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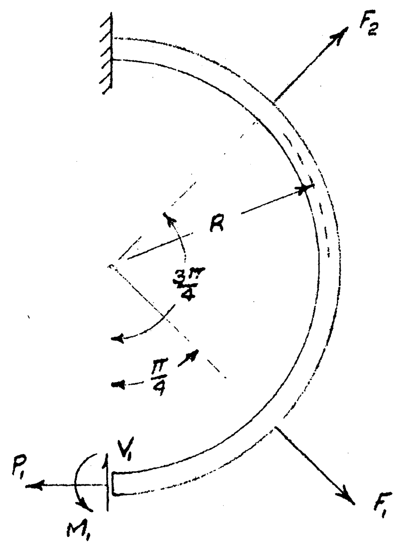
TITLE: DEFLECTION OF TAIL BEARING STRUCTURE

DEFLECTIONS BY CASTIGLIANOS THEORM



STRAIN ENERGY = U

$$U = \tfrac{1}{2}\int_0^{\ell} \frac{M^2}{EI}\, ds$$ (APPROX. – USING BENDING MOMENT ONLY)

DEFLECTION AT LOAD

$$\delta_F = \frac{\partial U}{\partial F} = \int_0^{\ell} \frac{M \frac{\partial M}{\partial F}}{EI}\, ds$$ (HERE $ds = Rd\theta$ WITH LIMITS FROM 0 TO π)

BOUNDARY CONDITIONS

$$\phi = \frac{\partial U}{\partial M_1} = 0$$

$$\delta_{P_1} = \frac{\partial U}{\partial P_1} = 0$$

$$\delta_{V_1} = \frac{\partial U}{\partial V_1} = 0$$

$\theta = 0$ TO $\theta = \frac{\pi}{4}$

$$M = M_1 - V_1 R \sin\theta - P_1 R(1 - \cos\theta)$$

$$\frac{\partial M}{\partial M_1} = 1$$

$$\frac{\partial M}{\partial V_1} = -R \sin\theta$$

$$\frac{\partial M}{\partial P_1} = -R(1 - \cos\theta)$$

$$\frac{\partial M}{\partial F_2} = 0$$

$$\frac{\partial M}{\partial F_1} = 0$$

$\theta = \frac{\pi}{4}$ TO $\theta = \frac{3\pi}{4}$

$$M = M_1 - V_1 R \sin\theta - P_1 R(1 - \cos\theta) + F_1 R \sin(\theta - \tfrac{\pi}{4})$$

$$\frac{\partial M}{\partial M_1} = 1$$

$$\frac{\partial M}{\partial V_1} = -R \sin\theta$$

$$\frac{\partial M}{\partial P_1} = -R(1 - \cos\theta)$$

$$\frac{\partial M}{\partial F_2} = 0$$

$$\frac{\partial M}{\partial F_1} = R \sin(\theta - \tfrac{\pi}{4})$$

$\theta = \frac{3\pi}{4}$ TO $\theta = \pi$

$$M = M_1 - V_1 R \sin\theta - P_1 R(1 - \cos\theta) + F_1 R \sin(\theta - \tfrac{\pi}{4}) + F_2 R \sin(\theta - \tfrac{3\pi}{4})$$

$$\frac{\partial M}{\partial M_1} = 1$$

$$\frac{\partial M}{\partial V_1} = -R\sin\theta$$

$$\frac{\partial M}{\partial P_1} = -R(1 - \cos\theta)$$

$$\frac{\partial M}{\partial F_2} = R\sin(\theta - \tfrac{3\pi}{4})$$

$$\frac{\partial M}{\partial F_1} = R\sin(\theta - \tfrac{\pi}{4})$$

SOLVING FOR M_1 IN TERMS OF V_1, P_1, F_1 & F_2

$$\phi = \frac{\partial U}{\partial M_1} = 0 = \int_0^{\pi} \frac{M \frac{\partial M}{\partial M_1}}{EI} R\,d\theta$$

$$0 = \int_0^{\pi} \frac{[M_1 - V_1 R\sin\theta - P_1 R(1 - \cos\theta)][1]\,R\,d\theta}{EI}$$

$$+ \int_{\pi/4}^{\pi} \frac{[F_1 R \sin(\theta - \frac{\pi}{4})]\,R\,d\theta}{EI} + \int_{3\pi/4}^{\pi} \frac{[F_2 R \sin(\theta - \frac{3\pi}{4})]\,R\,d\theta}{EI}$$

BUT

$$\sin(\theta - \tfrac{\pi}{4}) = (\sin\theta \cos\tfrac{\pi}{4} - \cos\theta \sin\tfrac{\pi}{4})$$

AND

$$\sin(\theta - \tfrac{3\pi}{4}) = (\sin\theta \cos\tfrac{3\pi}{4} - \cos\theta \sin\tfrac{3\pi}{4})$$

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SOLVING FOR M_1 IN TERMS OF V_1, P_1, F_1 & F_2 (Cont)

$$0 = \int_0^{\pi} \left[M_1 - V_1 R \sin\theta - P_1 R(1-\cos\theta)\right] R\, d\theta$$

$$+ \int_{\pi/4}^{\pi} F_1 R\left[\sin\theta \cos\tfrac{\pi}{4} - \cos\theta \sin\tfrac{\pi}{4}\right] R\, d\theta$$

$$+ \int_{3\pi/4}^{\pi} F_2 R\left[\sin\theta \cos\tfrac{3\pi}{4} - \cos\theta \sin\tfrac{3\pi}{4}\right] R\, d\theta$$

$$0 = \left\{ M_1\theta + V_1 R\cos\theta - P_1 R(\theta - \sin\theta)\right\}_0^{\pi}$$

$$+ \left\{ F_1 R\left[-\cos\tfrac{\pi}{4}\cos\theta - \sin\tfrac{\pi}{4}\sin\theta\right]\right\}_{\pi/4}^{\pi}$$

$$+ \left\{ F_2 R\left[-\cos\tfrac{3\pi}{4}\cos\theta - \sin\tfrac{3\pi}{4}\sin\theta\right]\right\}_{3\pi/4}^{\pi}$$

$$0 = \left\{M_1\pi - 2V_1R - P_1R\pi\right\} + \left\{0.707 F_1 R(2.414)\right\} + \left\{F_2 R(0.707)(-1+0.707+0.707)\right\}$$

$$0 = M_1\pi - 2V_1R - P_1R\pi + F_1R(1.7070) + F_2R(0.2927)$$

$$M_1 = \frac{2V_1R}{\pi} + P_1R - F_1R\left(\frac{1.7070}{\pi}\right) - F_2R\left(\frac{0.2927}{\pi}\right)$$

$$M_1 = \frac{R}{\pi}\left(2V_1 + P_1\pi - 1.7070F_1 - 0.2927F_2\right)$$

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<u>SOLVING FOR V_1 IN TERMS OF P_1, F_1 & F_2</u>

$$\delta_{V_1} = 0 = \frac{\partial U}{\partial V_1} = \int_0^{\pi} \frac{M \frac{\partial M}{\partial V_1}}{EI} R\,d\theta$$

$$0 = \int_0^{\pi} \left[M_1 - V_1 R\sin\theta - P_1 R(1-\cos\theta)\right]\left[-R\sin\theta\right] R\,d\theta$$

$$+ \int_{\frac{\pi}{4}}^{\pi} \left[F_1 R \sin(\theta - \tfrac{\pi}{4})\right]\left[-R\sin\theta\right] R\,d\theta$$

$$+ \int_{\frac{3\pi}{4}}^{\pi} \left[F_2 R \sin(\theta - \tfrac{3\pi}{4})\right]\left[-R\sin\theta\right] R\,d\theta$$

$$0 = \int_0^{\pi} \left[-M_1 R\sin\theta + V_1 R\sin^2\theta + P_1 R(\sin\theta - \tfrac{1}{2}\sin 2\theta)\right] d\theta$$

$$+ \int_{\frac{\pi}{4}}^{\pi} F_1 R\left[-\cos\tfrac{\pi}{4}\sin^2\theta + \sin\tfrac{\pi}{4}(\tfrac{1}{2}\sin 2\theta)\right] d\theta$$

$$+ \int_{\frac{3\pi}{4}}^{\pi} F_2 R\left[-\cos\tfrac{3\pi}{4}\sin^2\theta + \sin\tfrac{3\pi}{4}(\tfrac{1}{2}\sin 2\theta)\right] d\theta$$

$$0 = \left\{ M_1\cos\theta + V_1 R(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta) + P_1 R(-\cos\theta + \tfrac{1}{4}\cos 2\theta)\right\}_0^{\pi}$$

$$+ \left\{ F_1 R\left[-\cos\tfrac{\pi}{4}(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta) - \sin\tfrac{\pi}{4}(\tfrac{1}{4}\cos 2\theta)\right]\right\}_{\frac{\pi}{4}}^{\pi}$$

$$+ \left\{ F_2 R\left[-\cos\tfrac{3\pi}{4}(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta) - \sin\tfrac{3\pi}{4}(\tfrac{1}{4}\cos 2\theta)\right]\right\}_{\frac{3\pi}{4}}^{\pi}$$

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SOLVING FOR V_1 IN TERMS OF P_1, F_1 & F_2 (Cont)

$$0 = -2M_1 + V_1R\left(\frac{\pi}{2} - 0\right) + P_1R\left(2 + \frac{1}{4} - \frac{1}{4}\right)$$

$$+ F_1R\left[(-0.707)\left(\frac{3\pi}{8} + \frac{1}{4}\right) - (0.707)\left(\frac{1}{4} - 0\right)\right] + F_2R\left[+0.707\left(\frac{\pi}{8} - 0 - \frac{1}{4} - \frac{1}{4}\right)\right]$$

$$0 = -2M_1 + V_1R\frac{\pi}{2} + 2P_1R + F_1R(-1.1864) + F_2R(-0.0759)$$

$$M_1 = \frac{R}{\pi}\left(2V_1 + P_1\pi - 1.7070F_1 - 0.2927F_2\right)$$

SUBSTITUTING

$$0 = -\frac{2R}{\pi}\left(2V_1 + P_1\pi - 1.7070F_1 - 0.2927F_2\right) + V_1R\frac{\pi}{2} + 2P_1R$$

$$+ F_1R(-1.1864) + F_2R(-0.0759)$$

$$0 = -V_1\left(\frac{4R}{\pi} - \frac{R\pi}{2}\right) + P_1(2R - 2R) + F_1\left(\frac{3.414R}{\pi} - 1.1864R\right)$$

$$+ F_2(0.1863R - 0.0759R)$$

$$0 = -V_1(-0.2984) - 0.0997F_1 + 0.1104F_2$$

$$V_1 = 0.3341F_1 - 0.3700F_2$$

SOLVING FOR P_1 IN TERMS OF F_1 & F_2.

$$\delta_{P_1} = 0 = \frac{\partial U}{\partial P_1} = \int_0^{\pi} \frac{M \frac{\partial M}{\partial P_1}}{EI} R\,d\theta$$

$$0 = \int_0^{\pi} \left[M_1 - V_1 R \sin\theta - P_1 R(1-\cos\theta)\right]\left[-R(1-\cos\theta)\right] R\,d\theta$$

$$+ \int_{\frac{\pi}{4}}^{\pi} F_1 R\left[\sin\left(\theta - \tfrac{\pi}{4}\right)\right]\left[-R(1-\cos\theta)\right] R\,d\theta$$

$$+ \int_{\frac{3\pi}{4}}^{\pi} F_2 R\left[\sin\left(\theta - \tfrac{3\pi}{4}\right)\right]\left[-R(1-\cos\theta)\right] R\,d\theta$$

$$0 = \int_0^{\pi} \left[-M_1(1-\cos\theta) + V_1 R(\sin\theta - \tfrac{1}{2}\sin 2\theta) + P_1 R(1 - 2\cos\theta + \cos^2\theta)\right] d\theta$$

$$+ \int_{\frac{\pi}{4}}^{\pi} F_1 R\left[-\cos\tfrac{\pi}{4}(\sin\theta - \tfrac{1}{2}\sin 2\theta) + \sin\tfrac{\pi}{4}(\cos\theta - \cos^2\theta)\right] d\theta$$

$$+ \int_{\frac{3\pi}{4}}^{\pi} F_2 R\left[-\cos\tfrac{3\pi}{4}(\sin\theta - \tfrac{1}{2}\sin 2\theta) + \sin\tfrac{3\pi}{4}(\cos\theta - \cos^2\theta)\right] d\theta$$

SOLVING FOR P_1 IN TERMS OF F_1 & F_2 (Cont.)

$$0 = \left\{ -M_1(\theta - \sin\theta) + V_1R\left(-\cos\theta + \tfrac{1}{4}\cos 2\theta\right) + P_1R\left(\theta - 2\sin\theta + \tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right) \right\}_0^{\pi}$$

$$+ \left\{ F_1R\left[\cos\tfrac{\pi}{4}\left(-\cos\theta + \tfrac{1}{4}\cos 2\theta\right) + \sin\tfrac{\pi}{4}\left(\sin\theta - \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)\right] \right\}_{\frac{\pi}{4}}^{\pi}$$

$$+ \left\{ F_2R\left[-\cos\tfrac{3\pi}{4}\left(-\cos\theta + \tfrac{1}{4}\cos 2\theta\right) + \sin\tfrac{3\pi}{4}\left(\sin\theta - \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)\right] \right\}_{\frac{3\pi}{4}}^{\pi}$$

$$0 = \left\{ -M_1(\pi - 0) + V_1R\left(2 + \tfrac{1}{4} - \tfrac{1}{4}\right) + P_1R\left(\tfrac{3}{2}\pi\right) \right\}$$

$$+ \left\{ F_1R\left[-0.707\left(1 + 0.707 + \tfrac{1}{4}\right) + 0.707\left(-0.707 - \tfrac{3\pi}{8} + \tfrac{1}{4}\right)\right] \right\}$$

$$+ \left\{ F_2R\left[0.707\left(1 - 0.707 + \tfrac{1}{4} - 0\right) + 0.707\left(-0.707 - \tfrac{\pi}{8} - \tfrac{1}{4}\right)\right] \right\}$$

$$0 = -M_1\pi + 2V_1R + \tfrac{3}{2}\pi P_1R - 2.5396F_1R - 0.5703F_2R$$

SUBSTITUTING FOR M_1

$$0 = -R\left(2V_1 + P_1\pi - 1.7070F_1 - 0.2927F_2\right) + 2V_1R + \tfrac{3}{2}\pi P_1R - 2.5396F_1R - 0.5703F_2R$$

$$0 = \tfrac{\pi}{2}P_1 - 0.8330F_1 - 0.2776F_2$$

$$P_1 = 0.5303F_1 - 0.1767F_2$$

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$$\delta_{F_2} = \frac{\partial U}{\partial F_2} = \int_0^{\pi} \frac{M \frac{\partial M}{\partial F_2}}{EI} R\,d\theta$$

$$\delta_{F_2} = \frac{1}{EI}\int_0^{\frac{\pi}{4}} \left[M_1 - V_1 R \sin\theta - P_1 R(1-\cos\theta)\right]\left[0\right] R\,d\theta$$

$$+\frac{1}{EI}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left[M_1 - V_1 R \sin\theta - P_1 R(1-\cos\theta) + F_1 R \sin(\theta-\tfrac{\pi}{4})\right]\left[0\right] R\,d\theta$$

$$+\frac{1}{EI}\int_{\frac{3\pi}{4}}^{\pi} \left[M_1 - V_1 R \sin\theta - P_1 R(1-\cos\theta) + F_1 R \sin(\theta-\tfrac{\pi}{4}) + F_2 R \sin(\theta-\tfrac{3\pi}{4})\right]\left[R \sin(\theta-\tfrac{3\pi}{4})\right] R\,d\theta$$

$$\delta_{F_2} = \frac{R^2}{EI}\int_{\frac{3\pi}{4}}^{\pi} \Big\{ M_1\left(\cos\tfrac{3\pi}{4}\sin\theta - \sin\tfrac{3\pi}{4}\cos\theta\right) - V_1 R\left(\cos\tfrac{3\pi}{4}\sin^2\theta - \tfrac{1}{2}\sin\tfrac{3\pi}{4}\sin 2\theta\right)$$

$$- P_1 R\left[\cos\tfrac{3\pi}{4}\left(\sin\theta - \tfrac{1}{2}\sin 2\theta\right) - \sin\tfrac{3\pi}{4}\left(\cos\theta - \cos^2\theta\right)\right]$$

$$+ F_1 R\left[\cos\tfrac{3\pi}{4}\cos\tfrac{\pi}{4}\sin^2\theta - \tfrac{1}{2}\cos\tfrac{3\pi}{4}\sin\tfrac{\pi}{4}\sin 2\theta - \tfrac{1}{2}\cos\tfrac{\pi}{4}\sin\tfrac{3\pi}{4}\sin 2\theta + \sin\tfrac{3\pi}{4}\sin\tfrac{\pi}{4}\cos^2\theta\right]$$

$$+ F_2 R\left[\cos^2\tfrac{3\pi}{4}\sin^2\theta - \cos\tfrac{3\pi}{4}\sin\tfrac{3\pi}{4}\sin 2\theta + \sin^2\tfrac{3\pi}{4}\cos^2\theta\right]\Big\}\,d\theta$$

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$$\delta_{F_2} = \frac{R^2}{EI}\Big\{ M_1\left(-\cos\tfrac{3\pi}{4}\cos\theta - \sin\tfrac{3\pi}{4}\sin\theta\right) - V_1R\Big[\left(\cos\tfrac{3\pi}{4}\right)\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)$$

$$+\tfrac{1}{4}\sin\tfrac{3\pi}{4}\cos 2\theta\Big] - P_1R\Big[\cos\tfrac{3\pi}{4}\left(-\cos\theta + \tfrac{1}{4}\cos 2\theta\right) - \sin\tfrac{3\pi}{4}\left(\sin\theta - \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)\Big]$$

$$+F_1R\Big[\cos\tfrac{3\pi}{4}\cos\tfrac{\pi}{4}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right) + \tfrac{1}{4}\cos\tfrac{3\pi}{4}\sin\tfrac{\pi}{4}\cos 2\theta$$

$$+\tfrac{1}{4}\cos\tfrac{\pi}{4}\sin\tfrac{3\pi}{4}\cos 2\theta + \sin\tfrac{3\pi}{4}\sin\tfrac{\pi}{4}\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right)\Big]$$

$$+F_2R\Big[\cos^2\tfrac{3\pi}{4}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right) + \tfrac{1}{2}\cos\tfrac{3\pi}{4}\sin\tfrac{3\pi}{4}\cos 2\theta + \sin^2\tfrac{3\pi}{4}\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right)\Big]\Big\}\Bigg|_{3\pi/4}^{\pi}$$

$$\delta_{F_2} = \frac{R^2}{EI}\Big\{ M_1\left[(0.707)(-1+0.707) - 0.707(0-0.707)\right] - V_1R\left[(-0.707)\left(\tfrac{\pi}{8} - \tfrac{1}{4}\right) + 0.707\left(\tfrac{1}{4}\right)\right]$$

$$-P_1R\left[-0.707\left(1 - 0.707 + \tfrac{1}{4} - 0\right) - 0.707\left(0 - 0.707 - \tfrac{\pi}{8} - \tfrac{1}{4}\right)\right]$$

$$+F_1R\left[-0.500\left(\tfrac{\pi}{8} - \tfrac{1}{4}\right) - 0.500\left(\tfrac{1}{4} - 0\right) + 0.500\left(\tfrac{1}{4} - 0\right) + 0.500\left(\tfrac{\pi}{8} + 0 + \tfrac{1}{4}\right)\right]$$

$$+F_2R\left[0.500\left(\tfrac{\pi}{8} - 0 - \tfrac{1}{4}\right) - 0.500\left(\tfrac{1}{2} - 0\right) + 0.500\left(\tfrac{\pi}{8} + 0 + \tfrac{1}{4}\right)\right]$$

$$\delta_{F_2} = \frac{R^2}{EI}\left\{0.2927M_1 - 0.0759V_1R - 0.5703P_1R + 0.2500F_1R + 0.1427F_2R\right\}$$

Substituting for M_1

$$\delta_{F_2} = \frac{R^2}{EI}\Big\{\frac{0.2927R}{\pi}\left(2V_1 + P_1\pi - 1.7070F_1 - 0.2927F_2\right) - 0.0759V_1R - 0.5703P_1R$$

$$+0.2500F_1R + 0.1427F_2R\Big\}$$

$$\delta_{F_2} = \frac{R^3}{EI}\left\{+0.1164\,V_1 - 0.2776\,P_1 + 0.0910\,F_1 + 0.1154\,F_2\right\}$$

SUBSTITUTING FOR V_1 & P_1

$$\delta_{F_2} = \frac{R^3}{EI}\left\{-0.0193\,F_1 + 0.0256\,F_2\right\}$$

$$\delta_{F_1} = \frac{\partial U}{\partial F_1} = \int_0^{\pi} \frac{M\frac{\partial M}{\partial F_1}}{EI}\,R\,d\theta$$

$$\delta_{F_1} = \frac{1}{EI}\int_0^{\frac{\pi}{4}}\left[M_1 - V_1 R\sin\theta - P_1 R(1-\cos\theta)\right]\left[0\right] R\,d\theta$$

$$+\frac{1}{EI}\int_{\frac{\pi}{4}}^{\pi}\left[M_1 - V_1 R\sin\theta - P_1 R(1-\cos\theta) + F_1 R\sin(\theta-\tfrac{\pi}{4})\right]\left[R\sin(\theta-\tfrac{\pi}{4})\right] R\,d\theta$$

$$+\frac{1}{EI}\int_{\frac{3\pi}{4}}^{\pi}\left[F_2 R\sin(\theta-\tfrac{3\pi}{4})\right]\left[R\sin(\theta-\tfrac{\pi}{4})\right] R\,d\theta$$

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$$\delta_{F_1} = \frac{R^2}{EI}\int_{\pi/4}^{\pi}\Big\{ M_1\left(\sin\theta\cos\tfrac{\pi}{4} - \cos\theta\sin\tfrac{\pi}{4}\right) - V_1R\left(\cos\tfrac{\pi}{4}\sin^2\theta - \tfrac{1}{2}\sin\tfrac{\pi}{4}\sin 2\theta\right)$$

$$- P_1R\left[\left(\cos\tfrac{\pi}{4}\right)\left(\sin\theta - \tfrac{1}{2}\sin 2\theta\right) - \sin\tfrac{\pi}{4}\left(\cos\theta - \cos^2\theta\right)\right]$$

$$+ F_1R\left[\cos^2\tfrac{\pi}{4}\sin^2\theta - \sin\tfrac{\pi}{4}\cos\tfrac{\pi}{4}\sin 2\theta + \sin^2\tfrac{\pi}{4}\cos^2\theta\right]\Big\}\,d\theta$$

$$+ \frac{R^2}{EI}\int_{3\pi/4}^{\pi} F_2R\Big[\cos\tfrac{\pi}{4}\cos\tfrac{3\pi}{4}\sin^2\theta - \tfrac{1}{2}\cos\tfrac{\pi}{4}\sin\tfrac{3\pi}{4}\sin 2\theta - \tfrac{1}{2}\sin\tfrac{\pi}{4}\cos\tfrac{3\pi}{4}\sin 2\theta$$

$$+ \sin\tfrac{\pi}{4}\sin\tfrac{3\pi}{4}\cos^2\theta\Big]\,d\theta$$

$$\delta_{F_1} = \frac{R^2}{EI}\Big\{ M_1\left(-\cos\tfrac{\pi}{4}\cos\theta - \sin\tfrac{\pi}{4}\sin\theta\right) - V_1R\Big[\left(\cos\tfrac{\pi}{4}\right)\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)$$

$$+ \tfrac{1}{4}\sin\tfrac{\pi}{4}\cos 2\theta\Big] - P_1R\Big[\cos\tfrac{\pi}{4}\left(-\cos\theta + \tfrac{1}{4}\cos 2\theta\right)$$

$$- \sin\tfrac{\pi}{4}\left(\sin\theta - \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)\Big] + F_1R\Big[\cos^2\tfrac{\pi}{4}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)$$

$$+ \tfrac{1}{2}\sin\tfrac{\pi}{4}\cos\tfrac{\pi}{4}\cos 2\theta + \sin^2\tfrac{\pi}{4}\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right)\Big\}\Bigg|_{\pi/4}^{\pi}$$

$$+ \frac{R^2}{EI}\Big\{ F_2R\Big[\cos\tfrac{\pi}{4}\cos\tfrac{3\pi}{4}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right) + \tfrac{1}{4}\cos\tfrac{\pi}{4}\sin\tfrac{3\pi}{4}\cos 2\theta$$

$$+ \tfrac{1}{4}\sin\tfrac{\pi}{4}\cos\tfrac{3\pi}{4}\cos 2\theta + \sin\tfrac{\pi}{4}\sin\tfrac{3\pi}{4}\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right)\Big]\Big\}\Bigg|_{3\pi/4}^{\pi}$$

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$$\delta_{F_1} = \frac{R^2}{EI}\Big\{ M_1\big[(-0.707)(-1-0.707) - 0.707(-0.707)\big] - V_1 R\big[0.707(\tfrac{3\pi}{8} - 0 + \tfrac{1}{4})$$

$$+ 0.707(\tfrac{1}{4} - 0)\big] - P_1 R\big[0.707(1 + 0.707 + \tfrac{1}{4} - 0) - 0.707(0 - 0.707 - \tfrac{3\pi}{8} + \tfrac{1}{4})\big]$$

$$+ F_1 R\big[0.500(\tfrac{3\pi}{8} - 0 + \tfrac{1}{4}) + 0.500(\tfrac{1}{2}) + 0.500(\tfrac{3\pi}{8} + 0 - \tfrac{1}{4})\big]\Big\}$$

$$+ \frac{R^2}{EI}\Big\{ F_2 R\big[-0.500(\tfrac{\pi}{8} - 0 - \tfrac{1}{4}) + 0.500(\tfrac{1}{4}) - 0.500(\tfrac{1}{4}) + 0.500(\tfrac{\pi}{8} + 0 + \tfrac{1}{4})\big]\Big\}$$

$$\delta_{F_1} = \frac{R^2}{EI}\Big\{ 1.7070 M_1 - 1.1864 V_1 R - 2.5396 P_1 R + 1.4280 F_1 R + 0.2500 F_2 R \Big\}$$

$$M_1 = \frac{R}{\pi}(2V_1 + P_1\pi - 1.7070 F_1 - 0.2927 F_2)$$

$$P_1 = 0.5363 F_1 + 0.1767 F_2$$

$$V_1 = 0.3341 F_1 - 0.3700 F_2$$

Then:

$$M_1 = R(0.1996 F_1 - 0.1521 F_2)$$

SUBSTITUTING

$$\delta_{F_1} = \frac{R^3}{EI}(0.0256 F_1 - 0.0193 F_2)$$

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REQUIRED INERTIA OF SECTION IF $F_2 = 0$

$$\delta_{F_1} = 0.0256 \frac{R^3 F_1}{E I}$$

$$I = 0.0256 \frac{R^3 F_1}{E \, \delta_{F_1}}$$

LET $R = 47$ IN.

$E = 30 \times 10^6$ psi

$\delta_{F_1} = 0.030$ IN.

$F_1 = 367{,}000$ LBS.

THEN

$$I = \frac{0.0256 \, (47)^3 \, (367{,}000)}{(30 \times 10^6)(0.030)}$$

$$I = 1084 \text{ IN.}^4$$

SECTION ROUGHLY $8\frac{1}{2}'' \times 24''$

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TITLE DEFLECTION of TAIL BEARING STRUCTURE

A POSITIVE DEFLECTION AT F_1 CAUSES A NEGATIVE DEFLECTION AT F_2, BUT SUPPOSE THAT LOAD F_2 IS IMPOSED BY A HYDROSTATIC BEARING SUCH THAT $F_2 = F_0 \left(\frac{2h}{2h + \delta_{F_2}} \right)^3$. HERE F_0 IS THE INITIAL LOAD WHEN NO WIND MOMENT IS APPLIED, AND 'h' IS THE HYDROSTATIC FILM THICKNESS.

THE FOLLOWING IS A DEVELOPMENT OF THE LOAD-DEFLECTION RELATIONS AT F_1 AND INCORPORATING THE HYDROSTATIC FILM AT F_2.

WE HAVE THE EQUATIONS:

1) $\delta_{F_2} = \frac{R^3}{EI}(-0.0193 F_1 + 0.0256 F_2)$

2) $\delta_{F_1} = \frac{R^3}{EI}(0.0256 F_1 - 0.0193 F_2)$

3) $F_2 = F_0 \left(\frac{2h}{2h + \delta_{F_2}} \right)^3$

SOLVING FOR δ_{F_2} IN EQUATION (3):

$$\delta_{F_2} + 2h = 2h \sqrt[3]{F_0/F_2}$$

$$\delta_{F_2} = 2h \left[\sqrt[3]{F_0/F_2} - 1 \right]$$

SUBSTITUTING IN EQUATION (1):

$$2h\sqrt[3]{F_0} - \sqrt[3]{F_2}\,(2h) = \frac{R^3}{EI}(-0.0193 F_1 + 0.0256 F_2)\sqrt[3]{F_2}$$

FOR SIMPLICITY, LET

$a = \frac{R^3}{EI}$, $\sqrt[3]{F_2} = Y$, $b = 0.0193$, $C = 0.0256$

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$$\therefore \quad Y^4 + Y\left[\frac{2h}{ac} - \frac{bF_1}{c}\right] - \frac{2h}{ac}\sqrt[3]{F_o} = 0$$

NOW, LET

$$t = \frac{2h}{ac} - \frac{bF_1}{c} \quad , \quad n = \frac{2h}{ac}\sqrt[3]{F_o}$$

$$Y^4 + tY - n = 0$$

FOR SOLUTION of THIS QUARTIC EQUATION WE HAVE THE FOLLOWING BASE EQUATION:

$$Y^4 + qY^2 + rY + S = 0$$

$$(Y^2 + 2KY + \ell)(Y^2 - 2KY + m) = 0$$

$$64K^6 + 32qK^4 + 4(q^2 - 4s)K^2 - r^2 = 0$$

IN THIS CASE,

$$q = 0 \quad , \quad r = t \quad , \quad s = -n$$

$$64K^6 + 16nK^2 - t^2 = 0$$

$$K^6 + \frac{n}{4}K^2 - \frac{t^2}{64} = 0$$

LET $z = K^2$, AND APPLY CARDAN'S RULES FOR THE SOLUTION of A CUBIC EQUATION,

$$z^3 + \frac{n}{4}z - \frac{t^2}{64} = 0$$

IN THIS CASE,

$$b = 0 \quad , \quad c = \frac{n}{4} \quad , \quad d = -\frac{t^2}{64}$$

$$p = \frac{n}{4} \qquad j = -\frac{t^2}{64}$$

$$S = \frac{1}{27}p^3 + \frac{1}{4}j^2 \quad ; \quad S = \frac{1}{27}\left(\frac{n^3}{64}\right) + \frac{1}{4}\left(\frac{t^4}{64^2}\right)$$

FINALLY;

$$z = K^2 = \sqrt[3]{-\frac{1}{2}j + \sqrt{S}} + \sqrt[3]{-\frac{1}{2}j - \sqrt{S}}$$

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NOW WE CAN SOLVE FOR Y IN GENERAL TERMS.

$$(Y^2 + 2KY + l)(Y^2 - 2KY + m) = 0$$

$$Y = -K \pm \sqrt{K^2 - l} \quad ; \quad Y = K \pm \sqrt{K^2 - m}$$

$$l = \tfrac{1}{2}\left(q + 4K^2 - \frac{r}{2K}\right) = 2K^2 - \frac{r}{4K}$$

$$m = \tfrac{1}{2}\left(q + 4K^2 + \frac{r}{2K}\right) = 2K^2 + \frac{r}{4K}$$

$$Y = -K \pm \sqrt{\frac{r - 4K^3}{4K}} \quad ; \quad Y = K \pm \sqrt{\frac{r + 4K^3}{-4K}}$$

SINCE 'K' CAN BE EITHER POSITIVE OR NEGATIVE AND GIVE THE SAME 'Y' ROOTS, USE 'K' POSITIVE.

THEN:

$$① \; Y = -K \pm \sqrt{\frac{r - 4K^3}{4K}} \quad ; \quad ② \; Y = K \pm \sqrt{\frac{r + 4K^3}{-4K}}$$

SINCE 'K' IS POSITIVE AND 'r' IS ALWAYS A NEGATIVE VALUE, EQUATION ① HAS IMAGINARY ROOTS. TO FOLLOW, FROM EQUATION ② 'Y' HAS ONE POSITIVE ROOT.

$$Y = \sqrt[3]{F_2} = K + \sqrt{\frac{r + 4K^3}{-4K}}$$

WHERE:

$$K^2 = \sqrt[3]{-\tfrac{1}{2}j + \sqrt{s}} + \sqrt[3]{-\tfrac{1}{2}j - \sqrt{s}}$$

$$j = -\frac{t^2}{64} \qquad s = \frac{1}{27}\left(\frac{n^3}{64}\right) + \frac{1}{4}\left(\frac{t^4}{64^2}\right)$$

$$r = t = \frac{2h}{ac} - \frac{bF_1}{c} \qquad n = \frac{2h}{ac}\sqrt[3]{F_0}$$

$$a = \frac{L^3}{F_1}$$

$$c = 0.0256$$

$$b = 0.0193$$

F_2 CAN NOW BE DETERMINED IF F_1 IS KNOWN.

δ_{F_2} AND δ_{F_1} CAN THEN BE CALCULATED FROM EQUATION 1) & 2).

1) $\delta_{F_2} = \frac{R^3}{EI}\left(-0.0193\, F_1 + 0.0256\, F_2\right)$

2) $\delta_{F_1} = \frac{R^3}{EI}\left(0.0256\, F_1 - 0.0193\, F_2\right)$

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TITLE COMPUTATIONS OF TAIL BEARING DEFLECTION

FOR AN APPLIED LOAD, $F_1 = 350{,}000^{\#}$
AND AN INITIAL LOAD, $F_0 = 75{,}000^{\#}$
WHERE:

$I = 1000 \text{ IN.}^4$ $\qquad$ $E = 30 \times 10^6$ PSI

$R = 50$ IN. $\qquad$ $h = .007$

THE DEFLECTIONS δ_{F_1} & δ_{F_2} MAY BE CALCULATED.

$$r = t = \left(\frac{2h}{ac} - \frac{b}{c}F_1\right) = \left\{\frac{(14\times10^{-3})(30\times10^6)(10^3)}{(2.56\times10^{-2})(12.50\times10^4)} - \frac{1.93\times10^{-2}}{2.56\times10^{-2}}(35\times10^4)\right\}$$

$$r = t = 131.25\times10^3 - 263.9\times10^3$$

$$r = t = -13.265\times10^4$$

$$t^2 = 175.96\times10^8$$

$$t^4 = 30{,}962\times10^{16}$$

$$n = \frac{2h}{ac}\sqrt[3]{F_0} = \frac{(14\times10^{-3})(30\times10^6)(10^3)}{(12.50\times10^4)(2.56\times10^{-2})}\cdot\sqrt[3]{75\times10^3}$$

$$n = 131.25\times10^3 \times 42.19$$

$$n = 55.374\times10^5$$

$$-\frac{1}{2}j = \frac{t^2}{128} = \frac{175.96}{128}\times10^8 = 1.375\times10^8$$

$$S = \left(\frac{1}{27}\right)\frac{n^3}{64} + \left(\frac{1}{4}\right)\frac{t^4}{64^2} = \frac{53.374^3}{27\cdot64}\times10^{15} + \frac{309{,}620}{4\cdot64^2}\times10^{15}$$

$$S = 98.26\times10^{15} + 18.9\times10^{15}$$

$$S = 11.716\times10^{16}$$

$$\sqrt{S} = 3.42\times10^8$$

TITLE COMPUTATIONS OF TAIL BEARING DEFLECTION

$$K^2 = \sqrt[3]{-\tfrac{1}{2}j + \sqrt{S}} + \sqrt[3]{-\tfrac{1}{2}j - \sqrt{S}}$$

$$K^2 = (1.375\times10^8 + 3.42\times10^8)^{1/3} + (1.375\times10^8 - 3.420\times10^8)^{1/3}$$

$$K^2 = (479.5\times10^6)^{1/3} - (204.5\times10^6)^{1/3}$$

$$K^2 = 7.83\times10^2 - 5.89\times10^2$$

$$K^2 = 194$$

$$K = 13.93$$

$$K^3 = 2702.4$$

$$Y = 13.93 + \sqrt{\frac{-132{,}650 + 4(2702.4)}{-4(13.93)}}$$

$$Y = 13.93 + 46.77 = 60.70$$

$$\sqrt[3]{F_2} = Y = 60.7$$

$$F_2 = 223{,}650^{\#}$$

$$a = \frac{R^3}{EI} = \frac{50^3}{30\times10^6\times10^3}$$

$$a = 4.167\times10^{-6}$$

$$\delta_{F_1} = 4.167\times10^{-6}\left[0.0256(35\times10^4) - (0.0193)(223{,}650)\right]$$

$$\delta_{F_1} = (4.167\times10^{-6})(46.4\times10^2)$$

$$\delta_{F_1} = .0193''$$

$$\delta_{F_2} = 4.167\times10^{-6}\left[0.0256(223{,}650) - (0.0193)(35\times10^4)\right]$$

$$\delta_{F_2} = -(4.167\times10^{-6})(10.3\times10^2)$$

$$\delta_{F_2} = -.0043''$$

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TITLE COMPUTATIONS OF TAIL BEARING DEFLECTION

CHECK :

$$\delta_{F_2} = 2h\left[\sqrt[3]{F_0/F_2} - 1\right]$$

$$\delta_{F_2} = 14 \times 10^{-3}\left[\sqrt[3]{\frac{7.5 \times 10^4}{22.365 \times 10^4}} - 1\right]$$

$$\delta_{F_2} = 14 \times 10^{-3}(0.695 - 1)$$

$$\delta_{F_2} = -.0043$$

SUMMARY :

$F_1 = 350{,}000^{\#}$

$F_2 = 223{,}650^{\#}$

$\delta_{F_1} = 0.0193''$

$\delta_{F_2} = -0.0043''$

60

$.0178° \div 360$

$1.78 \times 10^{-2} \div 3.6 \times 10^{2}$

$\frac{6.4 \text{ seconds}}{3}$

$\frac{.0193}{600} = \frac{1.93 \times 10^{-2}}{6 \times 10^{2}}$

$.0322 \div \frac{180}{\pi}$

$\frac{3.22 \times 10^{-4} \times 1.8 \times 10^{2}}{\pi} = 1.78 \times 10^{-2}$

$.0178° \quad .322 \times 10^{-4}$

$.0322$

$.0178$