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MEMORANDUM

From: B.L. Ulich

Subject: Absolute Thermal Calibration of Spectral Line Observations

The thermal calibration signal in a millimeter wavelength spectral line receiver is generally derived from a rotating half-disk covered with absorbent material. When the chopper wheel is rotated, the synchronously detected power is proportional to the difference in the temperature of the absorbent material and the sky brightness temperature. We now derive the absolute temperature change seen by the receiver feed horn when the chopper is rotated, and thus the equivalent antenna temperature of the calibration signal. Since millimeter wavelength spectral line receivers are double sideband, we allow the possibility of different gains and atmospheric optical depths at the signal and image frequencies.

When the absorbent material is in front of the feed horn, the antenna temperature is:

$$T_A = G_S T_{AMB} + G_I T_{AMB} \qquad (1)$$

where T_A = Antenna temperature looking at absorbent material (^oK)

T_{AMB} = Ambient temperature (^oK)

G_S = Receiver gain in signal sideband

G_I = Receiver gain in image sideband

Since the receiver is near the ground, we can safely assume that the absorbent material is at ambient temperature. When the chopper wheel is not in front of the feed horn, the antenna temperature is:

$$T_A' = G_S \{\eta_L T_{SKY}(SIGNAL) + (1 - \eta_L) T_{AMB}\} + G_I \{\eta_L T_{SKY}(IMAGE) + (1- \eta_L) T_{AMB}\} \quad (2)$$

where T_A' = Antenna temperature looking at sky (^oK)

η_L = Antenna loss efficiency

T_{SKY} = Sky brightness temperature (^oK)

The antenna loss efficiency η_L includes ohmic loss, blockage by spars and radiometer, and spillover. It is assumed that all absorbed, blocked, or spilled over radiation falls on a blackbody at ambient temperature. The synchronously detected chopper wheel antenna temperature is given by

$$T_{CAL} = T_A - T_A' = G_S \eta_L \{ T_{AMB} - T_{SKY}(SIGNAL)\} + G_I \eta_L \{ T_{AMB} - T_{SKY}(IMAGE) \} \quad (3)$$

where T_{CAL} = Chopper wheel antenna temperature (^oK)

Neglecting the small effect of the 2.7^{o}K cosmic background radiation, the sky brightness temperature is adequately modeled by (Falcone _et al_., 1971, and Ulich, 1973)

$$T_{SKY} = T_M(1 - e^{-\tau x}) \quad (4)$$

where T_{SKY} = Sky brightness temperature (^{o}K)

T_M = Mean atmospheric temperature (^{o}K)

τ = Zenith optical depth of atmosphere

x = Air mass ($\approx$ secant of zenith angle)

The mean atmospheric temperature T_M depends slightly on the zenith optical depth (Kislyakov, 1966) since for large τ only the atmosphere near the antenna is sampled. In this analysis, however, we assume that T_M is independent of τ. The sky brightness temperatures in the two sidebands are given by

$$T_{SKY}(\text{SIGNAL}) = T_M(1 - e^{-\tau_S x}) \tag{5}$$

and
$$T_{SKY}(\text{IMAGE}) = T_M(1 - e^{-\tau_I x}) \tag{6}$$

where T_{SKY}(SIGNAL) = Sky brightness temperature in signal sideband (^{o}K)

T_{SKY}(IMAGE) = Sky brightness temperature in image sideband (^{o}K)

τ_S = Zenith optical depth in signal sideband

τ_I = Zenith optical depth in image sideband

Inserting Eq.5 and Eq.6 into Eq.3 we get for the chopper wheel antenna temperature

$$T_{CAL} = G_S\eta_L\,\{T_{AMB} - T_M(1 - e^{-\tau_S x})\} + G_I\eta_L\,\{T_{AMB} - T_M(1 - e^{-\tau_I x})\} \tag{7}$$

The antenna temperature of a spectral line in the signal sideband is given by

$$T_A'' = G_S\eta_B\eta_L T_B e^{-\tau_S x} \tag{8}$$

where T_A'' = Antenna temperature of the spectral line (^{o}K)

η_B = Beam coupling efficiency

T_B = Brightness temperature of the spectral line (^{o}K)

The beam coupling efficiency η_B is the normalized convolution of the antenna power pattern and the source brightness distribution. For sources very large compared to the half power beamwidth, such as the sky itself, $\eta_B = 1$. Note that the chopper wheel calibration and the line observation are assumed to be made at the same air mass x.

The directly measured quantity in a spectral line observation is the ratio R of the peak line intensity to the chopper wheel calibration signal

$$R = \frac{T_A''}{T_{CAL}} = \frac{G_S \eta_B \eta_L T_B e^{-\tau_S x}}{T_{CAL}} \qquad (9)$$

where R = Ratio of spectral line peak intensity to chopper wheel calibration signal

Solving for T_B we get

$$T_B = \frac{Re^{\tau_S x}}{\eta_B G_S \eta_L} \Big[G_S \eta_L \{T_{AMB} - T_M(1 - e^{-\tau_S x})\} + G_I \eta_L \{T_{AMB} - T_M(1 - e^{-\tau_I x})\} \Big]$$

$$= \frac{R}{\eta_B} \Big[(1 + \frac{G_I}{G_S})(T_{AMB} - T_M) e^{\tau_S x} + T_M + \frac{G_I}{G_S} T_M e^{(\tau_S - \tau_I)x} \Big] \qquad (10)$$

We define the absolute calibration temperature C as

$$C = T_M + \frac{G_I}{G_S} T_M e^{(\tau_S - \tau_I)x} + \left(1 + \frac{G_I}{G_S}\right)(T_{AMB} - T_M) e^{\tau_S x} \quad (11)$$

where C = Absolute calibration temperature (^oK)

Thus $$T_B = \frac{RC}{\eta_B} \quad (12)$$

The first term of C is the constant term independent of optical depth and air mass. The second term corrects for the different gains and zenith optical depths of the two sidebands. The last term accounts for the fact that the sky is always colder than the chopper wheel and thus the sky fails to emit enough radiation to completely correct for absorption of the spectral line radiation.

The mean atmospheric temperature T_M exhibits a small dependence on the surface ambient temperature. The relationship derived by Altshuler et al. (1968) is

$$T_M = 1.12\ T_{AMB} - 50 \quad (13)$$

Substituting Eq.13 into Eq.11 we get

$$C = (1.12T_{AMB} - 50) + \frac{G_I}{G_S}(1.12T_{AMB} - 50)e^{(\tau_S - \tau_I)x} + \left(1 + \frac{G_I}{G_S}\right)(50 - 0.12T_{AMB})e^{\tau_S x}. \quad (14)$$

Eq.14 should be used to derive the calibration temperature when T_{AMB}, G_I/G_S, τ_S, τ_I and x are known.

If $G_I = G_S$:

$$C = (1.12T_{AMB} - 50)(1 + e^{(\tau_S - \tau_I)x}) + (100 - 0.24T_{AMB})e^{\tau_S x} \quad (15)$$

If $G_I = G_S$ and $\tau_S = \tau_I = \tau$:

$$C = (2.24T_{AMB} - 100) + (100 - 0.24T_{AMB})e^{\tau x} \quad (16)$$

Eq.15 is valid when the signal and image gains are equal. Eq.16 holds for equal gains and equal optical depths in both sidebands. Note that in the center of a broad atmospheric window τ_S and τ_I are very nearly identical, but close to an atmospheric absorption line they can be quite different. In this case, the second term in Eq.14 may become appreciable, and values of C larger than $2T_{AMB}$ are possible.

The traditional chopper wheel calibration temperature of 400°K is incorrect, and the published antenna (or brightness) temperatures for the $^{12}C^{16}O$ J = 1 → 0 transition at 2.6 MM are considerably in error.

REFERENCES

ALTSHULER, E.E., FALCONE, V.J., AND WULFSBERG, K.N. (1968). Atmospheric effects on propagation at millimeter wavelengths. IEEE Spectrum 5, 83-90.

FALCONE, V.J., WULFSBERG, K.N., AND GITELSON, S. (1971). Atmospheric emission and absorption at millimeter wavelengths. Radio Science 6, 347-355.

KISLYAKOV, A.G. (1966). Effective path length and mean temperature of the atmosphere. Radiofizika 9, 451-461.

ULICH, B.L. (1973). Absolute brightness tempeature measurements at millimeter wavelengths. The University of Texas at Austin, Electrical Engineering Research Laboratory Technical Report No. NGL-006-73-1.