BASIC THEORY OF THE AUTOCORRELATION RADIOMETER

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INTRODUCTION

This article is a brief theoretical description of the autocorrelation radiometer. It is written in a form describing the mathematical operations performed by each part of the system starting at the antenna terminals and proceeding to a final computer output. Most of these operations are definitions or are stated without proof; the emphasis is to give an understanding of the function and design criterion for component parts of the system. The purpose is also to answer the questions, "What, exactly, does the system measure ? How is this affected by gain changes, bandpass variation, and unbalance in switching?"

The block diagram, Figure 1, and the first section on definitions summarize much of the paper and should be used as an aid to the reading of the remaining part of the paper.

DEFINITION OF TERMS

T(f) -- The power spectrum referred to the receiver input. It is the sum of receiver noise, T_R, and the antenna temperature, $T_A(f)$.

G(f) -- The power gain function of the receiver. It is assumed to be a bandpass function centered near 1420 mc, having 1 db bandwidth, B_1, and 20 db bandwidth, B_{20}. The spectrum, T(f), will be measured over the band B_1.

Sx(f) -- The video frequency spectrum of the video signal, x(t). It is the input spectrum, T(f), multiplied by G(f), and hetrodyned to low frequencies between 0 and B_{20}.

$$Sx(f) = G(f + f_0)\ T\ (f + f_0)$$

(In this equation, f is in the video frequency range; $f + f_0$ is approximately 1420 mc.)

$\bar{T}$ -- The average input temperature over the band determined by G(f). It is the quantity which is determined when the receiver is used as a single channel conventional radiometer.

$$\bar{T} = \frac{\int_0^\infty T(f)\, G(f)\, df}{\int_0^\infty G(f)\, df}$$

The spectrum which the autocorrelation system measures is necessarily normalized to $\bar{T}$. Thus, $\bar{T}$ must be measured separately as a scale factor.

y(t) -- The clipped video frequency signal. In terms of the video frequency signal, x(t),

$$y(t) = c \text{ when } x(t) > 0$$

$$y(t) = -c \text{ when } x(t) < 0$$

where c is an arbitrary constant. y(t) is a square wave of peak-to-peak amplitude, 2 c, and random zero crossings.

τ -- y(t) is sampled at intervals τ seconds apart giving samples $y(k\tau)$ where k is an interger. If τ is chosen so that $\tau = 1/2\ B_{20}$ the sampling process does not distort the spectrum which is to be measured.

$\rho_x(\alpha)$, $\rho_y(\alpha)$ -- The normalized autocorrelation functions of x(t) and y(t). The autocorrelation function, $R(\alpha)$ is defined as,

$$R_x(\alpha) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t-\alpha)dt$$

In words, $R_x(\alpha)$ is the time average of a signal, x(t), multiplied by itself delayed by α seconds. $R_x(o)$ is the mean square of x(t) and hence is equal to the time average power when x(t) is the voltage across a one ohm resistor. The normalized autocorrelation function is given by $\rho_x(\alpha) = R_x(\alpha)/R_x(o)$. The Fourier transform of the autocorrelation function is the power spectrum. The Fourier transform of the normalized autocorrelation function is the power spectrum normalized to have unit area. The autocorrelation function can be approximated as a sum, for example,

$$\rho_y(\alpha) = \frac{1}{V_o C^2} \sum_{k=1}^{V_o} y(k\tau)y(k\tau-\alpha)$$

V_m -- The output of the digital correlator is a set of N positive binary numbers, V_o, V_1, $V_2, \ldots, V_{N-1}$. Each of the numbers gives a point on the normalized autocorrelation function,

$$\rho_y(m\tau) = \frac{2V_m - V_o}{V_o} \qquad m = o,\ N - 1$$

Each V_m is accumulated in a counter which is read out at the end of an observation interval (say 10 seconds).

$\omega(m\tau)$ -- A set of numbers ($m = o, N - 1$) which are used to weight the autocorrelation function before Fourier transformation. The Fourier transform of $\omega(m\tau)$ is $W(f)$, which is the bandpass of the equivalent spectral resolving filter. That is, the spectrum that is measured, $T^*(f)$, is the convolution of $T(f)$ and $W(f)$. $W(f)$ is a function having a narrow main lobe of width $\Delta f \simeq 1/N\tau$ and spurious side lobes. $\omega(\tau)$ and $W(f)$ are chosen to give a compromise between narrow Δf and low side lobe level.

$s(f)$ -- The Fourier transform of $\rho_x(m\tau) \cdot \omega(m\tau)$. It is equal to the spectrum of $S_x(f)$ normalized to unit area and convolved with $W(f)$.

$$s(f) = \frac{S_x(f) * W(f)}{\int_o^\infty S_x(f)\, df}$$

$s_o(f)$-- The value of $s(f)$ when $T(f)$ is white noise. It is a measure of $G(f)$, the power gain function of the receiver.

RADIO FREQUENCY PORTION OF THE SYSTEM

The function of the radio frequency portion of the receiver is to select and amplify a band of frequencies centered about some desired frequency, and to hetrodyne this band down to video frequencies. Suppose this video signal is $x(t)$ and the receiver power gain is $G(f)$ having 1 db and 20 db bandwidths of B_1 and B_{20}. If all spurious responses are properly rejected the power spectrum, $S_x(f)$, of $x(t)$ is related to the power spectrum, $T(f)$, at the receiver input by,

$$S_x(f) = G(f + f_0)\ T(f + f_0) \tag{1}$$

where f_0 is a known sum of local oscillator frequencies (assuming all l.o.'s operate below signal frequency). $G(f + f_0)$ is known in that it can be measured by conventional frequency response techniques or, more accurately, by the use of the autocorrelation system with a $T(f + f_0)$ which is known (say white noise). The measurement of an unknown $T(f + f_0)$ is thus reduced to measuring the video spectrum $S_x(f)$.

The spectrum is measured in the bandwidth, B_1, whereas the frequency resolution is determined as a small fraction of B_{20}. Thus, for efficient use of the system,

B_1/B_{20} should be as close to unity as possible. Values of .8 or .9 are in order.

Since the one-bit autocorrelation analysis requires that the video signal have Gaussian statistics, care should be taken (as in any receiver) that the local oscillator signals are much larger (say 30 db) than the signal and that amplifiers are not saturating.

CLIPPING

In preparation for digitalization, the video signal, x(t), is "clipped", giving a signal, y(t), related to x(t) in the ideal case by,

$$y(t) = c \text{ when } x(t) > 0$$

$$y(t) = -c \text{ when } x(t) < 0$$

where c is an arbitrary constant which will be assumed equal to unity in further discussion.

The clipping operation can be made close to ideal by making the rms value of x(t) large ($\approx$ 10 volts) while keeping the clipping uncertainty levels small ($\approx$ 10 mv).

The clipping operation is carried out in practice by cascades of diode clippers and wide band amplifiers.

SAMPLING

The next step in the signal processing is to sample y(t) periodically in time giving discrete values, $y(i\tau)$, where i is an integer and τ is the sampling period. Each sample is +1 or −1 and can be easily stored, delayed, and multiplied in the digital correlator.

If this distortion due to sampling is to be 40 db below the measured spectrum, the sampling period, τ, should be equal to $.5/B_{20}$. In practice sampling is accomplished by the multiplication of y(t) by a periodic train of pulses, each having shape, p(t). Ideally, p(t) is an impulse. The effect of the finite width of p(t) is to multiply the true spectrum by the square of the magnitude of the Fourier transform of p(t). This effect can be made small by making p(t) short compared to $1/B_1$.

MODES OF OPERATION

The autocorrelation system can be used in two ways, which we will call total power mode and switched mode. These modes of operation are similar to those in the conventional radiometer but there are some differences which result because of the clipping operation.

In total power mode the system measures the spectrum of the receiver noise plus antenna temperature. In switched mode the receiver input is switched (at 10 cps to 400 cps rate) between the antenna and a noise source. The correlator is gated in phase with the front-end switch to add the autocorrelation function of the antenna signal plus receiver noise and subtract the autocorrelation function of the noise source signal plus receiver noise. Thus, in switched mode the system measures the difference of the spectra of the antenna signal and the noise source.

In both modes of operation the spectral measurement is highly independent of receiver gain. However, in total power mode the spectral measurement depends (through Equation 1) on the shape of the receiver power gain function G(f). This function, G(f) can be measured by the insertion of white noise into the receiver at some time before or after a spectral measurement is made. An error will result if G(f) changes between the time it is measured and the time the signal spectrum is measured. In switched mode, measurements of G(f) and spectral measurements are performed during short adjacent time intervals and thus there is little time for G(f) to change. The advantage of total power mode is that a high speed switch is not required and the rms deviation of the spectral measurement is halved, thus the sensitivity is doubled.

It is hard to say at this time which mode will be more useful. Provision should be made for the use of both.

Description of further processing of the signal will be divided according to mode.

TOTAL POWER MODE

In total power mode the output of the digital correlator is a set of N positive numbers, V_o, V_1, ..., V_{N-1}, which are related to the input samples, $y(i\tau)$ by the relation,

$$V_m = \sum_{c=1}^{V_o} \frac{1 + y(i\tau)y(i\tau + m\tau)}{2} \qquad m = o, N - 1 \qquad (2)$$

These numbers, when normalized, give the normalized autocorrelation function of y(t),

$$\rho_y(m\tau) = \frac{1}{V_o}\sum_{i=1}^{V_o} y(i\tau)y(i\tau + m\tau) = \frac{2V_m - V_o}{V_o} \tag{3}$$

$$n = o, N - 1$$

It is easily seen that $V_m \leq V_o$ and $\rho_y(m\tau) \leq \rho_y(o) = 1$. The rms deviation (due to finite duration of data) of V_m is approximately $\sqrt{V_o}$ and the rms deviation (normalized to the mean) of the spectrum is approximately $\sqrt{N/V_o}$.

V_o is simply equal to the total number of samples. For 10 mc sampling of 4 receivers, $4 \times 10^8 \simeq 2^{29}$ samples are counted in 10 seconds. 32-bit counters are planned for the NRAO system, thus up to 80 seconds can be allowed between readouts.

Since the rms deviation of V_m is $\sqrt{V_o}$ it can be seen that half of the bits of V_m are significant. The number of bits of V_o which should be used in further processing thus varies from about 13 to 17 depending on the sampling speed and the time between readouts.

The numbers, V_m, are fed into a computer, normalized according to (3), and then corrected by the following relation to give $\rho_x(m\tau)$, the normalized autocorrelation function of the unclipped signal, x(t),

$$\rho_x(m\tau) = \sin \frac{\pi}{2}\rho_y(m\tau) \tag{4}$$

$$m = o, N - 1$$

$\rho_x(m\tau)$ is now multiplied by a weighting function $\omega(m\tau)$. The purpose of this step is to provide a suitable spectral resolving filter. The Fourier transform, W(f), of $\omega(m\tau)$ [considering $\omega(-m\tau) = \omega(m\tau)$] is the bandpass of an equivalent spectral resolving filter. The weighting function, $\omega[m\tau]$, must be equal to unity for m = 0 and must be equal to zero for $m \geq N$. The choice of $\omega(m\tau)$ is governed by a compromise between having a narrow main lobe and having low spurious responses.

The Fourier transformation of $\omega(m\tau)\rho_x(m\tau)$ is performed by the computer, giving a spectral function, s(f),

$$s(f) = \tau\rho_x(o)\omega(o) + 2\tau\sum_{m=1}^{N-1} \rho_x(m\tau)\omega(m\tau)\cos 2\pi f m\tau \tag{5}$$

This function, s(f), is related to the input power spectrum, T(f), by the following relation,

$$s(f) = \frac{[T(f + f_0)G(f + f_0)]^*}{\int_0^\infty T(f + f_0)G(f + f_0)df} \tag{6}$$

where the superscript, *, denotes convolution with W(f), i.e.,

$$[T(f + f_0)G(f + f_0)]^* = \int_0^\infty T(\alpha)G(\alpha)W(f + f_0 - \alpha)d\alpha \tag{7}$$

This operation is similar to the one which arises in spatial coordinates when a source is scanned by an antenna having a gain function equal to W. If T(f)G(f) does not vary over the range where W(f) is non-zero, then

$$[T(f + f_0)G(f + f_0)]^* \approx T(f + f_0)G(f + f_0). \tag{8}$$

In words, Equation (6) states that the quantity that the autocorrelation system measures, s(f), is the input spectrum, $T(f + f_0)$ multiplied by the receiver power gain function, $G(f + f_0)$, smoothed by the filter function, W(f), and normalized to have unit area. The multiplication by G(f) and the normalization are easily removed by the measurement of two auxiliary quantities, $s_o(f)$ and $\bar{T}$.

The function, $s_o(f)$, is simply s(f) when the input, $T(f + f_0)$, is white noise. If we divide s(f) by $s_o(f)$ and assume that G(f) is smooth compared to W(f), we obtain the final result,

$$t(f) = \frac{s(f - f_0)}{s_o(f - f_0)} = \frac{T^*(f)}{\bar{T}} \tag{9}$$

(Note, in this equation, f is of the order of 1420 mc, whereas in previous equations f was in the video range.)

The quantity $\bar{T}$ is the input temperature averaged over the receiver bandpass and is the quantity which is measured by conventional single channel techniques.

$$\bar{T} = \frac{\int_0^\infty T(f)G(f)df}{\int_0^\infty G(f)df} \tag{10}$$

SWITCHED MODE

In the switched mode the receiver input is switched between the antenna terminals and a noise generator. The correlator is gated in phase with the switch to add the autocorrelation function, $\rho_{ys}(m\tau)$, which results when the receiver input is connected to the antenna terminals, and to subtract the autocorrelation function, $\rho_{yn}(m\tau)$, which results when the receiver input is connected to the noise source. Thus, in switched mode the output of the digital correlator is a set of N positive numbers, V_o, V_1, V_2, ... V_{N-1}, which are related to these autocorrelation functions by,

$$\Delta\rho_y(m\tau) = \rho_{ys}(m\tau) - \rho_{yn}(m\tau) = \frac{2V_m - 2V_o}{V_o} \qquad m = o, N-1 \tag{11}$$

The correlator will allow a short period of time (of the order of 1/20 to the switching period) after the switch has changed positions to permit transients in the system to dissapate. During this time the previous N samples of the antenna signal, which are stored in the correlator, would be shifted out; thus no error takes place because of cross-correlation of signal and noise samples. The switching cycle is precisely determined by counts of the sampling pulses. For example, if the sampling rate were 1 mc and a 100 cps switching rate were desired, 4500 products of samples of the antenna signal would be added to a V_m counter, the next 500 products would be discarded, 4500 products of samples of the noise source signal would be subtracted from V_m, and the next 500 products discarded.

The correction due to clipping for switched mode is given by,

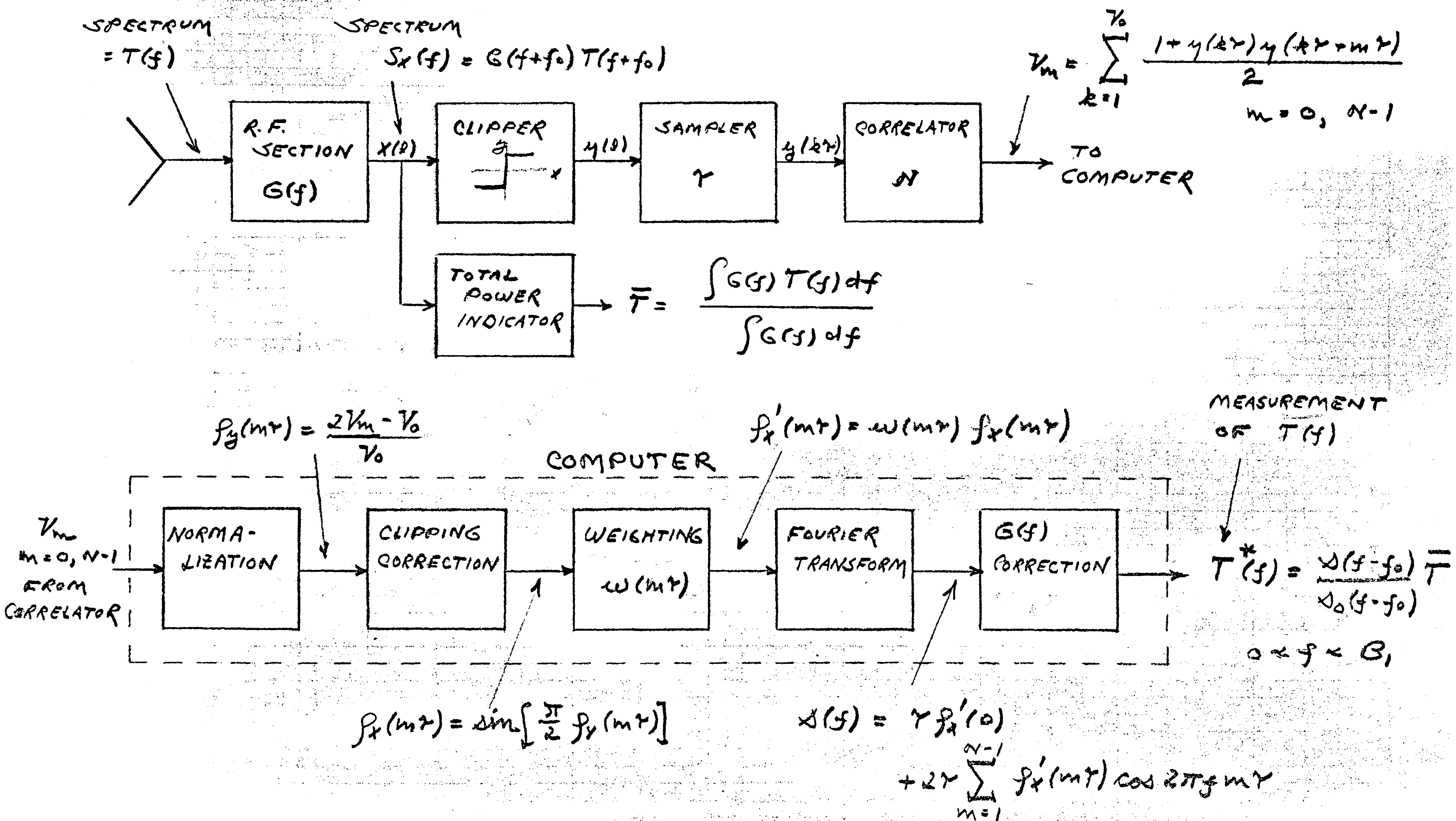
$$\Delta\rho_x(m\tau) = 2\sin\frac{\pi\Delta\rho_y(m\tau)}{4} \cdot \cos\frac{\pi[\Delta\rho_y(m\tau) + 2\rho_{yn}(m\tau)]}{4} \tag{12}$$

Note that $\rho_{yn}(m\tau)$ is needed for the correction; it is measured in total power mode with the switch locked at the noise generator position at same time close the observation time. It is also needed for the calculation of $s_o(f)$.

The weighting, Fourier transformation, and division by $s_o(f)$ operations are similar to those discussed under total power mode. The quantity which is determined (analogous to Equation 9) is

$$\Delta t(f) = \frac{T^*(f) - \bar{\bar{T}}}{\bar{\bar{T}}} \tag{13}$$

where $\bar{T}$ is the average input temperature previously discussed and defined by Equation (10).


SPECTRUM = T(f)
SPECTRUM
Sx(f) = G(f+f0) T(f+f0)
R.F. SECTION G(f)
x(t)
CLIPPER
y(t)
SAMPLER
y(kτ)
CORRELATOR N
TO COMPUTER
Vm = Σ (k=1 to V0) [1 + y(kτ) y(kτ + mτ)] / 2
m = 0, N-1
TOTAL POWER INDICATOR
T̄ = ∫G(f) T(f) df / ∫G(f) df
fy(mτ) = (2Vm - V0) / V0
COMPUTER
fx'(mτ) = w(mτ) fx(mτ)
MEASUREMENT OF T(f)
Vm m=0, N-1 FROM CORRELATOR
NORMA-LIZATION
CLIPPING CORRECTION
WEIGHTING w(mτ)
FOURIER TRANSFORM
G(f) CORRECTION
T*(f) = S(f-f0) / S0(f-f0) T̄
0 < f < B1
fx(mτ) = sin[π/2 fy(mτ)]
S(f) = τ fx'(0) + 2τ Σ (m=1 to N-1) fx'(mτ) cos 2πf mτ