AN EXPERIMENTAL CHECK OF ANTENNA TOLERANCE THEORY USING THE NRAO 85-FT. AND 300-FT. TELESCOPES

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Peter G. Mezger
National Radio Astronomy Observatory *
Green Bank, West Virginia

The antenna tolerance theory describes the effect of systematic and random mechanical deviations of an antenna on its electrical performance. The reflector surfaces of the NRAO 85-ft. and 300-ft. antennas have been measured at different elevation angles using photogrammetric methods. In the printed abstract of this paper it is shown in some detail how the deviations of the reflector from the best fitting paraboloid have been computed. Slide 1 shows how a mechanical deviation Δd of the reflector from the best fitting paraboloid ~~leads to~~ (is connected with) a phase error ($\Delta\delta$) of the plane wave in the aperture plane. The relation

$$\Delta\delta = \frac{2\pi}{\lambda}(2\Delta d) \tag{1}$$

is rigorously valid only for flat reflectors. The effect of these random reflector deviations on antenna pattern and aperture efficiency can be completely described when the RMS reflector deviation $\sqrt{\bar{d}^2}$ and hence the corresponding RMS phase error $\sqrt{\bar{\delta}^2}$ and the correlation length ℓ of the deviations are ~~shown~~ Known. The correlation length ℓ is defined as either the

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average distance at which the reflector deviations become statistically independent. Or if the random reflector deviations are described by a Fourier integral, ℓ has the meaning of a shortest spatial wavelength at which the Fourier spectrum has values (virtually) different from zero.

The basic work in antenna tolerance theory is due to John Ruze. Following his calculations one finds that the resulting power pattern of an antenna can be presented as the superposition of the diffraction pattern f_D and an error pattern f_E

(2) $$f(\theta,\varphi) = f_D(\theta,\varphi) + f_E(\theta)$$

The error pattern has approximately a gaussian shape. The integration of this antenna pattern yields for the antenna solid angle

(3) $$\Omega = \int \overline{f}\, d\Omega = \Omega_o + (e^{\delta^2} - 1)\, \Omega_o = \Omega_o e^{\delta^2}$$

where Ω_o is the antenna solid angle of the diffraction pattern alone. The gain of the antenna is then

(4) $$G = \frac{4\pi}{\Omega} = \frac{4\pi}{\Omega_o}\, e^{-\delta^2} = G_o e^{-\delta^2}$$

i.e., the gain G of an antenna with the square phase error $\overline{\delta}^2$ is ~~equal to~~ the product of the gain G_o of the undistrubed reflector multiplied by $e^{-\overline{\delta}^2}$. It is of interest to note that under practical conditions the gain reduction depends only on the square of the phase error and not on the correlation length. Considering the aperture efficiency η_A rather than the gain one finds

(5) $$\eta_A(z) = \eta_{A_o} \exp\{-\overline{\delta}^2\} = \eta_{A_o} \exp\left\{-16\pi^2 \frac{\overline{\alpha}^2}{\lambda^2}\right\}$$

η_{A_o} is the aperture efficiency of the undisturbed reflector.

Using radio astronomical techniques, which will be described in another paper given here tomorrow, we have measured the aperture efficiency of the NRAO 85-ft. and 300-ft. telescopes. Slide 2 shows the 85-ft. telescope, whose reflector is made of preshaped solid aluminum panels. This telescope has been operated at a shortest wavelength of 2-cm. Slide 3 shows the 300-ft. telescope, which is at the present the worlds largest partially steerable telescope. The surface of the 300-ft. reflector is made of aluminum mesh, the average size of which is about two and one half meters. This telescope has been operated at a shortest wavelength of 10-cm. In order to obtain the aperture efficiency of the undisturbed reflector, we have plotted the logarithm of the measured aperture efficiencies of each telescope as a function of the inverse square of the wavelength and fitted a straight line through these points using the method of least squares.

The intersection of these straight lines with the axis $1/\lambda^2 = 0$ yields aperture efficiences of 67% for the undisturbed 300-ft. telescope and 58% for the undisturbed 85-ft. telescope. With these values and the measured and weighted square deviations of the reflectors, we have computed the expected change in aperture efficiency with wavelength. These computed aperture efficiencies are shown as full curves in slide 4. In the case of the 300-ft. telescope, the weighted RMS reflector deviation is nearly independent on the telescope position and the measured values agree well with the computed curves. In the case of the 85-ft. telescope, the photogrammetric survey yields a weighted RMS deviation of 2.75-mm at zenith position and 4.17-mm at horizontal position of the reflector. The measured aperture efficiency values are considerably higher than the computed curves. The dashed curve gives the best fit to the measured values; the corresponding effective RMS deviation of the reflector is 1.75-mm. The downward pointing arrows at the points measured at 6-cm and 2-cm wavelength have the following meaning: Whereas all other points have been measured with an edge taper of the feed pattern of -18 db, the point at 6-cm has been measured with a TRG scalar feed designed for low spill over and high gain. The point at 2-cm wavelength has been measured with an edge taper of the feed pattern of -14 db.

A contour map representation of the deviations of the 85-ft. reflector is shown in slide 5. The shadowed areas correspond to deviations in a direction opposite to the focal point, whereas the deviations at zenith have random characteristics. The deviations measured at horizontal reflector position are rather large scale deflections which yield an astigmatism of the antenna.

Slide 6 shows the relative change of the aperture efficiency of the two NRAO 85-ft. telescopes as a function of zenith distance. The aperture efficiency of the 85/1-foot telescope changes by about 40% between zenith and horizon; that of the 85/2-foot telescope changes by only 10%. One can calculate the increase in the RMS deviation Δd which causes this gain variation from the relations

(6a)
$$\eta_A(z)/\eta_A(0) = \exp\left\{-16\pi^2 \frac{\Delta d(2d + \Delta d)}{\lambda^2}\right\}$$

and

(6b)
$$\Delta d \simeq \frac{\lambda^2}{2d} \frac{\log\left[\eta_A(0)/\eta_A(z)\right]}{16\pi^2} \quad \text{for } \Delta d \ll d$$

We have found in this way that the effective RMS reflector deviation of the 85/1-ft. telescope of 1.75-mm increases between zenith and horizon by 0.17-mm, The corresponding increase of the RMS deviation of the 85/2-ft. telescope is only 0.07-mm.

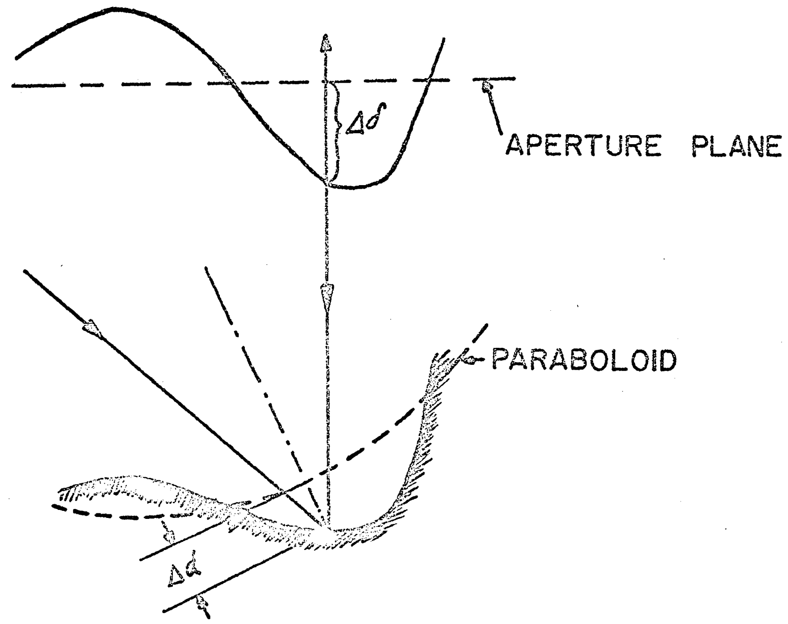
Whereas, the decrease in the aperture efficiency of a telescope depends only on the square reflector deviation, the shape of the error pattern is determined by both the square reflector deviation and the correlation length of these deviations. Slide 7 shows the error pattern of the 300-ft. telescope, computed for the measured RMS reflector deviation of 12.5-mm, a wavelength of 10-cm and various correlation lengths ℓ. With increasing correlation length the HPW of the error pattern becomes narrower and its peak value gets higher. The following method has been proven to be useful for the measurement of the correlation lengths. We measure first the aperture efficiency of the antenna with a point source and then with an extended source like the moon. The latter measurement yields a higher aperture efficiency since not only the central region of the main beam but also the region of the first sidelobes receive radiation power from the extended source. The difference between the two values of the aperture efficiency will be the greater for a given RMS reflector deviation the narrower the error pattern, and hence the longer the correlation length. In this way we found a correlation length of 2.5 meters for the 300-ft. telescope, which is about the average size of the reflector panels. The measured antenna pattern out to the fifth side lob[illegible] is shown as a dashed curve and is in good agreement with the [illegible] pattern computed for a correlation length of 2.5 meters. The fact that the even side lobes in the east-west direction are suppressed is due to the aperture blocking of the two feed support legs in north-south direction.

It has been found that even in cases of a very low aperture efficiency the shape of the main beam is virtually not affected by the random reflector deviations. As another result we have found that the aperture efficiency of the 85-ft. telescope changes at 2-cm wavelength by only 1.7% when the direction of polarization is changed by 180°, which means that the reflector deviations must really be randomly distributed. Our investigations of the NRAO telescopes has shown that the results of antenna tolerance theory are in good agreement with the experimental results. In addition, we have found that radio telescopes can still be useful observing instruments even if their RMS phase error is larger than unity, provided their characteristics are carefully measured.

$$\Delta\delta = \frac{2\pi}{\lambda}(2\Delta d)$$

$$\sqrt{\overline{d^2}} \rightarrow \sqrt{\overline{\delta^2}}$$

CORRELATION LENGTH e

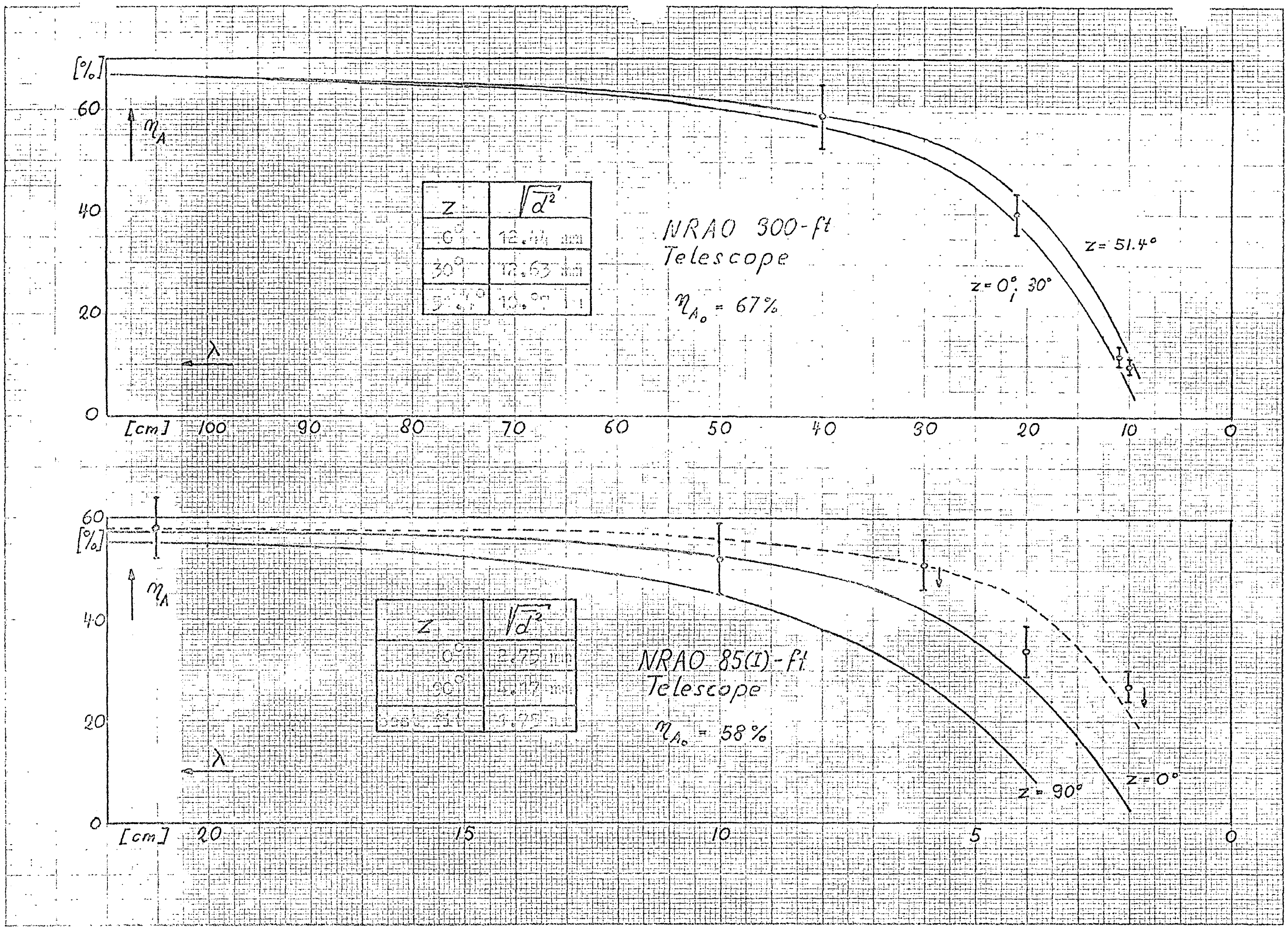


$$\overline{f(\theta, \phi)} = f_D(\theta, \phi) + f_E(\theta)$$

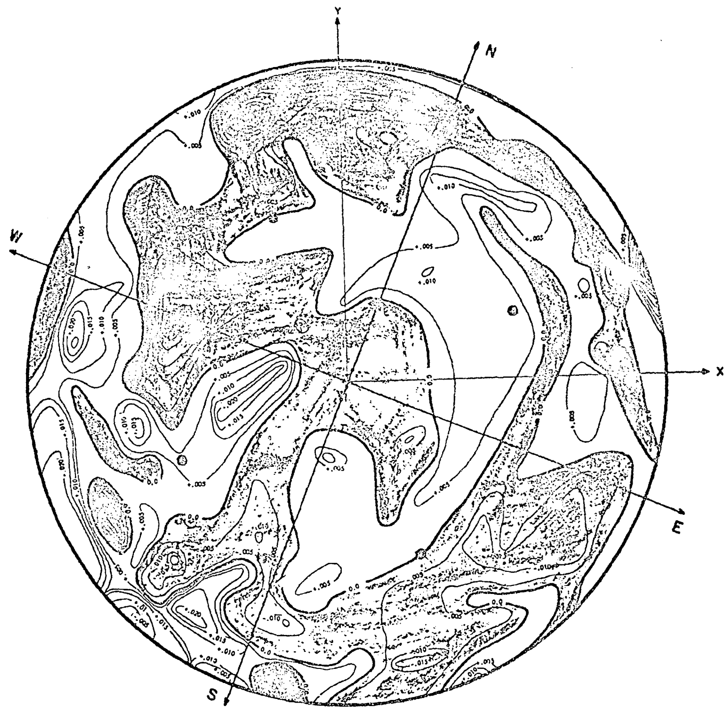
$$\Omega = \int \bar{f}\, d\Omega = \Omega_0 + (e^{\overline{\delta^2}} - 1)\,\Omega_0 = \Omega_0\, e^{\overline{\delta^2}}$$

$$G = \frac{4\pi}{\Omega} = \frac{4\pi}{\Omega_0}\, e^{-\overline{\delta^2}} = G_0\, e^{-\overline{\delta^2}}$$

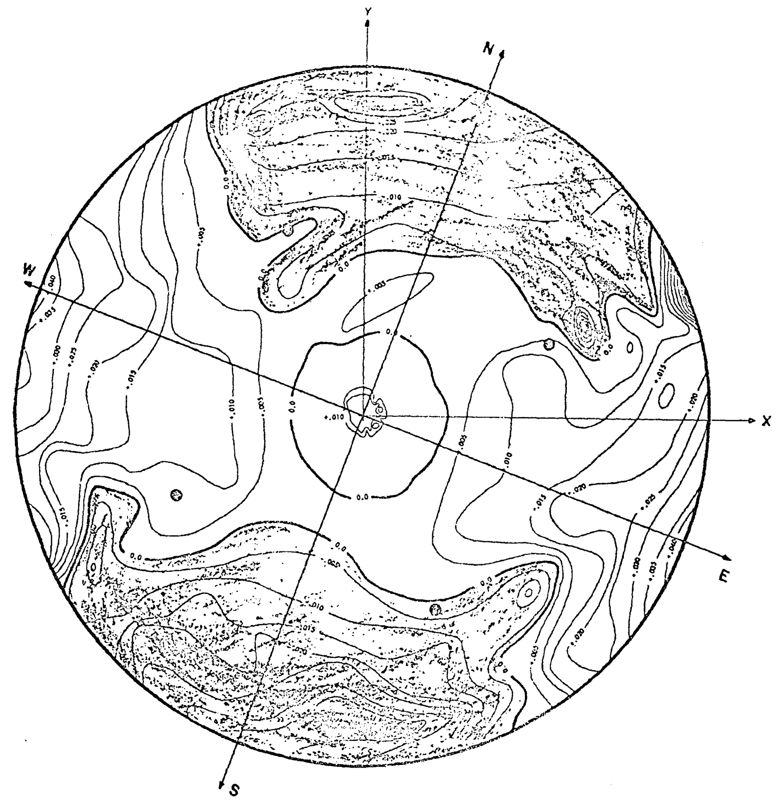
$$\eta_A(\lambda) = \eta_{A0} \exp\left\{-\overline{\delta^2}\right\} = \eta_{A0} \exp\left\{-16\pi^2 \frac{\overline{d^2}}{\lambda^2}\right\}$$



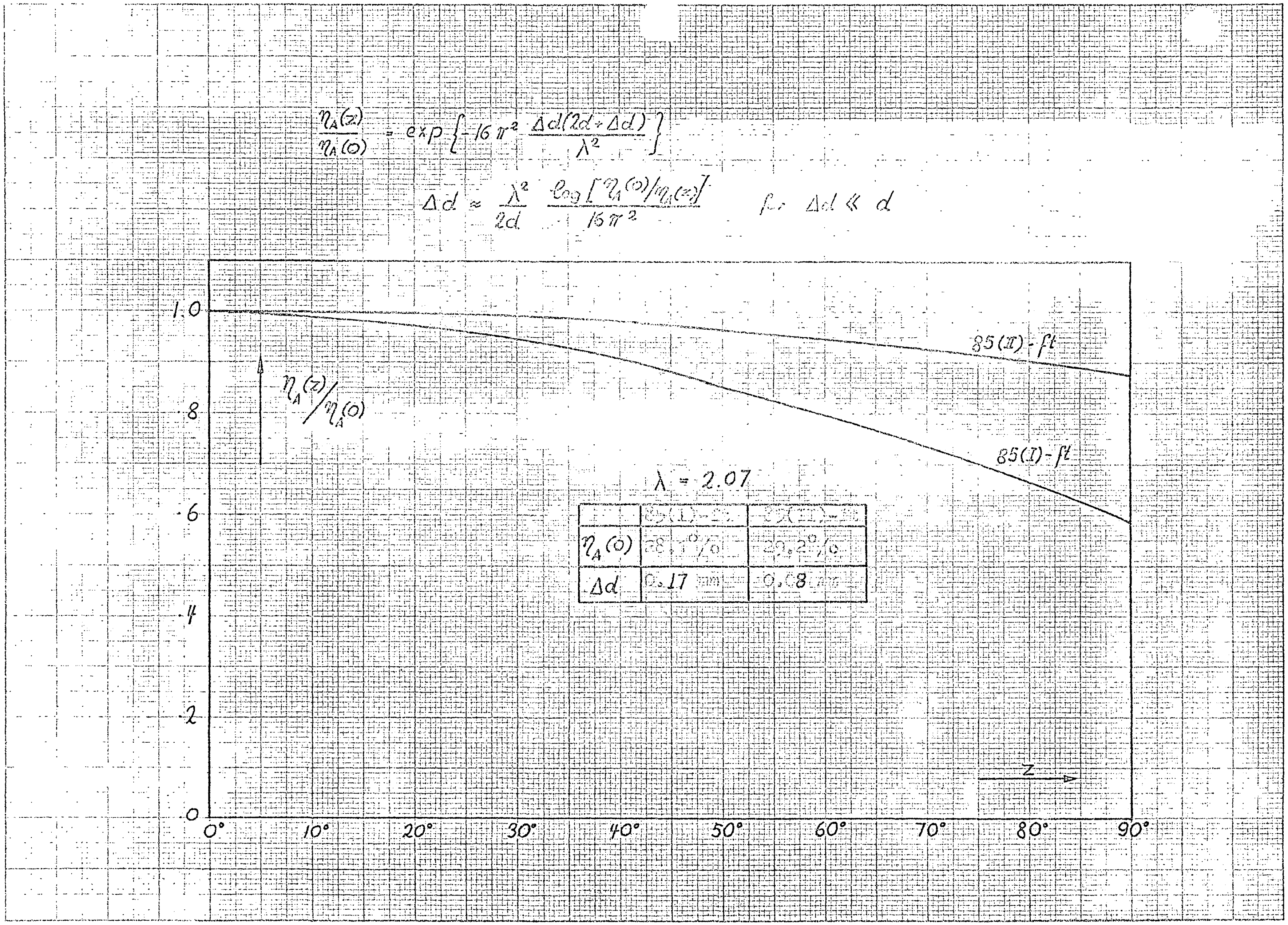
Aperture efficiency of the NRAO 300-foot and 85-foot telescopes as a function of wavelength.



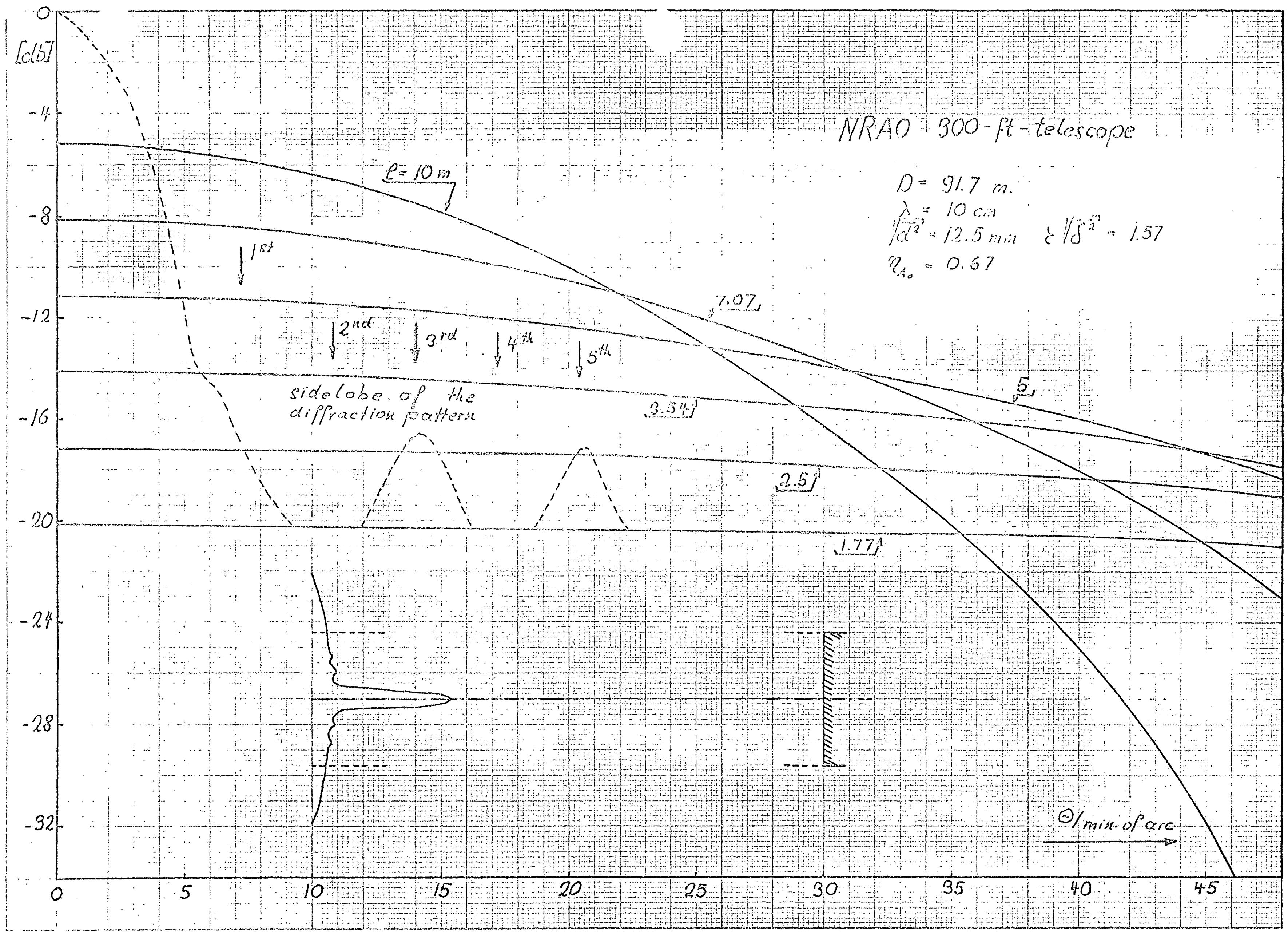
85(I)-FT. TELESCOPE AT z = 0°



85(I)-FT. TELESCOPE AT z = 90°



Relative change of aperture efficiency of the NRAO 85-foot telescope
as a function of zenith distance z.



Error pattern of the NRAO 300-foot telescope.