is +90°. The locations of the point source fringes are specified completely by the parameters of the interferometer (baseline length in wavelengths, hour angle and declination of the interferometer poles, and the difference in electrical path length from the local oscillator to the two mixers) and the source position (hour angle and declination).

If we find that the phase angles we derive from observations of a point source, using an adopted set of instrumental parameters and an adopted source position, are not always equal to zero, there are three possible explanations.

(a) One or more of the adopted instrumental parameters is incorrect;

(b) The adopted source position is in error;

(c) The local oscillator frequency is unstable, which means that the baseline length in wavelengths is variable.

The following discussion will treat the first two possibilities. The effects of the third are essentially outside the calibration problem.

In general, a non-zero phase angle found for a point source will be the sum of two parts, one constant and the other variable in time. We call the latter the <u>phase drift</u>, and this is the subject of the present report. We shall not consider the constant part further at present.

II. The Phase Drift Equation

The fringe pattern for a point source is

$$f(T) = A \cos \{ 2\pi [B_1 \sin \delta + B_2 \cos \delta \cos (T - \alpha - h) + B_3]\} \qquad (1)$$

where

$$B_1 = \frac{D}{\lambda} \sin d$$

$$B_2 = \frac{D}{\lambda} \cos d$$

B_3 = difference in electrical path length from the local oscillator to the two mixers, expressed in wavelengths.

and

$\frac{D}{\lambda}$ = baseline length in wavelengths

d = declination of the northern instrument pole

h = hour angle of the northern instrument pole

A = fringe amplitude

α = source right ascension

δ = source declination

T = local sidereal time

Equation (1) provides the phase reference used in the reduction of the observations for phase and amplitude. Clearly ($B_1 \sin \delta + B_3$) is a constant for a given source; thus errors in the values adopted for B_1, B_3, or δ will contribute to the constant component of an observed phase error. On the other hand, the second term in the argument of the fringe function is time dependent, and errors in the quantities comprising it will lead to a time dependent phase error; this is the phase drift. We shall assume that the sidereal time is always correct, and consider how errors in B_2, δ, α, and h enter into the phase drift.

A progressive phase drift implies that the reference fringe pattern has the wrong rate. It is easily shown from equation (1) that the fringe rate in cycles per radian is

$$R = B_2 \cos \delta \sin (T - \alpha - h) \qquad (2)$$

We relate an error in R to errors in the quantities on the right-hand side of (2) by differentiating logarithmically; this gives

$$\frac{\Delta R}{R} = \frac{\Delta B_2}{B_2} - \Delta\delta \tan\delta - (\Delta\alpha + \Delta h) \operatorname{ctn} \tau$$

where $\tau = T - \alpha - h$, and $\Delta\delta$, Δh, and $\Delta\alpha$ are in radians.

Now, a cumulative phase drift $\Delta\Phi$ (in degrees) developing in a time ΔT implies that

$$\frac{\Delta R}{R} = \frac{\Delta\Phi}{360^\circ} \cdot \frac{1}{R\Delta T}$$

Hence we have the phase drift equation

$$\boxed{\frac{\Delta B_2}{B_2} - \Delta\delta \operatorname{ctn} \delta - (\Delta\alpha + \Delta h) \operatorname{ctn} \tau = \frac{\Delta\Phi}{360\ R\Delta T}} \qquad (3)$$

Note that there is a constant phase drift due to errors in B_2 and δ, and a time varying phase drift due to errors in h and α.

Now we shall consider how equation (3) can be applied to refining the calibration of the interferometer from observations of a point source with an accurately known position, or to finding an accurate position for a point source when the instrumental parameters are known precisely.

III. The Determination of B_2 and h

Assume that we reduce observations of a point source whose position is known very accurately, using trial values of B_2 and h. The values chosen for B_1 and B_3 are unimportant; any phase drift will be due to the errors in B_2 and h. In this case $\Delta\delta = \Delta\alpha = 0$, and the phase drift equation reduces to

$$\frac{\Delta B_2}{B_2} - \Delta h \operatorname{ctn} \tau = \frac{\Delta\Phi}{360\ R\Delta T} \qquad (4)$$

This is the calibration equation. Our problem is simply to use it to refine the trial values of B_2 and h.

It is best to use the phase drifts at a number of hour angles, and to make a least squares solution of (4) for $\Delta B_2/B_2$ and Δh. This method offers high accuracy, and gives as a by-product a determination of the statistical reliability of the result.

While this is the preferable means of solution, there are two very simple but less accurate procedures worth noting. First, ctn $\tau = 0$ when $\tau = 90°$ (i.e., when the source is 6 hours from crossover). In this case

$$\frac{\Delta B_2}{B_2} = \frac{\Delta\Phi}{360\ R\Delta T}$$

This can be used for a quick check on B_2.

Second, one can determine the phase drift at two hour angles such that $\tau_2 = 180° - \tau_1$. Then ctn $\tau_2 = -$ctn τ_1; furthermore it follows from (2) that R is the same in both cases. We now have

$$\frac{\Delta B_2}{B_2} - \Delta h \text{ ctn } \tau_1 = \frac{\Delta\Phi_1}{360\ R\Delta T}$$

$$\frac{\Delta B_2}{B_2} + \Delta h \text{ ctn } \tau_1 = \frac{\Delta\Phi_2}{360\ R\Delta T}$$

whence

$$\frac{\Delta B_2}{B_2} = \frac{\Delta\Phi_2 + \Delta\Phi_1}{720\ R\Delta T}$$

$$\Delta h \text{ ctn } \tau_1 = \frac{\Delta\Phi_2 - \Delta\Phi_1}{720\ R\Delta T}$$

IV. <u>Determination of Source Position</u>

If we know ΔB_2 and Δh, we can use the phase drift observed on a point source whose position is not precisely known in order to refine its location. Equation (4) gives

$$\Delta\delta \tan\delta + \Delta\alpha \text{ ctn } \tau = \frac{\Delta B_2}{B_2} - \Delta h \text{ ctn } \tau - \frac{\Delta\Phi}{360\ R\Delta T} \qquad (5)$$

Again it is preferable to make the solution by least squares, although one can use short methods analogous to those discussed in the preceding section.