140-Foot Telescope Pointing Calibration

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Summary

Expressions for the pointing corrections necessary to compensate for encoder angular offsets, beam collimation errors, axis perpendicularity errors, polar axis orientation errors, reflector and yoke flexures, encoder eccentricities, and cyclic errors are derived.

The accuracy with which corrections must be computed, and specially the influence of atmospheric conditions, are discussed.

Optimum procedures for single source observations are analyzed, and an example of gaussian beam fitting is given.

A tentative list of calibration sources is included with some discussion on the overall observing procedure.

A performance index is defined and applied to the determination of the optimum observing wavelength. A programming example of pointing correction parameter fitting is included with an application to data prepared from the standard pointing correction curves, which are also reproduced.

Expressions for structural and thermal focal point changes are derived and discussed.

Definition of rectangular coordinate systems

Four cartesian coordinate systems will be defined. The axis orientations have been chosen so that the number of rotation matrices necessary to perform transformations between systems is minimized, while reasonably intuitive designations are preserved. Other choices to suit different tastes are possible without altering in substance the resulting pointing equations.

Astrometric system

- X_1 positive towards the source
- Y_1 on the hour angle semiplane through the source, oriented positive towards increasing declinations
- Z_1 perpendicular to the hour angle semiplane through the source, oriented positive towards increasing hour angles.

Intermediate equatorial system

X_2 positive towards the intersection of the hour angle semiplane through the source with the equator

Y_2 positive towards the north pole

Z_2 identical with Z_1

Equatorial system

X_3 positive towards the intersection of the meridian elevated semiplane with the equator

Y_3 identical with Y_2

Z_3 positive towards the west cardinal point

Altazimuth system

X_4 positive towards the south cardinal point

Y_4 positive towards the zenith

Z_4 identical with Z_3

Abbreviations

D	declination
H	hour angle
L	latitude
P	position angle
ΔD	error in declination
ΔH	error in hour angle
ΔP	error in position angle
s	sine
c	cosine
t	tangent

Conversion between systems

Transformations are given in one sense only. To express the inverse transformation note that the inverse of a rotation matrix is equal to its transpose (because rotation matrices are orthonormal).

Astrometric to intermediate equatorial

Rotation of -D in the counterclockwise sense around the Z_1 axis

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} cD & -sD & 0 \\ sD & cD & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Intermediate equatorial to equatorial

Counterclockwise rotation of H around the Y_2 axis

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} = \begin{bmatrix} cH & 0 & -sH \\ 0 & 1 & 0 \\ sH & 0 & cH \end{bmatrix} \cdot \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}$$

Equatorial to altazimuth

Counterclockwise rotation of -(90-L) around the Z_3 axis

$$\begin{bmatrix} X_4 \\ Y_4 \\ Z_4 \end{bmatrix} = \begin{bmatrix} sL & -cL & 0 \\ cL & sL & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}$$

Derivation of error terms

Notice that the various error constants that appear may be often given an arbitrary sign. A basic physical interpretation for an error is given, but other error terms may have the same functional form and will eventually be added together in a single term for computation. The hour angle errors are better expressed as $\Delta H.\ cD$, the angular error in the sky (error along the Z_1 axis of the astrometric system).

Index error in declination

Corresponds to an arbitrary rotation of the encoder relative to the shaft on which it is mounted.

$$\Delta D = A_1$$

$$\Delta H.cD = 0$$

Index error in hour angle

Similar in concept to the declination term

$$\Delta D = 0$$

$$\Delta H.cD = A_2.cD$$

Collimation error

Due to the error in perpendicularity between the antenna beam axis and the declination axis. The angular error in hour angle ΔH.cD is constant and equal to the collimation error

$$\Delta D = 0$$

$$\Delta H.cD = A_3$$

Perpendicularity error

Caused by the error in perpendicularity between the declination axis and the hour angle axis. Consider a perfect telescope in which the perpendicularity error is zero. We may transform the beam of this ideal telescope into the beam of the real telescope by a rotation vector of magnitude equal to the perpendicularity error angle, oriented along X_2 in the intermediate equatorial system. The components of this vector in the astrometric system are the pointing errors (ΔP, ΔH cD, ΔD) and may be computed as follows

$$\begin{bmatrix} \Delta P \\ \Delta HcD \\ \Delta D \end{bmatrix} = \begin{bmatrix} cD & sD & 0 \\ -sD & cD & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_4 \\ 0 \\ 0 \end{bmatrix}$$

so that

$$\Delta D = 0$$

$$\Delta HcD = -sD.A_4$$

where the minus sign may be absorbed in the arbitrary constant A_4

Polar axis orientation errors

These are due to the polar axis not pointing precisely to the true celestial pole. As indicated for the preceding case, we transform between the ideal beam and the actual beam by applying a suitable rotation vector. In this case the equatorial system is the most convenient and the rotation vector is represented in it by $(A_5, 0, A_6)$. Transforming this vector into the astrometric system we obtain:

$$\begin{bmatrix} \Delta P \\ \Delta H\ cD \\ \Delta D \end{bmatrix} = \begin{bmatrix} cH\ cD & sD & sH\ cD \\ -cH\ sD & cD & -sH\ sD \\ -sH & 0 & cH \end{bmatrix} \cdot \begin{bmatrix} A_5 \\ 0 \\ A_6 \end{bmatrix}$$

and

$$\Delta D = -sH\ A_5 + cH\ A_6$$

$$\Delta H\ cD = -cHsD\ A_5 - sHsD\ A_6$$

Refraction

Ilif and Holt (Journal of Research of the National Bureau of Standards, 67 D, 31, 1963) have investigated atmospheric refraction at 1.9 cm wavelength with a radio sextant tracking the sun. They arrive at the following empirical expression for the refraction angle r in degrees:

$$r = \left(\frac{180}{\pi} \times 10^{-6}\right) \left[\cot h - \frac{D}{(h+E)^F} \right] N_s - \frac{A}{(h+B)^C}$$

where h is the altitude in degrees, A, B, C, D, E, F are positive empirical constants, and N_s is the surface refractivity ($N_s = (n - 1) \times 10^6$, n index of refraction at the surface).

Substituting typical values

$$r = \left(\frac{180}{\pi} \times 10^{-6}\right) \left[\cot h - \frac{43}{(h+0.4)^{2.64}}\right] \cdot 325 \ldots$$

$$- \frac{40}{(h+2.7)^4}$$

The second and third terms in this formula are small correction terms. For the example given above we have

Elevation	2nd and 3rd Terms
20°	- 1."54
30°	- 0."47
40°	- 0."15

We may neglect these terms for practical purposes although there would be no difficulty in including them if it were necessary.

To a very good approximation assume then the refraction angle proportional to the tangent of the zenith distance. The direction of the zenith is given in the altazimuth system by the unit vector (0, 1, 0).

Transforming this vector to the astrometric system we have

$$\begin{bmatrix} X_z \\ Y_z \\ Z_z \end{bmatrix} = \begin{bmatrix} sL\ cHcD - cLsD & -sLcHsD - cLcD & -sLsH \\ cL\ cHcD + sLsD & -cLcHsD + sLcD & -cLsH \\ sH\ sD & -sHsD & cH \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_z \\ Y_z \\ Z_z \end{bmatrix} = \begin{bmatrix} cL\ cH\ cD + sL\ sD \\ -cL\ cH\ sD + sL\ cD \\ -\ cL\ sH \end{bmatrix}$$

Refraction displaces the antenna beam towards the zenith by an amount proportional to the tangent of the zenith distance (tZD)

$$tZD = \frac{\sqrt{Y_z^{\ 2} + Z_z^{\ 2}}}{X_z}$$

and corresponding errors in declination and hour angle are given by

$$\Delta D = A_7 \cdot tZD \cdot \frac{Y_z}{\sqrt{Y_z^{\ 2} + Z_z^{\ 2}}}$$

$$\Delta H.cD = A_7 \cdot tZD \cdot \frac{Z_z}{\sqrt{Y_z^{\ 2} + Z_z^{\ 2}}}$$

or substituting

$$\Delta D = A_7 \cdot \frac{-cL\ cH\ sD + sL\ cD}{cL\ cH\ cD + sL\ sD}$$

$$\Delta H\ cD = A_7\ \frac{-\ cL\ sH}{cL\ cH\ cD + sL\ sD}$$

Reflector flexure

We assume that the North-South bending is proportional to the North-South component of gravity, and similarly for the East-West bending, for any orientation. These assumptions should be good if the structure is not suffering from significant buckling or other structural hysteresis effects.

Because of the symmetry of the structure (the asymmetry introduced by the declination drive is negligible with good balancing) the N-S pointing error is due exclusively to the N-S component of the gravitational loading (and similarly for the E-W pointing error).

We have already computed the direction cosines for the zenith in connection with the refraction section above. Therefore we may write directly

$$\Delta D = A_8\ (-cL\ cH\ sD + sL\ cD)$$

$$\Delta H\ cD = A_9\ (\ -\ cL\ sH)$$

Fork flexure

We use arguments analogous to those employed in the case of the reflector flexure. The components of the unit vector directed to the nadir are (0, -1, 0) in the altazimuth system, and are given in the intermediate equatorial system by

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} sLcH & cLcH & sH \\ -cL & sL & 0 \\ -sLsH & -cLsH & cH \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} cL\ cH \\ sL \\ -cL\ sH \end{bmatrix}$$

Consider now how the distortions of the yoke affect the pointing of the reflector. The reflector is structurally connected to the yoke through the declination bearings and the declination drive. These three points determine a plane. Changes in the orientation of this plane will produce pointing errors. The pointing errors can be conveniently represented by a rotation vector with components (X_r, Y_r, Z_r) in the intermediate equatorial system. Since the yoke is symmetric with respect to the X_2, Y_2 plane, the X_2 and Y_2 components of gravity will contribute only to the Z_r component. The Z_2 component of gravity (antisymmetric with respect to the plane of symmetry) contributes only to the X_r and Y_r components. We have then:

$$\begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix} = \begin{bmatrix} A_{10} & sH \\ A_{11} & sH \\ A_{12} & cH \end{bmatrix}$$

The latitude factors and signs have been absorbed in the arbitrary constants.

Transforming these vectors to the astrometric system, the pointing errors become:

$$\begin{bmatrix} \Delta P \\ \Delta H cD \\ \Delta D \end{bmatrix} = \begin{bmatrix} cD & sD & 0 \\ -sD & cD & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A_{10} & sH \\ A_{11} & sH \\ A_{12} & cH \end{bmatrix}$$

and

$$\Delta D = A_{12}\ cH$$

$$\Delta H cD = -A_{10}\ sD\ sH + A_{11}\ cD\ sH$$

Encoder eccentricity errors

Due to eccentricity between the rotor and stator windings of the inductosyns. The error is sinusoidal with the angle.

$$\Delta D = A_{13}\ sD + A_{14}\ cD$$

$$\Delta H cD = A_{15}\ sH\ cD + A_{16}\ cH\ cD$$

Cyclic errors

Due to the intrinsic encoder cyclic error with a period equal to the angular spacing of the inductosyn poles (1° in our case). This error always exists in some small amount and can be magnified by any significant unbalance between the excitations of the sine and cosine windings.

$$\Delta D = A_{17} \sin \frac{180}{\pi} D + A_{18} \cos \frac{180}{\pi} D$$

$$\Delta HcD = A_{19} \sin \frac{180}{\pi} H \cdot cD + A_{20} \cos \frac{180}{\pi} H \cdot cD$$

Summary of pointing corrections

Let us summarize now all the correction terms described above, consolidating all terms of the same functional form, and renumbering sequentially the arbitrary constants. Note that the "empirical terms" in Pauliny Toth's memo of May 7, 1969 are all recovered in this treatment.

Declination terms $\Delta D = \sum$

A_1	Indexing
$A_2\ sH + A_3\ cH$	Polar axis orientation, Yoke flexure
$A_4 \dfrac{-cL\ cH\ sD + sL\ cD}{cL\ cH\ cD + sL\ sD}$	Refraction
$A_5\ cH\ sD$	Reflector flexure
$A_6\ sD + A_7\ cD$	Eccentricity, reflector flexure
$A_8 \sin \frac{180}{\pi} D + A_9 \cos \frac{180}{\pi} D$	Cyclic error

Hour angle terms $\Delta H \; cD = \sum$

$A_{10} \; cD$	Indexing
A_{11}	Collimation
$A_{12} \; sD$	Perpendicularity
$A_2 \; cH \; sD + A_{14} \; sH \; sD$	Polar axis orientation, Yoke flexure
$A_4 \dfrac{-cL \; sH}{cL \; cH \; cD + sL \; sD}$	Refraction
$A_{15} \; sH$	Reflector flexure
$A_{16} \; cD \; cH + A_{17} \; cD \; sH$	Yoke flexure, eccentricity
$A_{18} \sin \frac{180}{\pi} H \cdot cD + A_{19} \cos \frac{180}{\pi} H \cdot cD$	Cyclic error

Accuracy in the computation of error terms

If we aim for an rms pointing accuracy of 1/10 of the beamwidth at 2 cm, and allow for a large number of random contributions to the pointing error budget (say 16), single contributions should be computed with an rms accuracy of 3 arcseconds.

With this criterion, some error terms may be neglected. Residual periodic errors, in the inductosyn angular encoders, are expected to be under 3 seconds. However, because of difficulties in accurately balancing the drives to the quadrature windings for various temperatures and over a long period of time, it is desirable to preserve a capability to search for periodic errors. It is desirable to have integration times of a fraction of the time required to vary the hour angle by 1° (4 minutes of time), at least for a subset of the observations.

Influence of atmospheric conditions

The angle of refraction is affected by atmospheric temperature, pressure, and humidity. Froome and Essen (The Velocity of Light and Radio Waves, Academic Press 1969, p. 24) give expressions for the index of refraction as a function of atmospheric conditions. A simplified formula (but quite accurate for ordinary conditions and frequencies below 30 GHz) is the following

$$N = (n-1) \times 10^6 = \frac{103\ P_1}{T} + \frac{86}{T}\left(1 + \frac{5750}{T}\right) P_2$$

where T is the absolute temperature, P_1 is the partial pressure of dry air, and P_2 is the partial pressure of water vapor (expressed in mm of mercury).

For a total pressure of 760 mm, 20°C temperature, and 50% relative humidity, (8.9 mm water vapor pressure, refer to psychrometric chart).

$$\frac{dN}{dp} = 0.35$$

where p is the total pressure

$$\frac{dN}{dp_2} = 6.1$$

$$\frac{dN}{dT} = -1.3$$

Under these conditions the refractivity is equal to 320.
The refraction angle is approximately given by

$$r = (n-1) \cdot \mathrm{tg}\ z$$

and its derivatives are

$$\frac{dr}{dp} = 0.07\ \mathrm{tg}\ z \text{ arcseconds/mm total pressure}$$

$$\frac{dr}{dp_3} = 1.28\ \mathrm{tg}\ z \text{ arcseconds/mm water vapor pressure}$$

$$\frac{dr}{dT} = 0.26\ \mathrm{tg}\ z \text{ arcseconds/°C}$$

For a 3" error in the refraction angle, at altitudes of not less than 30°, we can tolerate errors of 25 mm in the total pressure, 1.3 mm in the water vapor pressure, and 6.7°C in the temperature.

Referring to the psychrometric chart, the wet minus dry bulb temperature difference has to be known with an accuracy of about 2°F or 10% in the relative humidity.

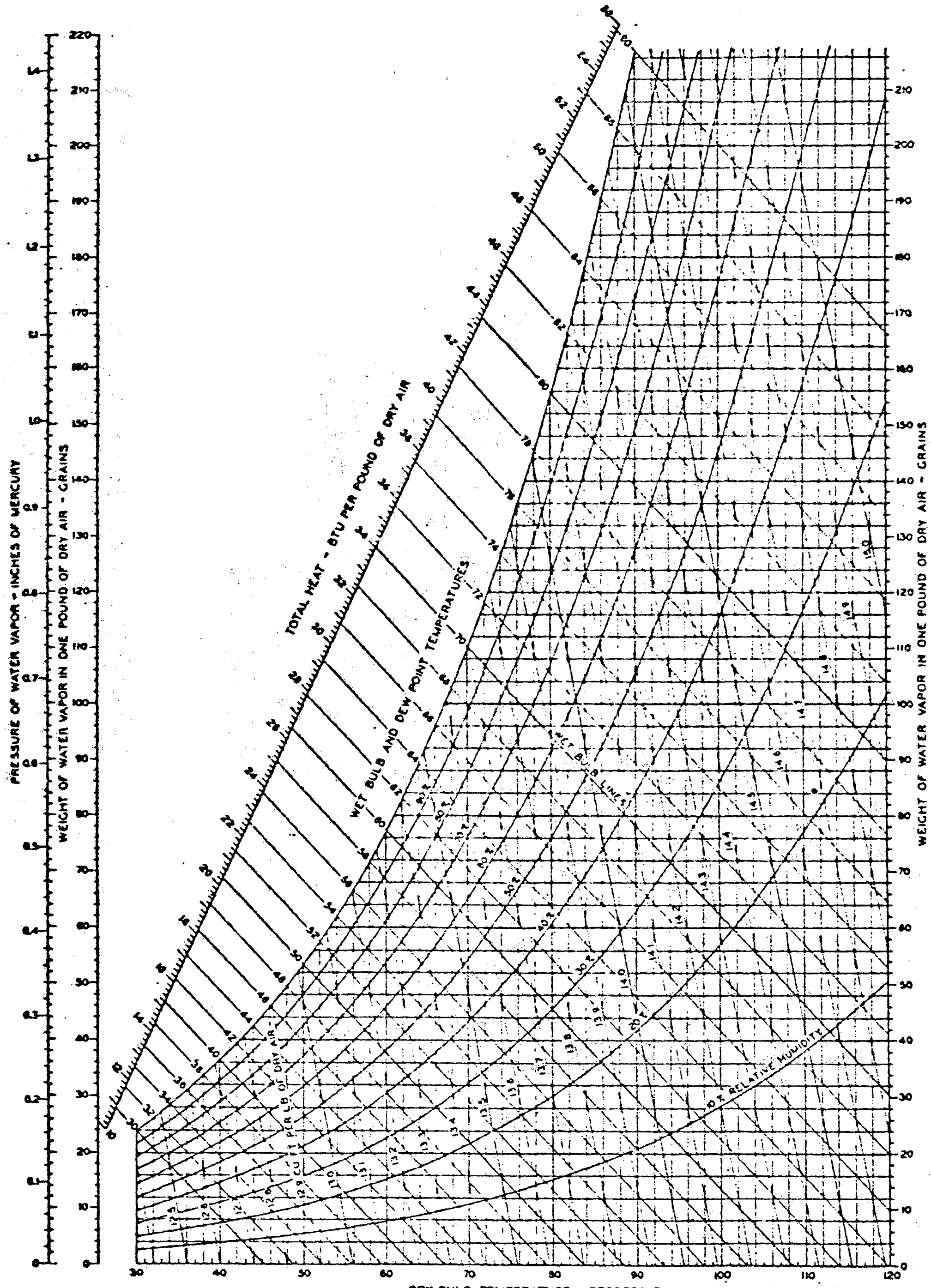


Fig. 1. Psychrometric properties of air at 29.92 inches of mercury absolute pressure.

Observing strategy for single sources

The shape of the main antenna beam can be closely approximated by a Gaussian function of two variables.

For our application, the following are a sufficient number of parameters, to be determined from observations

a) amplitude of the radio source emission

b) antenna beam width, beam assumed centrally symmetric

c) angular offsets of the beam; these are the pointing errors of main interest

d) baseline level at the beam center

e) baseline slopes in declination and hour angle

The antenna beam width should be retained as a free parameter since gravitational and other distortions will change its value with orientation and time. Baseline slopes may be significant because of ground noise pickup through the sidelobes.

We may derive some basic useful relations by considering a one dimensional Gaussian function as follows

$$y = a + b\, e^{-\frac{x^2}{2\sigma^2}}$$

where

y – receiver output

a – baseline level

b – source amplitude

σ – standard deviation (HPBW = 2.35 σ)

x – angle between the source and the beam center

then

$$\frac{dy}{dx} = -\frac{x}{\sigma^2}\, y$$

$$\frac{d^2y}{dx^2} = \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) y$$

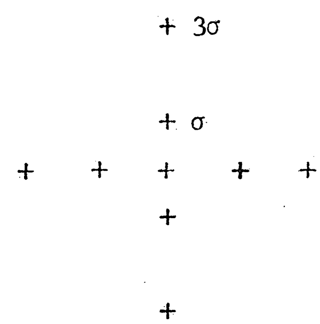
$$\frac{dy}{d\sigma} = \frac{x^2}{\sigma^3}\, y$$

$$\frac{d^2y}{d\sigma^2} = \left(\frac{x^4}{\sigma^6} - \frac{3x^2}{\sigma^4}\right) y$$

From the preceding, it may be concluded that the receiver output displays maximum sensitivity to variations of the angular offsets for $x = \sigma$, and to variations of the beamwidth for $x = \sqrt{3}\,\sigma$

A good determination of the baseline levels and slopes should be made from measurements far from the beam center. An angular offset equal to 1% of the beamwidth (3 arcseconds at 6 cm) will produce output changes of the order of magnitude of 1% of the peak response. For angles of more than 3σ, the amplitude response of the main beam becomes much less than 1%, and this is the minimum distance for good baseline determinations.

Based on the preceding criteria, many observing arrangements are possible. Take as a good example the one represented by the following diagram where the crosses indicate angular offsets from the nominal radio source position at the center.



Angular offsets of more than 3σ may be undesirable since distant confusing sources could be picked up. Scanning modes where the telescope continuously moves across the radio source are possible. Servo transients occur near the beginning and end of a scan, and data taking should begin after the servo settles to a steady velocity. This approach seems more complex to program in real time and later analyze.

Crosses with only 4 points symmetrically distributed around the center are undesirable as they lead to ill determined solutions for the angular offsets.

Several schemes have been tried in numerical simulation. The program reproduced below shows one such example for a case of 4 points asymmetrically distributed about the center.

```
C GA
      DIMENSION
     X OFFSET(2),CR(5),C(50),OFF(10),F(5),
     X WORKA(10),WORKB(10),DELP(10)
      NRAND=720225
      HPBW=0.1
      ARAN=0.2
      DO 120 NSORC=1,10
      CALL RANDU(NRAND,NRAND,FRAND)
      SIGMA=HPBW/2.35+(2*FRAND-1)*HPBW*ARAN
      CALL RANDU(NRAND,NRAND,FRAND)
      HC=FRAND*180-90
      CALL RANDU(NRAND,NRAND,FRAND)
      DC=FRAND*120-30
      CALL RANDU(NRAND,NRAND,FRAND)
      HS=HC+(2*FRAND-1)*SIGMA*ARAN/COS(DC)
      CALL RANDU(NRAND,NRAND,FRAND)
      DS=DC+(2*FRAND-1)*SIGMA*ARAN
      AMP=1.+(2*FRAND-1)*ARAN
      CALL RANDU(NRAND,NRAND,FRAND)
      BASE=(2*FRAND-1)*ARAN
      CALL RANDU(NRAND,NRAND,FRAND)
      DCE=DC
      HCE=HC
      BASEE=0
      AMPE=1
      SIGMAE=HPBW/2.35
C
      OFF(1)=0
      OFF(2)=0
      OFF(3)=HPBW/2
      OFF(4)=0
      OFF(5)=-OFF(3)
      OFF(6)=0
      OFF(7)=0
      OFF(8)=HPBW/(2*COS(DCE/57.3))
      OFF(9)=10*OFF(3)
      OFF(10)=0
      PRINT 130,BASE,AMP,SIGMA,DS,HS
C
      DO 120 NITER=1,10
      DO 100 I=1,5
      CALL COPY(OFF,I,OFFSET,2,5,0)
      D=DC+OFFSET(1)
      H=HC+OFFSET(2)
      HA=(HCE-H)*COS(D/57.3)
      EXPONE=-(HA**2+(DCE-D)**2)/(2*SIGMAE**2)
      CR(1)=1
      CR(2)=EXP(EXPONE)
      CR(3)=-AMPE*CR(2)*(DCE-D)/SIGMAE**2
      CR(4)=-AMPE*CR(2)*HA/SIGMAE**2
      CR(5)=AMPE*CR(2)*(-2*EXPONE/SIGMAE)
      HA=(HS-H)*COS(D/57.3)
      EXPON=-(HA**2+(DS-D)**2)/(2*SIGMA**2)
      F(I)=BASE+AMP*EXP(EXPON)-BASEE-AMPE*EXP(EXPONE)
      DO 110 J=1,5
      CALL LOC(I,J,IR,5,5,0)
      C(IR)=CR(J)
```

```
  110 CONTINUE
  100 CONTINUE
C
      CALL MINV(C,5,DET,WORKA,WORKB)
      CALL GMPRD(C,F,DELP,5,5,1)
      BASEE=BASEE+DELP(1)
      AMPE=AMPE+DELP(2)
      DCE=DCE+DELP(3)
      HCE=HCE+DELP(4)
      SIGMAE=SIGMAE+DELP(5)
      PRINT 130,BASEE,AMPE,SIGMAE,DCE,HCE,NITER,DET
  130 FORMAT(1X,5E12.5,I5,4E12.5)
  120 CONTINUE
C
      STOP
      END
```

-0.44561E-01	0.11153E [illegible]	[illegible].61774E-01	0.24747E 02	[illegible]8436E 02	
-0.44561E-01	0.10954E 01	0.57581E-01	0.24746E 02	0.68434E 02	1-0.62376E 04
-0.44561E-01	0.11142E 01	0.61491E-01	0.24747E 02	0.68436E 02	2-0.26722E 04
-0.44561E-01	0.11153E 01	0.61773E-01	0.24747E 02	0.68436E 02	3-0.20127E 04
-0.44561E-01	0.11153E 01	0.61774E-01	0.24747E 02	0.68436E 02	4-0.19734E 04
-0.44561E-01	0.11153E 01	0.61774E-01	0.24747E 02	0.68436E 02	5-0.19733E 04
-0.44561E-01	0.11153E 01	0.61774E-01	0.24747E 02	0.68436E 02	6-0.19733E 04
-0.44561E-01	0.11153E 01	0.61774E-01	0.24747E 02	0.68436E 02	7-0.19733E 04
-0.44561E-01	0.11153E 01	0.61774E-01	0.24747E 02	0.68436E 02	8-0.19733E 04
-0.44561E-01	0.11153E 01	0.61774E-01	0.24747E 02	0.68436E 02	9-0.19733E 04
-0.44561E-01	0.11153E 01	-0.61774E-01	0.24747E 02	0.68436E 02	10-0.19733E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	
0.16922E-01	0.93088E 00	0.54233E-01	0.95112E 00	-0.11244E 02	1-0.62372E 04
0.16922E-01	0.93511E 00	0.59202E-01	0.94998E 00	-0.11243E 02	2-0.21273E 04
0.16922E-01	0.93551E 00	0.59735E-01	0.94982E 00	-0.11243E 02	3-0.14368E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	4-0.13774E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	5-0.13768E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	6-0.13769E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	7-0.13768E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	8-0.13768E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	9-0.13768E 04
0.16922E-01	0.93552E 00	0.59740E-01	0.94982E 00	-0.11243E 02	10-0.13768E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	
-0.15161E 00	0.10573E 01	0.36459E-01	0.85772E 01	-0.46638E 02	1-0.62370E 04
-0.15161E 00	0.10674E 01	0.36443E-01	0.85769E 01	-0.46638E 02	2-0.76030E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	3-0.80716E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	4-0.80757E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	5-0.80757E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	6-0.80757E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	7-0.80757E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	8-0.80757E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	9-0.80757E 04
-0.15161E 00	0.10676E 01	0.36438E-01	0.85769E 01	-0.46638E 02	10-0.80757E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70777E 02	
-0.10600E 00	0.10345E 01	0.42131E-01	0.84766E 02	0.70780E 02	1-0.62371E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70780E 02	2-0.70772E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70779E 02	3-0.70909E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70779E 02	4-0.70863E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70779E 02	5-0.70827E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70779E 02	6-0.70783E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70778E 02	7-0.70747E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70778E 02	8-0.70716E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70778E 02	9-0.70669E 04
-0.10600E 00	0.10352E 01	0.42122E-01	0.84766E 02	0.70778E 02	10-0.70661E 04
-0.14975E 00	0.94630E 00	0.46254E-01	0.45733E 02	-0.10682E 01	
-0.14975E 00	0.91691E 00	0.45708E-01	0.45733E 02	-0.10734E 01	1-0.62370E 04
-0.14975E 00	0.94401E 00	0.46258E-01	0.45733E 02	-0.10690E 01	2-0.43262E 04
-0.14975E 00	0.94613E 00	0.46254E-01	0.45733E 02	-0.10687E 01	3-0.46076E 04
-0.14975E 00	0.94628E 00	0.46254E-01	0.45733E 02	-0.10684E 01	4-0.46510E 04
-0.14975E 00	0.94629E 00	0.46254E-01	0.45733E 02	-0.10683E 01	5-0.46556E 04
-0.14975E 00	0.94630E 00	0.46254E-01	0.45733E 02	-0.10683E 01	6-0.46564E 04
-0.14975E 00	0.94630E 00	0.46254E-01	0.45733E 02	-0.10682E 01	7-0.46506E 04
-0.14975E 00	0.94630E 00	0.46254E-01	0.45733E 02	-0.10682E 01	8-0.46506E 04
-0.14975E 00	0.94630E 00	0.46254E-01	0.45733E 02	-0.10682E 01	9-0.46506E 04
-0.14975E 00	0.94630E 00	0.46254E-01	0.45733E 02	-0.10682E 01	10-0.46506E 04
0.81320E-01	0.11422E 01	0.48235E-01	0.15754E 02	0.34800E 02	
0.81320E-01	0.11137E 01	0.48066E-01	0.15754E 02	0.34801E 02	1-0.62376E 04
0.81320E-01	0.11422E 01	0.48221E-01	0.15754E 02	0.34800E 02	2-0.59226E 04
0.81320E-01	0.11422E 01	0.48235E-01	0.15754E 02	0.34800E 02	3-0.60317E 04
0.81320E-01	0.11422E 01	0.48235E-01	0.15754E 02	0.34800E 02	4-0.60234E 04
0.81320E-01	0.11422E 01	0.48235E-01	0.15754E 02	0.34800E 02	5-0.60235E 04
0.81320E-01	0.11422E 01	0.48235E-01	0.15754E 02	0.34800E 02	6-0.60235E 04

Distribution of calibration radio sources in the sky

There are at least two possible approaches. One is to observe extensively a small number of bright radio sources with accurately known positions. Another is to observe a large number of sources with comparatively few observations per source.

The first approach is desirable insofar as less slewing time is required, the observing procedure is made simpler, and only a few bright and well known sources are needed. In an altazimuth instrument, where large excursions in both altitude and azimuth are possible while tracking a single source, the first approach may be practical. In an equatorial telescope, the need to obtain a good sampling in declination requires that a substantial number of radio sources be observed. The declination calibration of the 300 foot transit telescope presents in this respect an analogous problem.

Subject to further detailed selection, we could take for a starting point the list of sources contained in Mike Davis' memorandums of September 14 and October 25, 1971. These lists are reproduced below.

There is no simple way to plan in advance a detailed sequence of observations since almost certainly it will be necessary to adapt to circumstances of the moment. While observing, the declination and hour angles should be plotted and observing scheduled to make sure there are no large gaps in the coverage of the sky. This should be done separately during the day and night times, since we have reason to expect better pointing at night and it will be desirable to make separate day and night time analysis.

Calibrators

Source	Ref.	Source	Ref.
3C 8	CT	3C 288	RR
0019-00	RR	3C 289	RR
3C 12	CT	1345+12	RR
3C 20	C	1354+19	CT
3C 22	CT	3C 298	NRI
0106+01	NRI	3C 299	NRI
3C 55	C	3C 303	CT
0229+13	NRI	3C 305	C
3C 79	CT	3C 309.1	NRI
NRAO 140	NRI	3C 317	CT
CTA 26	CT	3C 318	RR
3C 94	CT	3C 319	C
0357-16	CT	3C 323.1	NRI
3C 137	CT	3C 324	RR
3C 153	RR	3C 330	CT
3C 166	C	3C 336	NRI
3C 171	CT	3C 341	CT
3C 173.1	CT	3C 338	C
3C 175.1	C	3C 343	NRI
3C 179	CT	3C 343.1	NRI
3C 180	CT	3C 345	NRI
3C 194	RR	1645+17	RR
3C 196	RR	3C 351	C
3C 200	RR	1732-09	CT
4C 39.25	RR	1756+13	CT
3C 231	CT	3C 371	NRI
3C 245	RR	3C 379.1	C
3C 249.1	NRI	3C 388	CT
3C 254	RR	3C 390	RR
1127-14	NRI	3C 401	RR
3C 263	NRI	3C 409	NRI
3C 265	C	3C 411	CT
3C 268.1	CT	3C 422	CT
3C 273	OPT	3C 427.1	NRI
1229-02	OPT	3C 429	CT
3C 275.1	RR	3C 433	CT
3C 277.1	RR	2127+04	NRI
3C 280	NRI	3C 438	CT
3C 287	NRI	2216-03	OPT
3C 286	NRI	3C 446	NRI
		CTA 102	NRI

C = Cambridge
RR = Malvern
CT = Caltech
NRI = NRAO interferometer
OPT = Optical

Calibrators

3C 8	1229-02
3C 12	1237-10
3C 20	3C 275.1
3C 22	3C 277.1
0106+01	3C 280
3C 43	1306-09
3C 45	3C 287
3C 47	3C 288
3C 52	3C 289
3C 79	1345+12
0310-15	3C 295
4C 32.14	3C 298
CTA 26	3C 299
3C 93	3C 300
3C 93.1	3C 303
3C 94	3C 305
0357-16	3C 318
3C 131	3C 319
3C 132	1523+03
3C 137	3C 323.1
3C 166	3C 324
3C 171	3C 332
3C 173.1	1621-11
3C 175.1	3C 336
0735-17	3C 343
3C 194	3C 343.1
3C 196	1645+17
4C 39.25	3C 351
3C 231	1730-13
3C 244.1	1732-09
3C 245	3C 381
3C 254	3C 390
1127-14	3C 409
1138+01	3C 411
3C 263.1	3C 422
3C 265	3C 424
3C 267	3C 429
3C 268.1	3C 430
3C 268.3	3C 433
3C 273	2127+04
	2216-03

Optimum observing wavelength

A number of factors affect the choice of an operating wavelength for pointing calibration purposes. Let us define a performance index, to allow a quantitative comparison between wavelengths, as follows:

$$PI \propto \frac{1}{HPBW} \times \frac{1}{T \text{ system noise}} \times \text{Bandwidth}^{1/2} \times \text{Mean Source}$$

Flux density x Gain loss factor x Beam distortion factor

The following proportionalities hold for the individual factors

$HPBW \propto \lambda$

T system noise $\propto$ cnt. for centimetric wavelengths

(Provided receivers of comparable quality are available, about 150°K)

Bandwidth $\propto \lambda^{-1}$

Flux density $\propto \lambda^{0.6}$

(Average spectral index for centimetric wavelengths, Toth, Kellerman, Davis, External Galaxies and Quasi Stellar Objects, p. 444, 1972)

Gain loss factor $\propto e^{-\left(\frac{\lambda_o}{\lambda}\right)^2}$

where λ_o represents a short wavelength limit of the telescope.

It is difficult to agree on particular numbers. 2 cm can be considered reasonable for the 140 foot telescope.

Beam distortion $\propto e^{-\left(\frac{\lambda_o}{\lambda}\right)^2}$

A beam distortion factor is required because a distorted beam shape will introduce systematic errors in the Gaussian fitting process. It is difficult to define precisely. Tentatively take the same expression as for the gain loss.

Combining all the above expressions we have

$$P.I \; \alpha \; \lambda^{-0.9} \; e^{-2\left(\frac{\lambda_o}{\lambda}\right)^2}$$

Tabulating this expression for some wavelengths of interest, with an arbitrary proportionality factor:

λ cm	PI
21	6
11	11
6	16
3	15
2	7

The best wavelength appears to be 6 cm.

Example of least squares parameter fit

A program listing is reproduced next to provide an example of application of the previous theory to fit the standard pointing curves, also reproduced for reference.

Different fits have been tried and the one given as an example does not include all the terms previously discussed. The rms of the residuals is 5" in declination and 8" in hour angle, for points in the range 60° to -10°, and 4^h to -4^h in hour angle. 35 points, as indicated in the curves, were included. No detailed description of the program will be provided at this time since it is not definitive and representative of the final formats.

The rms errors are comparable to those obtained by Pauliny Toth, who used observational data and obtained, as should be expected, somewhat larger errors.

```
C POCO
      DIMENSION C(1000),CA(50),CT(1000),CTC(1000),
     X WORKA(50),WORKB(50),
     X PES(100),
     X CLS(1000),
     X DELA(2),DELT(50),PARM(50)
      REAL L
      READ 200,NO
  200 FORMAT(I3)
      PRINT 200,NO
      RASE=4.85E-6
      L=35/57.3
      NP=13
      NRC=0
C
      DO 100 NCO=1,NO
      READ 210,D,H,DELD,DELH
  210 FORMAT(4F10.5)
      PRINT 211,NCO,D,H,DELD,DELH
  211 FORMAT(I5,4F10.5)
      D=D/57.3
      H=H*15/57.3
      DELD=DELD/(60*57.3)
      DELH=DELH*COS(D)/(240*57.3)
      CA(1)=1
      CA(2)=0
      CA(3)=0
      CA(4)=-COS(D)
      CA(5)=0
      CA(6)=-1
      CA(7)=0
      CA(8)=SIN(D)
      CA(9)=-SIN(H)
      CA(10)=COS(H)*SIN(D)
      CA(11)=COS(H)
      CA(12)=SIN(H)*SIN(D)
      CA(13)=(-COS(L)*COS(H)*SIN(D)+SIN(L)*COS(D))/
     X (COS(L)*COS(H)*COS(D)+SIN(L)*SIN(D))
      CA(14)=-COS(L)*SIN(H)/
     X (COS(L)*COS(H)*COS(D)+SIN(L)*SIN(D))
      CA(15)=-COS(L)*COS(H)*SIN(D)+SIN(L)*COS(D)
      CA(16)=0
      CA(17)=0
      CA(18)=-COS(L)*SIN(H)
      CA(19)=0
      CA(20)=SIN(D)*SIN(H)
      CA(21)=SIN(D)
      CA(22)=0
      CA(23)=COS(D)
      CA(24)=0
      CA(25)=0
      CA(26)=COS(H)
      IF(NRC.EQ.0) GO TO 150
      GO TO 160
  150 CALL MCPY(CA,CT,2,NP,0)
      GO TO 161
  160 CONTINUE
      CALL RTIE(C,CA,CT,NRC,NP ,0,0,2)
```

```
  161 CONTINUE
      NRC=NRC+2
      CALL MCPY(CT,C,NRC,NP ,0)
      DELT(NRC-1)=DELD
      DELT(NRC)=DELH
  100 CONTINUE
C
C
      CALL GMTRA(C,CT,NRC,NP )
      CALL GMPRD(CT,C,CTC,NP ,NRC,NP )
      CALL MINV(CTC,NP ,DET,WORKA,WORKB)
      PRINT 112,DET
  112 FORMAT(6E12.4)
      CALL GMPRD(CTC,CT,CLS,NP,NP,NRC)
      CALL GMPRD(CLS,DELT,CTC,NP,NRC,1)
C
      DO 110 I=1,NP
      CTCA=CTC(I)/4.85E-6
      PRINT 111,I,CTCA
  111 FORMAT(I3,F14.3)
  110 CONTINUE
      CALL GMPRD(C,CTC,RES,NRC,NP,1)
      CALL GMSUB(DELT,RES,RES,NRC,1)
      RMSH=0
      RMSD=0
      DO 180 I=1,NU
      RMSD=RMSD+RES(2*I -1)**2
      RMSH=RMSH+RES(2*I )**2
  180 CONTINUE
      RMSD=RMSD/NU
      RMSH=RMSH/NU
      RMSD=SQRT(RMSD)/4.85E-6
      RMSH=SQRT(RMSH)/4.85E-6
      DO 191 I=1,NU
      RESA=RES(2*I-1)/BASE
      RESB=RES(2*I)/BASE
      PRINT 192,I,RESA,RESB
  192 FORMAT(I5,2F12.3)
  191 CONTINUE
      PRINT 190,RMSD,RMSH
  190 FORMAT(2F12.3)
      STOP
      END
```

35

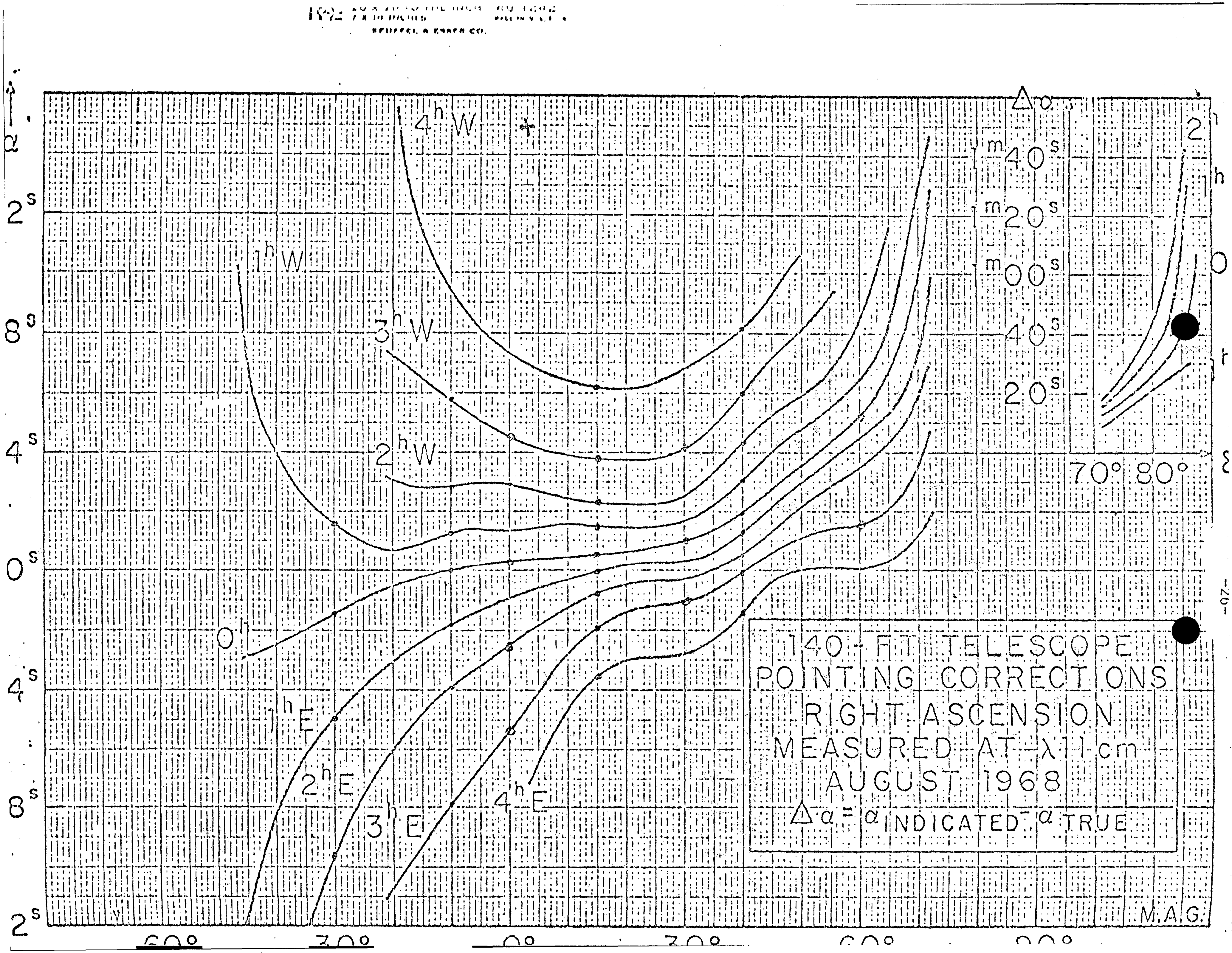
1	60.00000	0.0	1.40000	5.20000
2	60.00000	-3.00000	2.10000	1.60000
3	40.00000	4.00000	1.90000	8.20000
4	40.00000	3.00000	1.60000	6.00000
5	40.00000	2.00000	1.40000	4.30000
6	40.00000	1.00000	1.40000	3.00000
7	40.00000	0.0	1.40000	2.00000
8	40.00000	-1.00000	1.60000	1.30000
9	40.00000	-2.00000	1.60000	0.50000
10	40.00000	-3.00000	2.20000	-0.10000
11	40.00000	-4.00000	2.50000	-1.40000
12	30.00000	3.00000	1.50000	4.20000
13	30.00000	0.0	1.30000	1.00000
14	30.00000	-3.00000	2.20000	-1.00000
15	15.00000	4.00000	2.30000	6.20000
16	15.00000	3.00000	1.60000	3.90000
17	15.00000	2.00000	1.30000	2.30000
18	15.00000	1.00000	1.10000	1.50000
19	15.00000	0.0	1.10000	0.50000
20	15.00000	-1.00000	1.20000	0.0
21	15.00000	-2.00000	1.50000	-0.80000
22	15.00000	-3.00000	2.00000	-1.90000
23	15.00000	-4.00000	2.50000	-3.50000
24	0.0	3.00000	1.50000	4.50000
25	0.0	2.00000	1.20000	3.00000
26	0.0	0.0	1.00000	0.40000
27	0.0	-2.00000	1.40000	-2.50000
28	0.0	-3.00000	2.10000	-5.40000
29	-10.00000	3.00000	1.60000	3.80000
30	-10.00000	2.00000	1.20000	2.30000
31	-10.00000	1.00000	1.00000	1.60000
32	-10.00000	0.0	1.00000	0.50000
33	-10.00000	-1.00000	1.20000	-0.30000
34	-10.00000	-2.00000	1.50000	-1.90000
35	-10.00000	-3.00000	2.00000	-3.50000

0.4511E 03

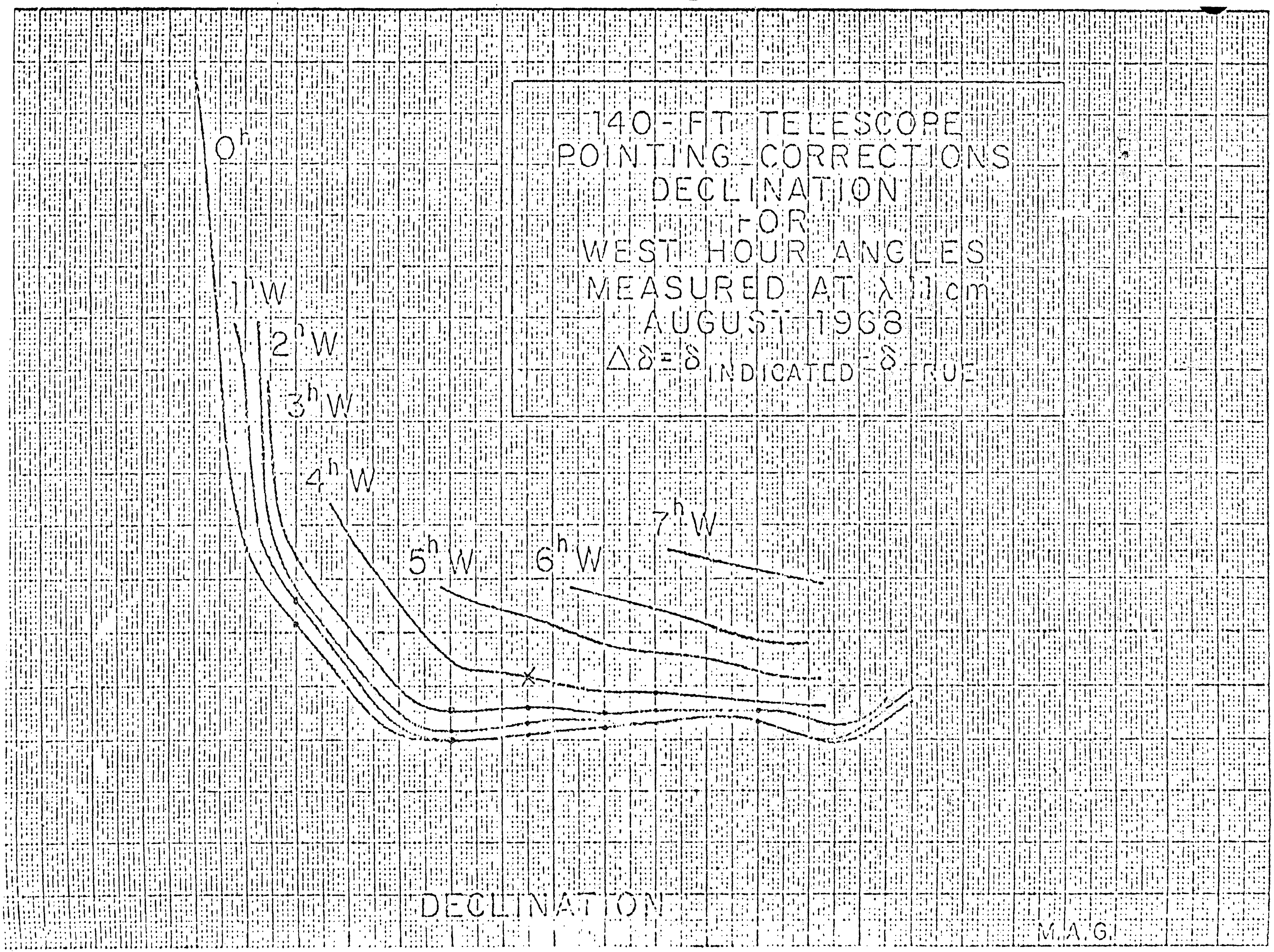
1	302.616
2	52.239
3	-77.283
4	1.888
5	13.066
6	-199.422
7	-31.443
8	-155.312
9	-39.875
10	186.535
11	-127.181
12	56.472
13	-23.982

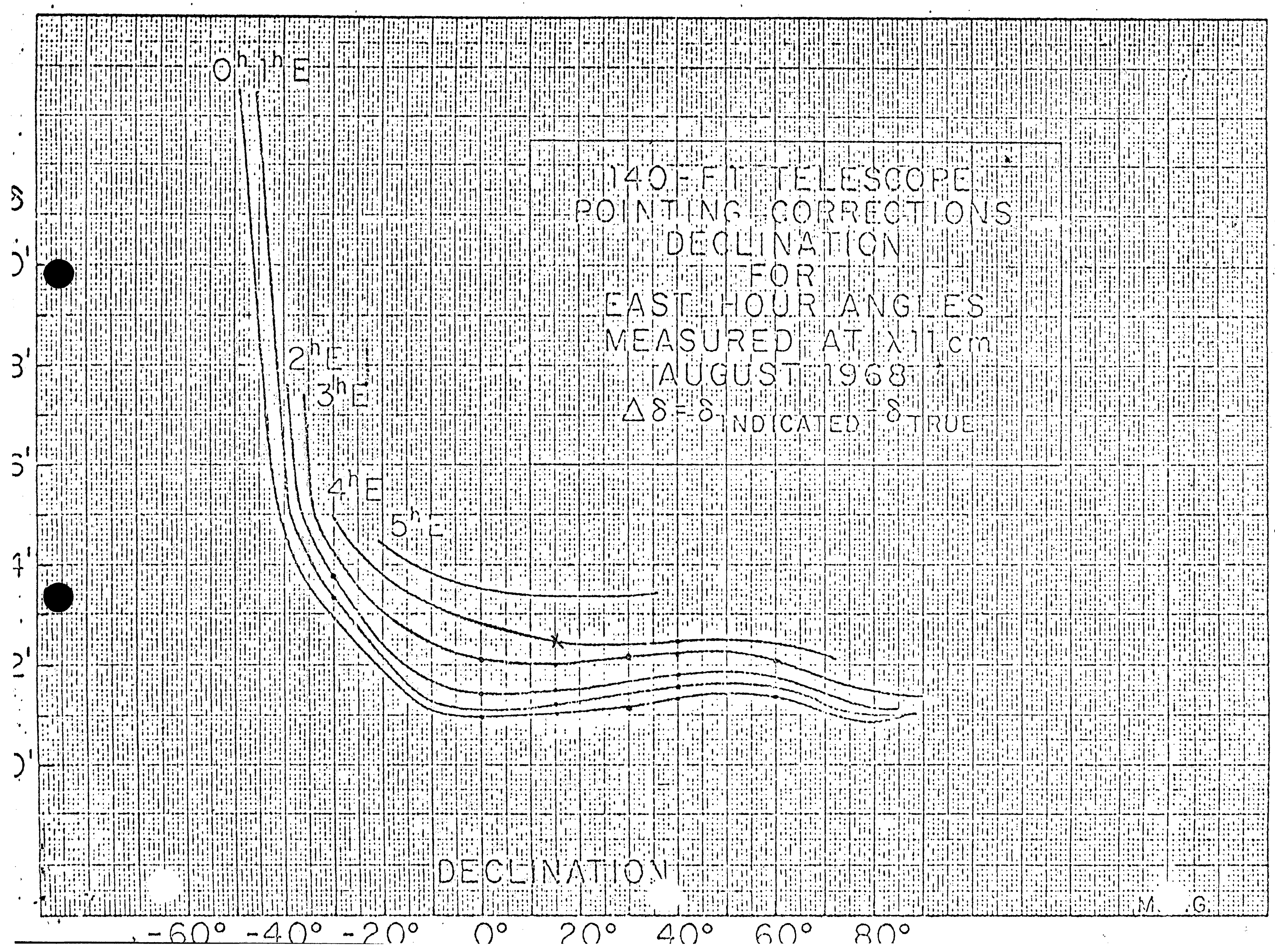
1	-8.335	-5.477
2	-3.763	-0.214
3	-3.229	11.423
4	-2.650	2.245
5	-1.220	-2.764
6	5.672	-4.229
7	4.881	-3.142
8	8.316	0.548
9	4.705	-2.355
10	7.586	6.049
11	1.465	2.087
12	-6.317	-7.407
13	3.813	-5.058
[illegible]	[illegible]	[illegible]

16	2.[illegible]	2.[illegible]
17	-1.196	6.626
18	-2.397	1.440
19	-1.821	-0.771
20	-5.744	5.823
21	-7.266	-7.152
22	-4.240	4.186
23	-4.626	-3.297
24	-7.740	4.837
25	-3.498	6.212
26	-1.217	4.929
27	-9.568	-7.278
28	2.696	-34.514
29	2.582	-14.327
30	1.230	-7.540
31	3.067	5.191
32	4.785	3.985
33	5.711	7.978
34	1.157	9.386
35	1.023	6.079
	5.519	8.368



4hW
3hW
2hW
1hW
0h
1hE
2hE
3hE
4hE
2s
8s
4s
0s
4s
8s
2s
Δα
1m40s
1m20s
1m00s
40s
20s
70° 80°
2h
1h
0
140-FT TELESCOPE
POINTING CORRECTIONS
RIGHT ASCENSION
MEASURED AT λ11cm
AUGUST 1968
Δα = α INDICATED - α TRUE
M.A.G.


140-FT TELESCOPE
POINTING CORRECTIONS
DECLINATION
FOR
WEST HOUR ANGLES
MEASURED AT λ 11cm
AUGUST 1968
Δδ = δ INDICATED − δ TRUE
0h
1hW
2hW
3hW
4hW
5hW
6hW
7hW
DECLINATION
M.A.G.



140-FT TELESCOPE
POINTING CORRECTIONS
DECLINATION
FOR
EAST HOUR ANGLES
MEASURED AT λ11cm
AUGUST 1968
Δδ = δ INDICATED - δ TRUE
0h 1h E
2h E
3h E
4h E
5h E
DECLINATION
-60° -40° -20° 0° 20° 40° 60° 80°

Structural and thermal focal point changes

Focusing is only indirectly related to the pointing calibration, but an understanding of the causes of focal point changes is nevertheless desirable to obtain observations in minimum time and with the highest quality. Eventually, an automatic focusing mode is desirable and in that respect it is important to be able to estimate the best focusing, as accurately as possible.

Gravitational deformations and thermal deformations are the main causes of focus changes. If the gravitational loads are decomposed along the three axis of the astrometric system, it is possible to determine that the Y and Z loadings, antisymmetric with respect to the planes of symmetry, will produce no changes in focal point position. This assumes that reactions at the declination drive are small, so that the asymmetry introduced is negligible (this assumption is normally well justified). The x component of the gravitational load does produce focal changes of the form

$$\Delta f = A_n \cos z$$

where A_n is a constant and z is the zenith distance. Referring to the section on refraction, we can express the cos z and substitute as follows

$$\Delta f = A_n \; (cL\; cH\; cD + sL\; sD)$$

Typically gravitational changes will be of the order of 1 inch. Thermal effects may be quite important. No detailed computations have been carried out for the 140' telescope but representative values can be extracted from 65 meter telescope design (von Hoerner and Herrero, 65 Meter Report No. 37, February 20, 1971).

Focal point motion can be as large as 1 mm per °F of axial temperature gradient across the reflector structure (this thermal mode is the most important). With temperature gradients of up to 10°F in the daytime, changes of 10 mm may occur.

To correct for these effects, measurements with temperature sensors in the telescope structure are necessary. This is a routine procedure with the 36 ft. telescope in Tucson. Present temperature sensors at the 140' telescope were installed with a statistical analysis of temperature differences in mind, and although not optimally located for gradient measurements, will be useful.

Mezger et al, August 1966 report on 140' tests, gives some curves of focal length changes as a function of the coordinates, (Fig. 10). Their usefulness is very limited because no thermal measurements were taken simultaneously.

Thermal focus changes can be set proportional to the observed temperature gradient across the thickness of the backup structure, neglecting other small thermal effects.