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AN ARRAY FOR HIGH-RESOLUTION MAPPING OF EXTRAGALACTIC RADIO SOURCES

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I. Introduction

We shall consider the general design of an antenna array for the specific purpose of mapping high surface brightness extragalactic radio sources to an angular resolution of the order of 10" of arc. It is assumed that the reader is familiar with the concepts of aperture synthesis. For the sake of brevity, the various equations are given without their derivations. These are available in the author's notes.

Technical problems of the array are not treated at all. Our present purpose is to evolve a basic design from the astronomical objectives alone.

The design of an antenna array for a particular type of observation requires attention to a number of questions. The most important of these are:

(a) Astronomical — resolution and minimum flux level, range of source sizes, size of the area to be mapped around each source.

(b) Instrumental — attainable receiver performance, means of combining and recording signals, allowable complexity in the system.

(c) Practical — convenience and speed of completing an observing program, most efficient utilization of the system, cost.

The following factors describe the geometrical characteristics of an array:

(a) The number of antenna stations and their placement with respect to each other.

(b) The sizes of the individual elements.

(c) The degree of steerability and movability of the elements.

(d) The number of elements.

Of these, (a) and (b) are rather narrowly defined by the objectives of the observing program. On the other hand, there is a fair latitude of choice in (c) and (d).

The interrelationships between the various factors are shown in Fig. 1. The basic assumptions are that the Fourier components of the brightness distribution are recorded individually, and that a single observation suffices to fix the value of a component. The diagram is drawn specifically for the case of extragalactic sources. One might connect the boxes differently for other types of program.

Extragalactic sources, as a group, have a number of properties which bear on the design of the array:

(a) The great majority are small in angular extent. Nearly all are under 5' of arc in size, with perhaps 2' being representative for the few hundred brightest. This means that we need map only a small area for each source; it also means that we must attain an effective resolution much less than 1' in order to obtain sufficient detail.

(b) Most are quite faint, requiring high sensitivity.

(c) They are numerous and fairly evenly distributed over the sky. Thus an efficient observing program does not necessarily imply a high degree of steerability in the array.

(d) Most are at high galactic latitudes, so we can disregard possible complications arising from galactic radiation.

(e) They do not exhibit rapid secular variation in their properties, so real-time operation of the array is not important.

(f) Many or most of them have polarization properties which differ from one part of the source to another. For this reason the feeds of the elements should be designed with polarization studies in mind. It also means that the array should be operable at at least two frequencies; another reason is that the brightness distributions of some sources probably vary with frequency.
While these considerations are important, they will not be treated further in the present report.

II. Requirements to be Met by the Array

We shall now establish the basic specifications of the array from the astronomical requirements.

A. Sensitivity: The required sensitivity is set, within reasonable limits, by the number of sources we wish to be able to observe. Let us adopt an operating wavelength of 10 cm and a southern declination limit of -30°. We assume that the majority of the sources fainter than 10 flux units are extragalactic. The number of sources with flux densities $\geq$ S (in flux units) at λ 10 cm, in the 31000 square degrees north of $\delta = -30°$, is approximately*

$$N(S) = 453 \; S^{-1.8} \tag{1}$$

Of course, we cannot set in advance a definite number of sources to observe. But it is clear that we shall want to be capable of reaching several hundred. In view of (1), this should be possible if we can measure 1 flux unit sources with sufficient accuracy. Thus we place our sensitivity limit at 1 flux unit.

B. Angular Resolution: As we have stated above, 2' of arc is probably a representative order of size for the sources we wish to study. The number of picture elements in a map is approximately

$$\left(\frac{\text{angular diameter of source}}{1/2 \text{ x beamwidth}}\right)^2 .$$

The greater the number of picture elements, the more detailed the map. A synthesized beamwidth of 10", which is about 1/10 the diameter of a typical source, will give the order of 500 picture elements. This should afford adequate detail for the purposes now foreseeable. Thus we adopt 10" of arc for the synthesized half-power beamwidth. The sidelobes of the synthesized beam must be low, of course. It seems reasonable to require that the first sidelobe be at least 20 db below the peak response.

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* An extrapolation from the 3C catalogue, following von Hoerner (Publ. NRAO, No. 2 (1961)), assuming that a typical source has a spectral index of -0.8 and that the slope of the (log N, log S) relation is -1.8.

C. Field of View: Unless we synthesize a filled aperture, the reconstituted radiation pattern will have multiple responses analogous to those of a diffraction grating. The filled aperture is not necessary for our purposes, and we simply need to ensure that the multiple responses are separated widely enough that only one at a time is looking at the source. As we have said, most extragalactic sources are smaller than 5' of arc. This we adopt as the minimum allowable separation of the main lobes of the ultimate radiation pattern.

D. Maximum Observing Time per Source: This is largely a matter of choice. We shall set the arbitrary but reasonable requirement that it be possible to complete the mapping of a source in not more than 30 observing days.

III. The Distribution of Fourier Components on the Transform Plane

In mapping a radio source by aperture synthesis, we measure Fourier components of the brightness distribution. After enough components have been obtained to define the brightness distribution with the required resolution, the observed two-dimensional transform is inverted. The result is the desired map. The Appendix gives a brief summary of the procedure.

Each Fourier component is the complex correlation coefficient $\tilde{V}$ of the signals received at two separated antennas. It is convenient in practice to represent the components in terms of a phase Φ and an amplitude V. The location of a component in the transform plane is determined by the physical separation P of the phase centers of the antennas (in wavelengths), the azimuth A of the line joining them, the latitude φ of the observing site, and the hour angle t and declination δ of the source at the time of observation.

Let us specify positions on the transform plane in terms of Cartesian coordinates u and v. These are expressed in cycles per radian. We orient the coordinates in such a way that the positive v-axis corresponds to north on the celestial sphere. That is, a component lying on the v-axis is measured when the projections of the two array elements on the celestial sphere lie on the same meridian. The crests of the corresponding interference fringes are perpendicular to this meridian. Although the transform plane is infinite, one half of it is redundant since the correlation coefficients at (u, v) and at (-u, -v) are indistinguishable. Thus it is sufficient to consider only a semi-infinite plane bounded by a straight line passing through the origin (u = 0, v = 0). In our case, we take this line to be the v-axis, and only positive values of u will be considered. Then u and v are determined uniquely by P, A, φ, t, and δ. The relationship is

$$u = \left|(\sin A \cos t - \sin \varphi \cos A \sin t) P\right| \tag{2a}$$

$$v = \{\cos \varphi \cos A \cos \delta + \sin \delta (\sin \varphi \cos A \cos t + \sin A \sin t)\} P \tag{2b}$$

Our observations refer to that part of the transform plane which is bounded by the lines $u = 0$, $u = u_{max}$, $v = v_{max}$, $v = -v_{max}$ (Fig. 2). The north-south and east-west beamwidths (in radians) of the synthesized radiation pattern are given by

$$\left.\begin{aligned} \beta_{NS} &= \frac{k}{v_{max}} \\ \beta_{EW} &= \frac{k}{u_{max}} \end{aligned}\right\} \tag{3}$$

The factor k depends on the weighting given to the amplitudes of the various components in the Fourier inversion process to control the sidelobe level. Weighting functions which hold maximum sidelobes to 20 db below peak response correspond to $k \approx 1.15$.

The density with which we must fill in the working area on the transform plane is determined by the field of view requirement. If the field of view is to be ψ (radians), then we have to measure Fourier components at intervals of

$$\Delta u = \Delta v = \psi^{-1} \tag{4}$$

We can now find the total number of Fourier components which we need to measure given a beamwidth and a field of view. This is

$$H = \left(\frac{u_{max}}{\Delta u} + 1\right)\left(\frac{2v_{max}}{\Delta v} + 1\right)$$

or, if $\beta_{NS} = \beta_{EW} = \beta$,

$$H = \left(\frac{\psi k}{\beta} + 1\right)\left(\frac{2\psi k}{\beta} + 1\right) \quad (5)$$

With the adopted values $\psi = 5'$, $\beta = 10''$, and $k = 1.15$, we have

$$H = 2485.$$

IV. Source Tracks in the Transform Plane

It can be shown that (2a) and (2b) are the parametric equations of an ellipse, with t as the parameter. Thus, if two array elements with fixed values of P, A, and φ are allowed to follow a source at declination δ as the earth rotates, they will measure Fourier components at successive points on an ellipse in the transform plane. This ellipse has the following properties:

(a) Its major axis is parallel to the u-axis.

(b) Its center is at $u = 0$, $v = P \cos \varphi \cos A \cos \delta$.

(c) Its major semi-axis is $P\sqrt{1 - \cos^2 \varphi \cos^2 A}$.

(d) Its eccentricity is $\cos \delta$; i.e., its axial ratio is $\sin \delta$.

From this, we have two useful results: a single fixed pair of array elements can work over a considerable arc in the transform plane, and the nature of the coverage thus obtained depends strongly on the declination of the source. At $\delta = 0°$, the ellipse degenerates to a straight line parallel to the u-axis, while at $\delta = 90°$ it is actually a circle.

We shall be concerned with the rate of motion on the elliptical tracks when we determine the integration time available for observation of a single Fourier component in Section VI.

V. A Simple Array Configuration

A T-shaped arrangement of equally-spaced stations, such as that shown in Fig. 3, can provide the necessary distribution of Fourier components from meridian transit observations alone. The antenna stations are spaced at intervals of $1/\psi$ wavelengths along two mutually perpendicular arms*. The signal at each element in the N-S arm is simultaneously correlated with that at each element in the E-W arm. Every such pair gives a unique Fourier component.

Let us label each station in the E-W and N-S arms with integers m and n, respectively. It is easy to show that the values of u and v corresponding to a pair of stations (m, n) are, for t = 0,

$$\left.\begin{aligned} u_{mn} &= m\psi^{-1} \\ v_{mn} &= n\psi^{-1} \cos(\varphi - \delta) \end{aligned}\right\} \qquad (6)$$

These equations define a rectangular grid in the transform plane.

Let there be M stations in each branch of the E-W arm and N stations in the N-S arm, not counting the center station at the intersection of the two arms. If M = N, we shall require a total of 3M + 1 antenna stations. Now $M = \psi u_{max}$, so

$$M = \frac{k\psi}{\beta} \qquad (7)$$

by equation (3). Using the previously adopted values of k, ψ, and β, we get M = 35, and the total number of array stations required is 106.

Now we can set the dimensions of the array. The station spacing corresponding to a 5' field of view is 688 λ, or 226 ft. for 10 cm wavelength. Then each of the three branches of the T must be 7910 ft. long. The arrangement of stations is shown in Fig. 4. The overall size of the array is about 1.5 by 3 miles.

An array of 106 elements probably would be too costly to be feasible. We must think in terms of using a smaller number of antennas which can be moved from one station to another fairly easily. In the following discussion, we shall assume that N-S and E-W arms

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* Ignoring the foreshortening of the north-south arm due to zenith distance. In reality the beamwidth in the N-S direction will be increased by sec $(\varphi - \delta)$ unless the spacings in the N-S arm are increased by the same factor.

each contain the same number of elements; this gives the maximum number of simultaneous inter-arm pairs. If we have Q elements in all, the number of such pairs is

$$\nu = \frac{1}{4} Q^2 \tag{8}$$

From (5) and (7), we see that the total number of Fourier components to be measured is

$$H = (M + 1)(2M + 1).$$

The number of element arrangements needed to observe H components ν at a time is H/ν. This is also the number of observing days needed to map a source, using meridian transit observations only. For $M = 35$,

$$H/\nu = \frac{10224}{Q^2} \tag{9}$$

In Section II, we set the arbitrary requirement that no more than 30 days of observing time be taken to complete the mapping of a source. Thus the minimum number of elements required for our purpose is 20*.

Before proceeding to the next section, a comment should be made regarding the pair $m = 0$, $n = 0$. Since two antennas cannot occupy the same station simultaneously, the measurement requires that the signal from one be split and fed to two amplifiers, with subsequent correlation of the two outputs. While the correlation coefficient is necessarily equal to unity, the measurement is not trivial, since the actual level of the correlator output provides a reference for normalizing the other components. It also gives the flux density of the source. This is the hard way, however. In practice it would be simpler to measure the flux density in the usual way and determine the normalizing factor from it (see Appendix, equation (A2)).

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* Equation (9) actually gives $Q = 19$, but our discussion assumes that there is an even number of elements. Hence 20 is the least number that will serve. This gives $H/\nu = 27$.

VI. Element Diameter

The maximum available integration time for a particular Fourier component is the time required for the response of the corresponding pair of elements to traverse a distance $1/\psi$ in the transform plane. For meridian observations, this is

$$\tau_{mn} = \frac{13751}{\sqrt{n^2 \sin^2 \varphi + m^2 \sin^2 \delta}} \tag{10}$$

sidereal seconds. The shortest available integration time is that for the outermost stations of the array:

$$\tau_{MM} = \frac{13751}{M\sqrt{\sin^2 \varphi + \sin^2 \delta}}$$

if M = N.

Let us assume a latitude of 38° N and a declination of +70° (we would seldom work north of this). Then, for the 106-station array, the available integration time is always at least

$$\tau_{MM} = 350 \text{ sec.}$$

Assuming an aperture efficiency of 50%, the minimum element diameter (in feet) is

$$D = 275\left(\frac{T_R}{S_m}\frac{R}{V}\right)^{1/2}\left(\tau_{MM}B\right)^{-1/4} \tag{11}$$

where

T_R = system noise temperature
S_m = minimum flux density in flux units
B = bandwidth
R = desired minimum ratio of integrated fringe amplitude to r.m.s. noise
V = visibility amplitude corresponding to R.

We have already decided on $S_m = 1$. Adopting $T_R = 100$ °K, $\tau_{MM} = 350$ sec, $B = 10^7$ cps, R = 5, V = 0.1, we get

$$D = 80.0 \text{ ft.}$$

for the minimum acceptable element diameter.

This result assumes that the array is capable of tracking for at least 6 minutes in hour angle. It also assumes that 100 °K system noise temperatures will be feasible by the time the array could go into operation.

VII. Secondary Lobes

The complete synthesized radiation pattern consists of a rectangular grid of 10" beams weighted by the voltage radiation pattern of a single element. Figure 5 shows the voltage pattern for an 80-foot paraboloid at 10 cm wavelength, assuming an aperture tapering identical to that used with the NRAO 85-foot antenna at 10 cm. The half-field beamwidth is $\sqrt{2}$ times the half-power beamwidth, or 25' of arc.

Figure 6 shows the distribution of array lobes within the primary envelope. There are 61 inside the 10% contour and 21 inside the 50% contour. These numbers are large enough that we must consider the likelihood of having a serious confusion problem. From equation (1) and the primary beam function, we find that there is about 1 chance in 1500 of having a random source of at least 1 flux unit inside the 50% contour. This suggests that confusion is not likely to be a problem. In cases where there is reason to suspect confusion, one can settle the matter by making a second set of observations at some hour angle off the meridian; this rotates and stretches the secondary lobe distribution without greatly altering the principal lobe. The best hour angle for doing this depends on the declination of the source.

Clearly, one must be cautious when observing a source which is within half a degree of another known source. The best procedure to use in such cases depends on the individual circumstances of each — the relative separations, orientations, and intensities of the two sources; whether the second source is extended; and the declination of the field.

These considerations make it desirable, although not absolutely necessary, that the array elements be steerable over a fairly wide range of hour angle. Polar mountings would be the most convenient to use. The elements are small enough to make such mountings practical.

VIII. Summary

We have arrived at an array with the following characteristics, assuming operation at λ 10 cm:

Configuration: Symmetrical T, with E-W arm 15820 ft., N-S arm 7910 ft. long.

Number of Antenna Stations: 106, spaced at intervals of 226 ft.

Elements: 80-foot paraboloids, capable of being moved freely from one station to another. Preferably fully steerable, on polar mounts.

Number of Elements: At least 20.

Such an array can satisfy the astronomical requirements laid down in Section II.

An instrument of this kind would have a great deal of versatility. For example, by using the E-W arm alone, one can synthesize a fan beam 5" (E-W) by 17' (N-S). Obliquely oriented fan beams can be realized by operating off the meridian. There is also the well-known advantage that the arms can be extended easily for higher resolution.

APPENDIX

Let a discrete source have a true brightness temperature distribution $T_B(x, y)$ at wavelength λ, where x and y are Cartesian coordinates measured in radians. The Fourier transform of this distribution is

$$C\vec{\vec{V}}(u, v) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T_B(x, y)e^{-j2\pi(ux + vy)}dxdy \qquad \text{(A1)}$$

where u and v are the Cartesian coordinates of a point in the Fourier transform plane, expressed in cycles per radian, and C is a normalizing factor to make $\vec{\vec{V}}(0, 0) = 1 + j0$. The value of C depends on the flux density of the source:

$$C = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T_B(x, y)dxdy = \frac{\lambda^2 S}{4\pi k} \qquad \text{(A2)}$$

where k is Boltzmann's constant and S is the flux density at wavelength λ.

In aperture synthesis, one observes the Fourier components $\vec{\vec{V}}(u, v)$, and recovers the brightness distribution by inverting the transform. If $\vec{\vec{V}}(u, v)$ were known for all u and v, the result would be

$$T_B(x, y) = C\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \vec{\vec{V}}(u, v)e^{j2\pi(ux + vy)}dudv \qquad \text{(A3)}$$

and this would be an exact description of the source. In practice, however, we measure $\vec{\vec{V}}(u, v)$ only at discrete points in the transform plane, and only up to some finite distance from the origin ($u = 0$, $v = 0$). Therefore we must approximate (A3) by

$$T_B'(x, y) = C\sum_{u=-u_{max}}^{u_{max}}\sum_{v=-v_{max}}^{v_{max}} \vec{\vec{V}}(u, v)e^{j2\pi(ux + vy)}\Delta u\Delta v \qquad \text{(A4)}$$

where Δu and Δv are the sampling intervals in the two coordinates. Since $T_B(x, y)$ is real, its Fourier transform is symmetrical about the origin: $\tilde{\tilde{V}}(u, v) = \tilde{\tilde{V}}(-u, -v)$. The smaller Δu and Δv, and the larger u_{max} and v_{max}, the better the approximation of $T_B'(x, y)$ to $T_B(x, y)$. Because (A4) is a series instead of an integral, $T_B'(x, y)$ is periodic in x and y while $T_B(x, y)$ is not; hence a radiation pattern synthesized in this way has multiple responses. Also, the fact that u_{max} and v_{max} are finite instead of infinite means that the resolving power is finite instead of infinite, and that sidelobes are introduced.

It is desirable to control the sidelobe level, although this can be done only at the expense of some resolution. Thus we replace $\tilde{\tilde{V}}(u, v)$ by $W(u, v)\, \tilde{\tilde{V}}(u, v)$ in the summation, where $W(u, v)$ is a weighting function analogous to the aperture taper of a paraboloidal antenna.

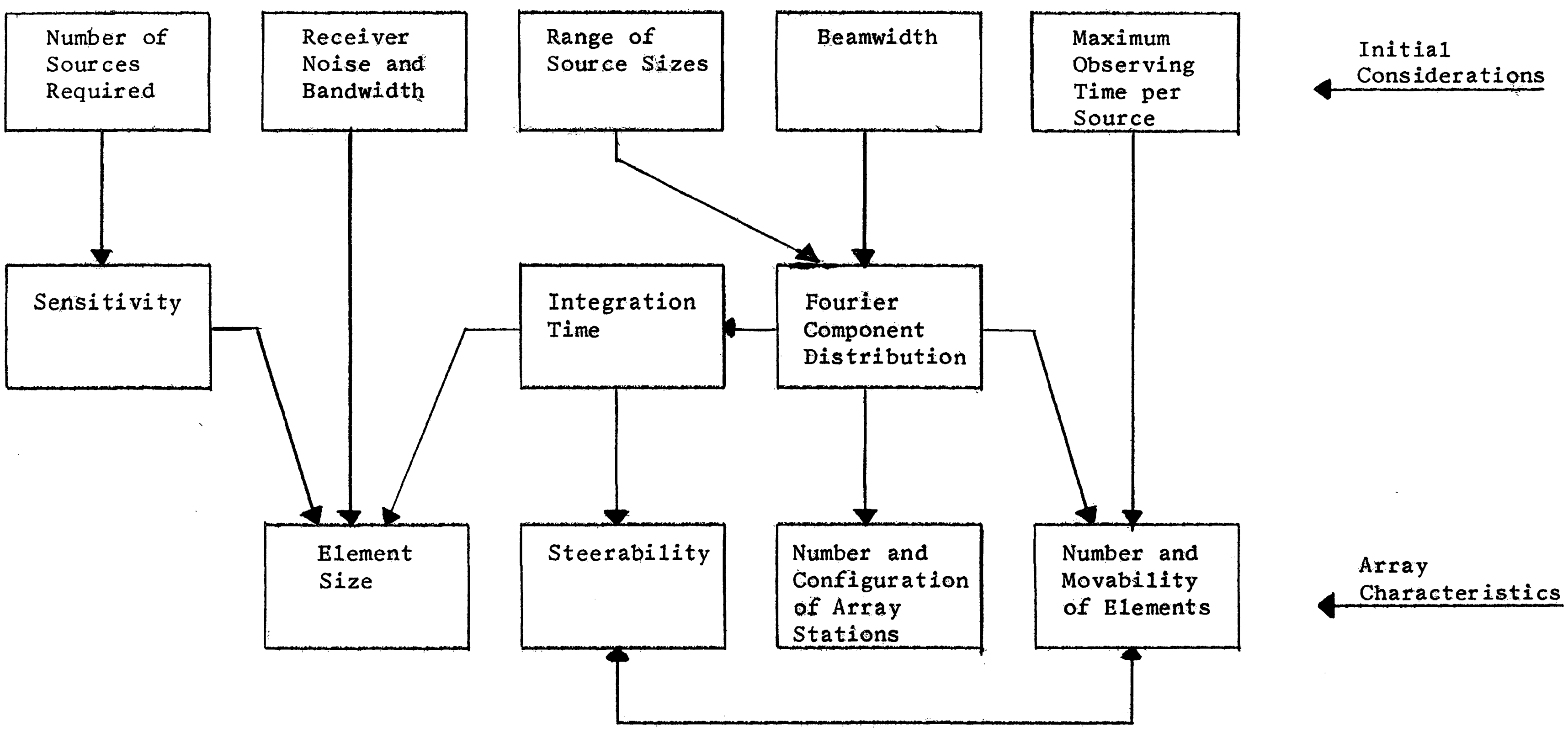

Fig. 1. Procedure for Array Design

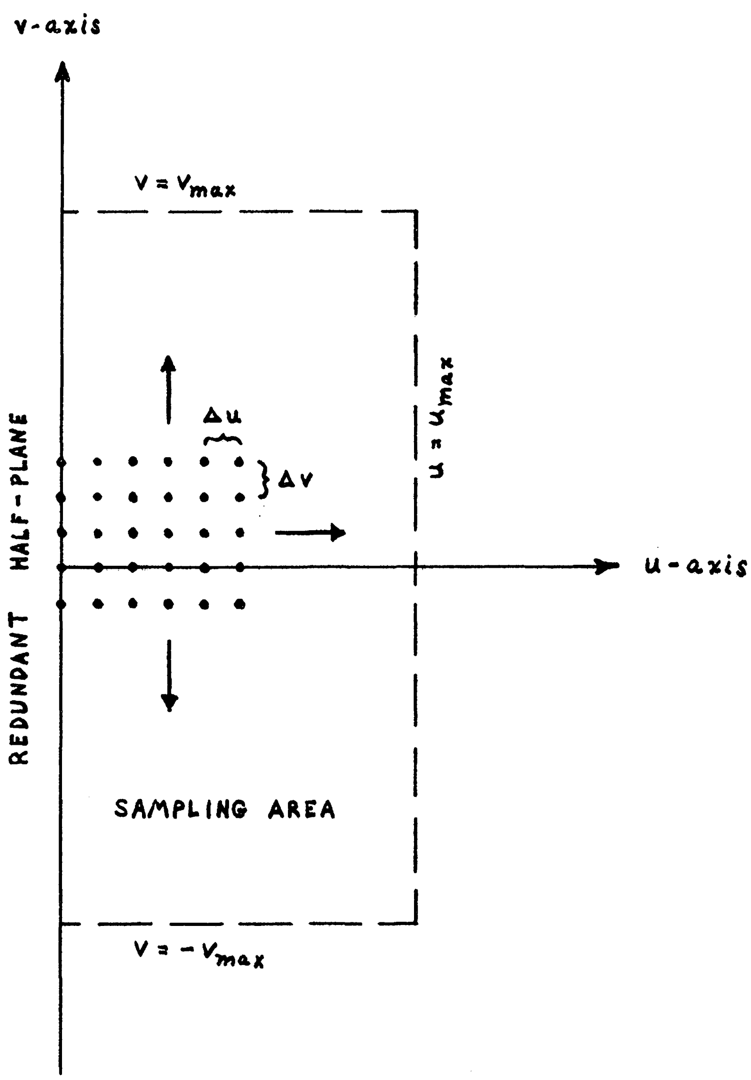

Fig. 2. Distribution of measured components in the Fourier transform plane.

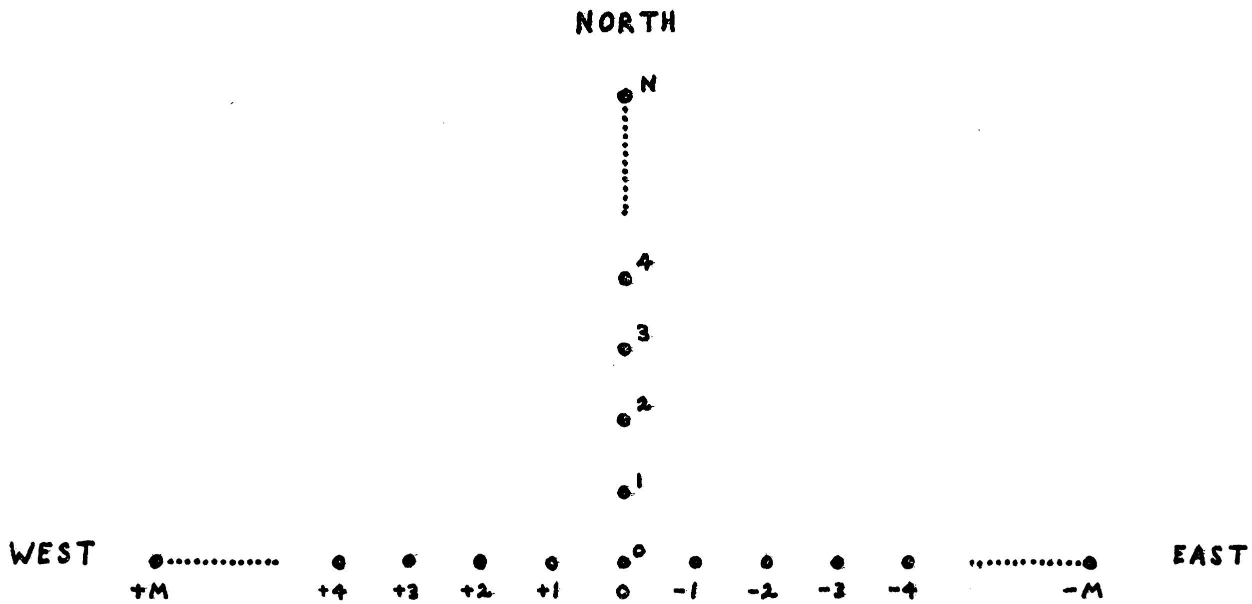

Fig. 3. Layout and numbering of array stations.

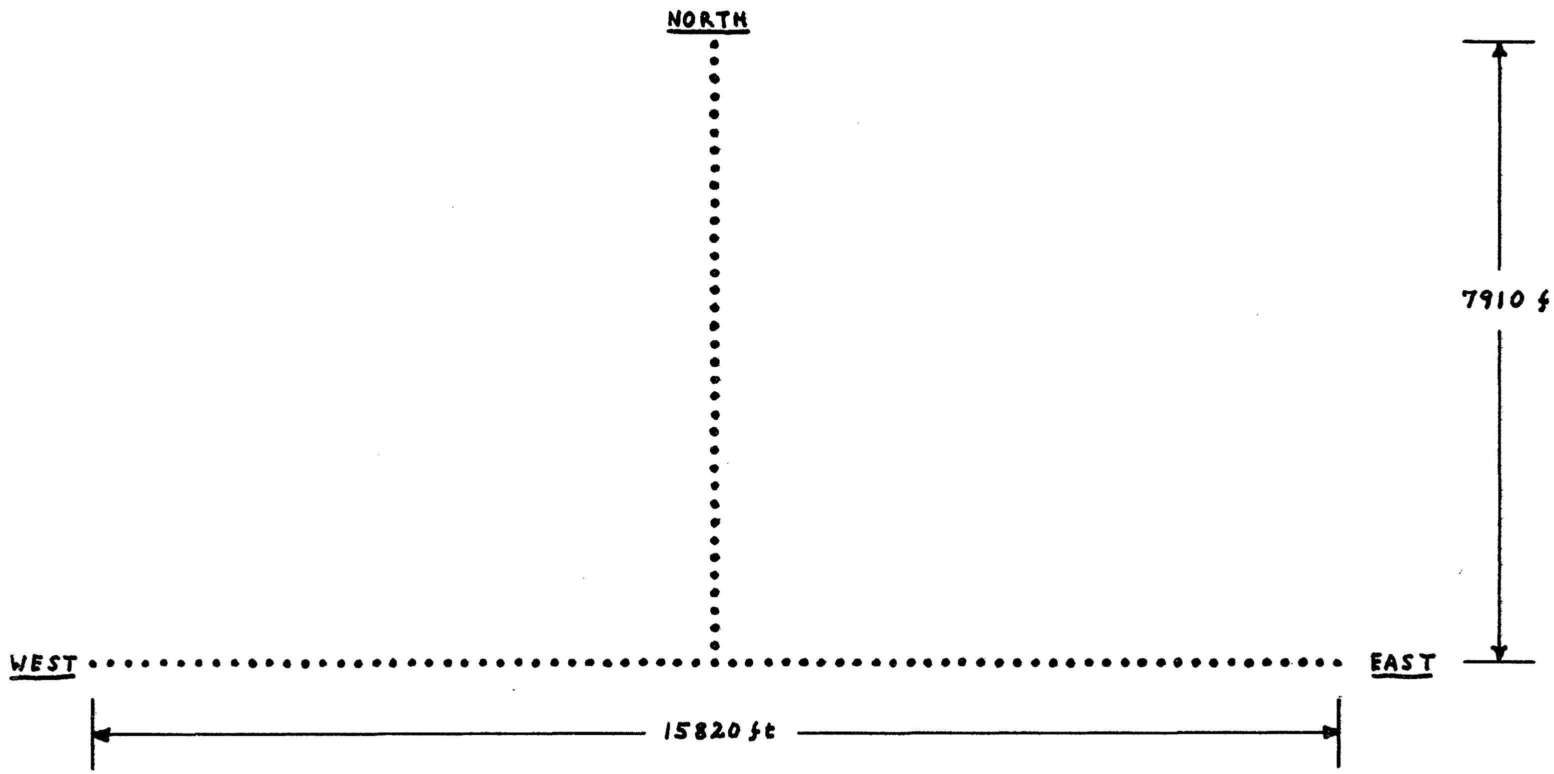

Fig. 4. Array plan. The stations are at intervals of 226 ft.

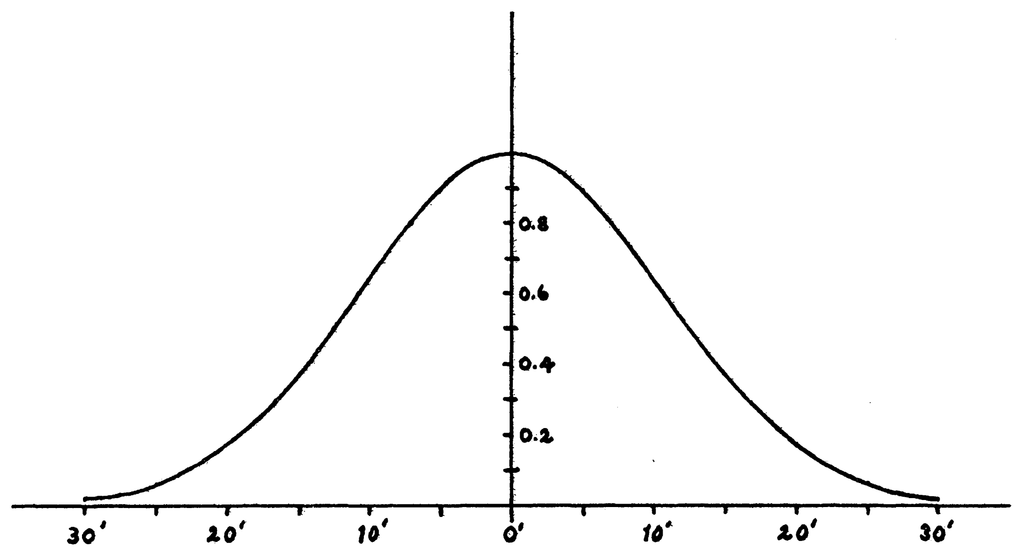

Fig. 5. Voltage radiation pattern of an 80-foot paraboloid.

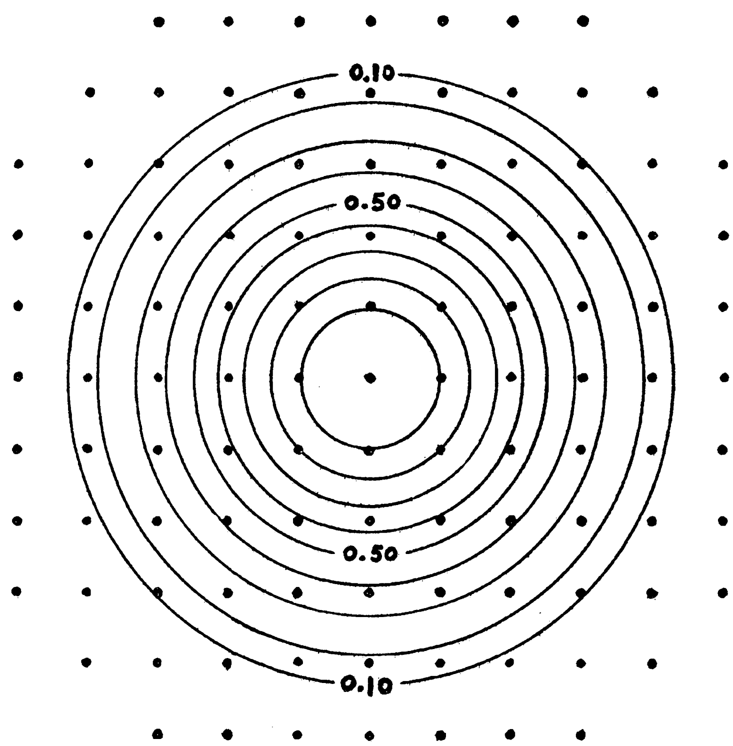

Fig. 6. Distribution of array lobes within the primary pattern.