Elaboration to Ref. 8 in Keen's report (No. 14)

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(around 11/14/63)

PART A

<u>The Basic Two Element System</u>

The basic two element system is shown in Fig. A(1).

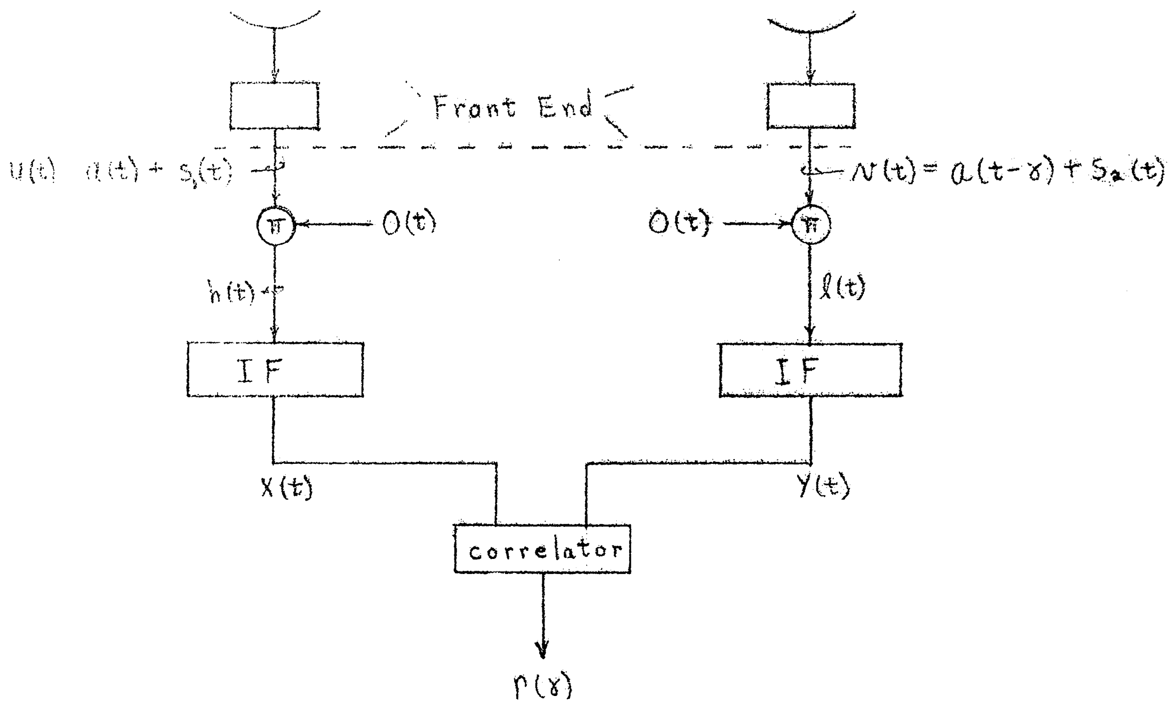


Fig. A(1)

The purpose of this section is to analyze the basic system shown above. The correlator itself is of interest here only to the extent that, by assumption, $\Gamma(\gamma) = \overline{x(t)y(t)}$. The extent to which this can be accomplished by each of the following techniques is discussed in the indicated sections.

Part C,B Analogue -- direct multiplication followed by integrator or fitter

Part D Manybit digital -- numerical average of sampled values of x(t) and y(t).

Part E Onebit digital -- numerical average of sampled values of clipped x(t) and y(t).

The object of this section is to obtain the properties of the correlator inputs and to relate these to the system parameters.

a(t) and s(t)

a(t) represents the voltage produced at mixer input due to source radiation striking the antenna. The notation as a voltage at this point has more of a symbolic than physical significance since the term applied here is somewhat ambiguous. The power spectrum of a(t), A(w), however, is well defined, and provides a clearer physical description. a(t) is mathematically defined as a random variable ranging in value from $-\infty$ to ∞ and having a joint probability density function $P_a(a_t A_{t+\tau})$. The assumption of infinite range is usually made in random signal analysis and is kept physically reasonable by choosing a density function which favors realizable values. As also usually done, a(t) is assumed to have Gaussian statistics in the sense that the joint probability density function $P_a(A_t, A_{t+\tau})$ is the bivariate Gaussian distribution.

$$P_a(a_t, a_{t+\tau}) = \frac{1}{2\pi\sigma_a^2 \sqrt{1-\rho_a^2(\tau)}} \exp\left\{-\frac{a_t^2 - 2\rho_a(\tau) a_t a_{t+\tau} + a_{t+\tau}^2}{2\sigma_a^2 \sqrt{1-\rho_a^2(\tau)}}\right\}$$

Such a density can be partially justified since the signals under consideration are a result of radiation which evolves as the effective superposition of a large number of random processes. In such case the resulting density can be shown to be Gaussian almost independent of the form of the original density functions.

The forms of the power spectrum of the source radiation is well known in radio astronomy and is found to depend on the type of mechanism producing the radiation. With the exception of neutral hydrogen radiation, this spectrum in all known cases may be assumed flat over the range of receiver bandpasses of interest in this paper. The exception shall be considered later as a special case. The power transfer characteristics of the antenna and r.f. amplifiers may also be assumed flat over the receiver bandpass. It follows then that the spectrum of a(t), A(w) is flat of height determined by the product of radiation intensity, antenna gain and receiver front end gain, and of width determined by receiver front end bandpass. The spectrum of A(w) is shown in Fig. A(2). Only the positive side of the double sided power spectrum is shown. Because the double sided spectrum is used, the effective power density is A(w) + A(-w) or T_a°K/rad.

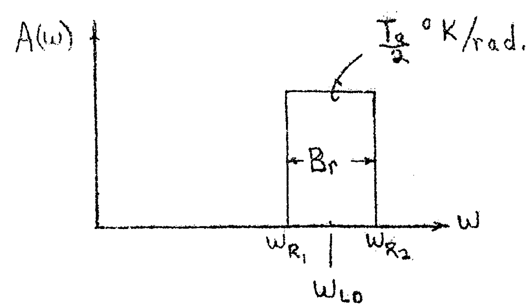


Fig. A(2)

The assumption of vertical sides on the bandpass need not be made and, in fact, is not very realistic. The form of the sides, however, is found to be of little importance as long as the I.F. bandwidth is much less than the front end bandwidth, a situation almost always encountered in practice.

s(t) represents the voltage produced at the mixer input which may be attributed to system noise. System noise here is used in the normal sense to indicate undesired signal. It includes atmospheric as well as receiver noise, but is to omit any disturbance which is coherent between the two antennas. It does not therefore include radiation due to unknown sources in a sidelobe of the correlated pair antenna pattern but would include the same radiation if it appeared in a sidelobe of only one of the antennas. Similarly, the coherent portion of atmospheric noise having space coherence over distances comparable to the baseline length is not represented in s(t). Coherent disturbances shall be dealt with later.

s(t) is also assumed to have Gaussian statistics with the joint probability density function having the bivariate Gaussian distribution. A partial justification of this assumption would be the discussion relating to the same for a(t). The argument is less rigorous as applied here since s(t) does not necessarily represent only the sum of a large number of uncorrelated process. The assumption is, nevertheless, the best available and is ~~usually~~ used largely on this account.

The power spectrum of s(t), S(w) is also assumed flat and to have a bandwidth determined by the receiver frontend bandpass. Although S(w) is known to increase with frequency about 5% over a 10 Mc/s bandwidth, the amount is insignificant for bandwidths under consideration.

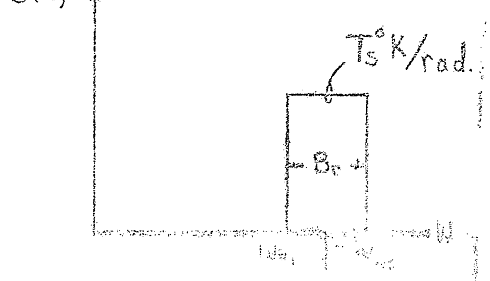


Fig. A(3)

Local Oscillator

The local oscillator signal, O(t) is represented by a sine wave of unit peak amplitude and angular frequency W_{1o}. For the time being the same oscillator signal is assumed at each mixer, excluding any possibility of phase shift or instability between the two. The effect of the latter shall be considered as a special case in a later section.

The oscillator may be considered in the frequency domain as having a power spectrum represented by a delta function located at W_{1o} and having an area of (1/2)°K.

I.F. Power Response

F(w) is defined as the I.F. ~~power response~~ filter transform and f(t), the I.F. impulse response. In this section two forms of F(w) shall be considered; rectangular and Gaussian, denoted $F_1(w)$ and $F_2(w)$ respectively. Although both forms shall be considered with respect to the analogue correlator, only the rectangular bandpass shall be considered for the digital correlators, Parts D and E. This is because a band limited signal is required in the digital systems and a precision filter having extremely sharp sides would follow the I.F. amplifiers.

$\left| F(w) \right|$ in all cases is normalized to unity at the center I.F. frequency, w_o.

$$\left\| F_1(w) \right\| = 1 \text{ for } w_o - \frac{B_{IF}}{2} \geq w \leq w_o + \frac{B_{IF}}{2} \text{ and 0 elsewhere}$$

$$\left\| F_2(w) \right\| = \exp - \frac{(w - w_o)^2}{2(.425\ B_{IF})^2}$$

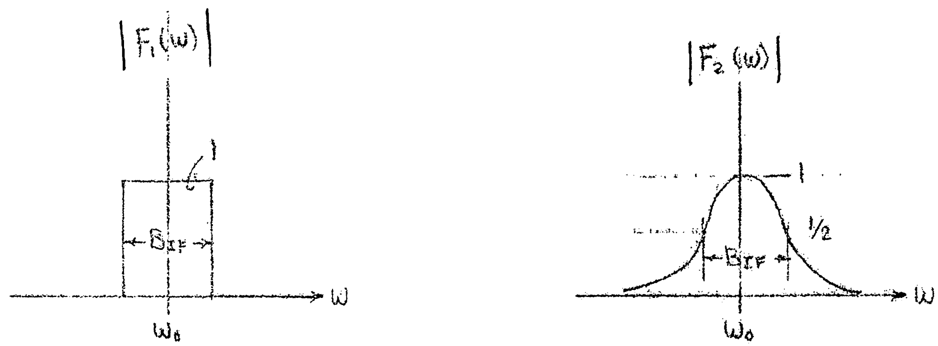


Fig. A(4)

System Description

$$u(t) = a(t) + s_1(t) \qquad v(t) = a(t - \gamma) + s_2(t)$$

$$U(w) = A(w) + S_1(w) \qquad V(w) = A(w) + S_2(w)$$

$$P_{vu}(\tau) = 2\int_0^{\infty} U(w)\ \cos wt dw \qquad P_{vv}(\tau) = 2\int_0^{\infty} V(w)\ \cos wt dw$$

$$P_{uv}(\tau) = \overline{u(t)\ v(t - \tau)} = \overline{\left[a(t) + s_1(t)\right]\left[a(t - \tau - \gamma) + s_2(t)\right]}$$

$$= \overline{a(t)\ a(t - \tau - \gamma)} = P_{aa}\ (\tau + \gamma)$$

$$\Phi_{uv}(w) = \frac{1}{2\pi}\int_0^{\infty} P_{uv}(\tau)\ \cancel{\cos w\tau}\, e^{-j\omega\tau} d\tau = \frac{1}{2\pi}\int_0^{\infty} P_{aa}\ (\tau + \gamma)\ e^{-jw\tau} d\tau = e^{j\gamma w}\ A(w)$$

$$h(t) = u(t)\ O(t) = \left[a(t) + s_1(t)\right]\ \sin W_{1o}t; \quad l(t) = v(t)\ O(t)$$

$$l(t) = \left[a\ (t - \gamma) + s_2(t)\right] \sin w_{1o}t$$

$$H(w) = U(w) * O(w) \qquad L(w) = V(w) * O(w)$$

$$P_{hh}(\tau) = \overline{\left\{\left[a(t) + s_1(t)\right]\ \sin w_{1o}t\right\}\left\{\left[a(t - \tau) + s_1(t - \tau)\right]\ \sin w_{1o}(t - \tau)\right\}} = \frac{1}{2}\ P_{uu}(\tau)\ \cos w_{1o}\tau$$

$$P_{11}(\tau) = \overline{\left\{\left[a\ (t - \gamma) + s_2(t)\right]\ \sin w_{1o}t\right\}\ \left[a(t - \gamma - \tau) + s_2(t - \tau)\right]\ \sin w_{1o}(t - \tau)} = \frac{1}{2}\ P_{vv}(\tau)\ \cos w_{1o}\ \tau$$

$$P_{hl}(\tau) = \overline{\left\{\left[a(t) + s_1(t)\right] \sin w_{lo} t\right\} \left[a(t - \gamma - \tau) + s_2(t - \tau)\right]}$$

$$\overline{\sin w_{lo}(t - \tau)} = \frac{1}{2} P_{uv}(\tau) \cos w_{lo}\tau = \left(\frac{1}{2}\right) P_{aa}$$

$$(\tau + \gamma) \cos w_{lo}\tau$$

$$\Phi_{hl}(w) = \frac{1}{2} \int_{-\infty}^{\infty} P_{hl}(\tau) e^{-jw\tau} d\tau = \left(\frac{1}{4}\right) e^{j\gamma w} \left\{\cos w_{lo}\gamma \left[A(w-w_{lo}) + A(w + w_{lo})\right] - j \sin w_{lo}\gamma \left[A(w-w_{lo}) + A(w + w_{lo})\right]\right\}$$

$$X(t) = \int_0^t h(\delta) f_x(t - \delta) d\delta \qquad Y(t) = \int_0^t l(\delta) f_y(t - \delta) d\delta$$

$$X(w) = H(w) \left|F_x(w)\right|^2 \qquad Y(w) = L(w) \left|F_y(w)\right|^2$$

Here $F_x(w)$ and $F_y(w)$ are used to denote the IF filter in the left and right channel respectively.

$$P_{xx}(\tau) = 2\int_0^\infty X(w)\cos w\tau\, dw \qquad P_{yy}(\tau) = 2\int_0^\infty Y(w)\cos w\tau\, dw$$

$$P_{xx}(\tau) = \int_0^\infty H(w)\left|F_x(w)\right|^2 \cos w\tau\, dw \qquad P_{yy}(\tau) = \int_0^\infty L(w)\left|F_y(w)\right|^2 dw$$

The cross correlation $P_{xy}(\tau)$ shall be obtained by the inverse Fourier transform of the cross power spectrum $\emptyset_{xy}(w)$. As shown in the appendix, $\emptyset_{xy}(w) = \overline{F_x}(w)\ F_y(w)\ \emptyset_{hl}(w)$. In the present case the same IF filter is assumed in each channel.

$$F_x(w) = F_y(w) = F(w); \quad \emptyset_{xy}(w) = \left|F(w)\right|^2 \emptyset_{hl}(w)$$

hence

$$\emptyset_{xy}(w) = \frac{T_a e^{j\gamma w}}{2} \left|F(w)\right|^2 \cos w_{lo}\gamma \quad \text{if} \quad B_{rf} > B_{IF}$$

$$P_{xy}(\tau) = \int_{-\infty}^{\infty} \emptyset_{xy}(w)\ e^{-jw\tau}\ dw$$

The correlator defines $r(\gamma)$ by $r(\gamma) \equiv x(t)y(t) = P_{xy}(0)$

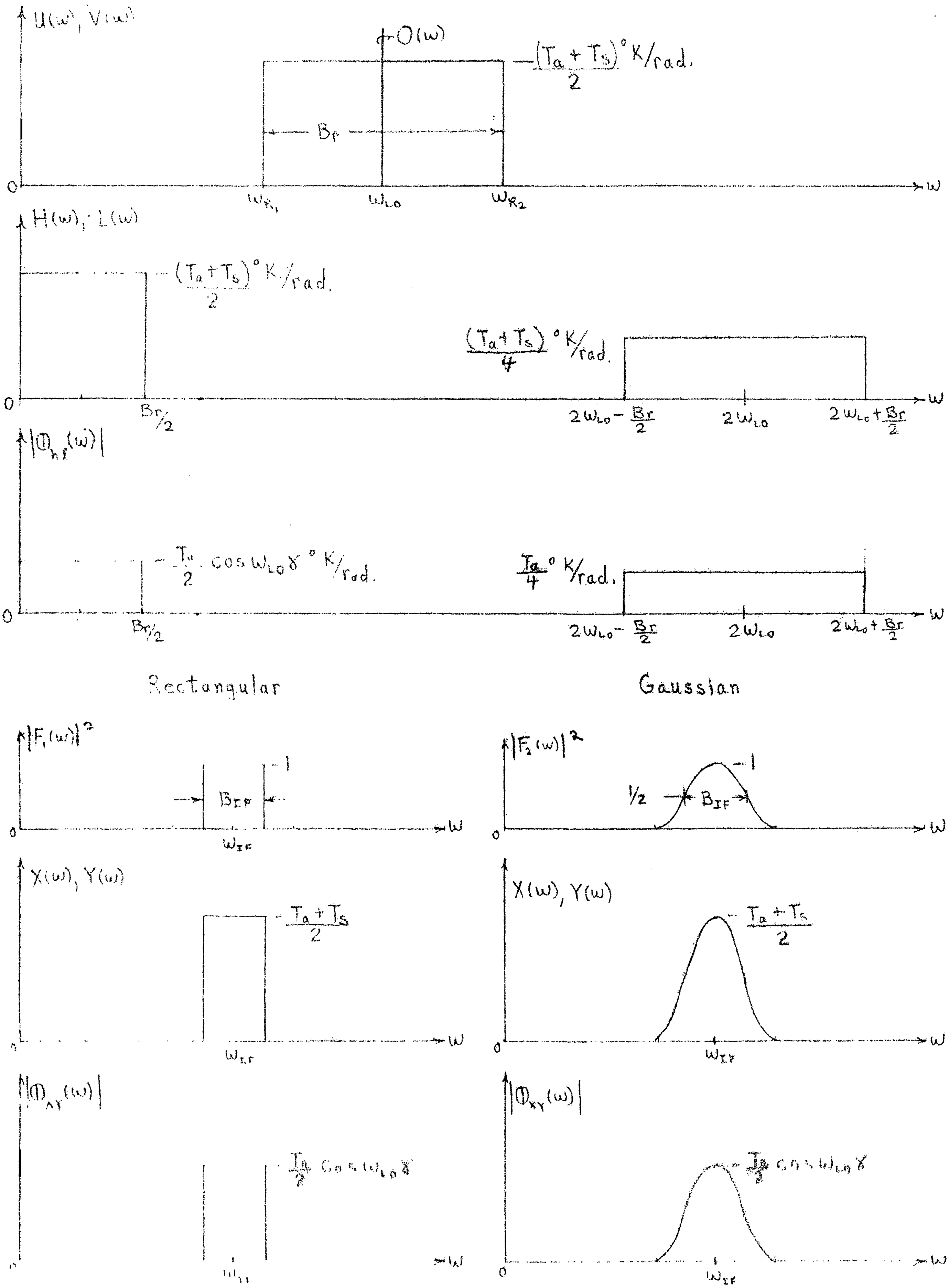
hence, $r(\gamma) = \int_{-\infty}^{\infty} \emptyset_{xy}(w)\ dw = (T_a)\ \cos w_{lo}\gamma \int_0^\infty \left|F(w)\right|^2 \cos \gamma w dw$

Rectangular bandpass

$$r(\gamma) = T_a\ B_{IF} \left(\frac{\sin \frac{B_{IF}}{2}\gamma}{\frac{B_{IF}}{2}\gamma}\right) \cos w_{IF}\gamma \quad \cos w_{lo}\gamma$$

Gaussian bandpass

$$r(\gamma) = \sqrt{2\pi}\ T_a\ (.425\ B_{IF})\ \exp - \left[\frac{(.425\ B_{IF})^2\ \gamma^2}{2}\right] \cos w_{IF}\gamma \cos w_{lo}\gamma$$



U(ω), V(ω)
O(ω)
(Ta + Ts)/2 °K/rad.
Br
ωR1
ωLO
ωR2
ω
0
H(ω), L(ω)
(Ta + Ts)/2 °K/rad.
(Ta + Ts)/4 °K/rad.
Br/2
2ωLO − Br/2
2ωLO
2ωLO + Br/2
|Φhl(ω)|
Ta/2 cos ωLO γ °K/rad.
Ta/4 °K/rad.
Rectangular
Gaussian
|F1(ω)|²
|F2(ω)|²
1
1/2
BIF
ωIF
X(ω), Y(ω)
(Ta + Ts)/2
|ΦXY(ω)|
Ta/2 cos ωLO γ

The above expressions shall be termed the normal fringe pattern for a rectangular and Gaussian bandpass, respectively. Since $w_{lo} >> w_{IF}$ and $w_{IF} > B$, both expressions may be expressed as $(T_a B_{IF} \cos w_{lo} \gamma)$ multiplied by an envelope function.

$$r(\gamma) = E(\gamma)\, T_a\, B_{IF} \cos w_{lo} \gamma \quad \text{where } E(\gamma) \text{ is the envelope function.}$$

$$E(\gamma) = \left(\frac{\sin \frac{B_{IF}\,\gamma}{2}}{\frac{B_{IF}\,\gamma}{2}} \right) \cos w_{IF} \gamma \quad \text{for a rectangular bandpass}$$

$$E(\gamma) = 1.07 \exp - \left[\frac{(.425\, B_{IF})^2 \gamma^2}{2} \right] \cos w_{IF}\gamma \text{ for a Gaussian bandpass.}$$

Lobe Switching - RF Phase Delay

Lobe switching may be accomplished by inserting a 90° phase shift in the oscillator signal to either mixer.

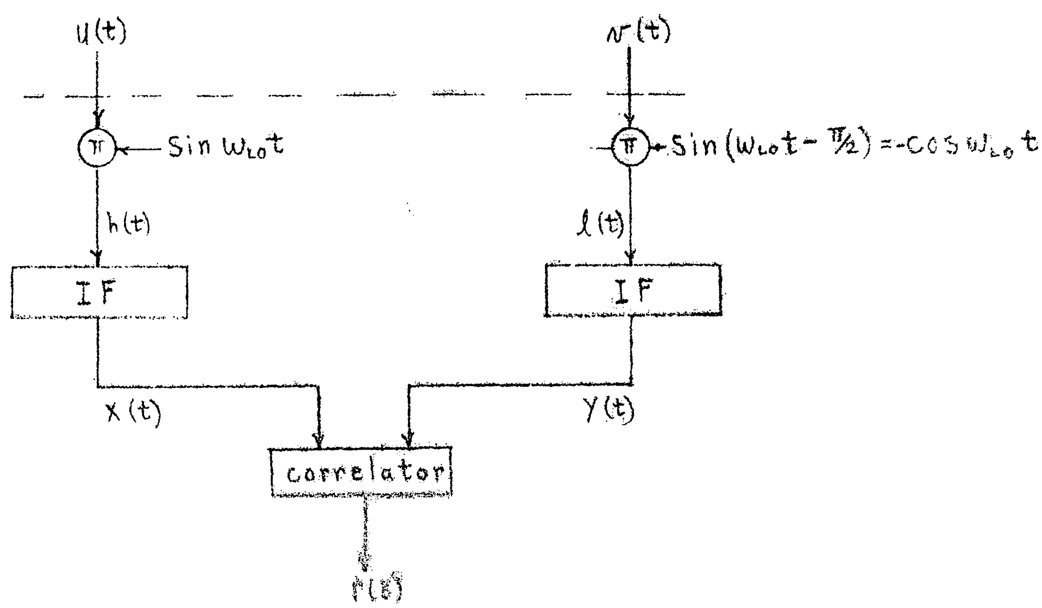
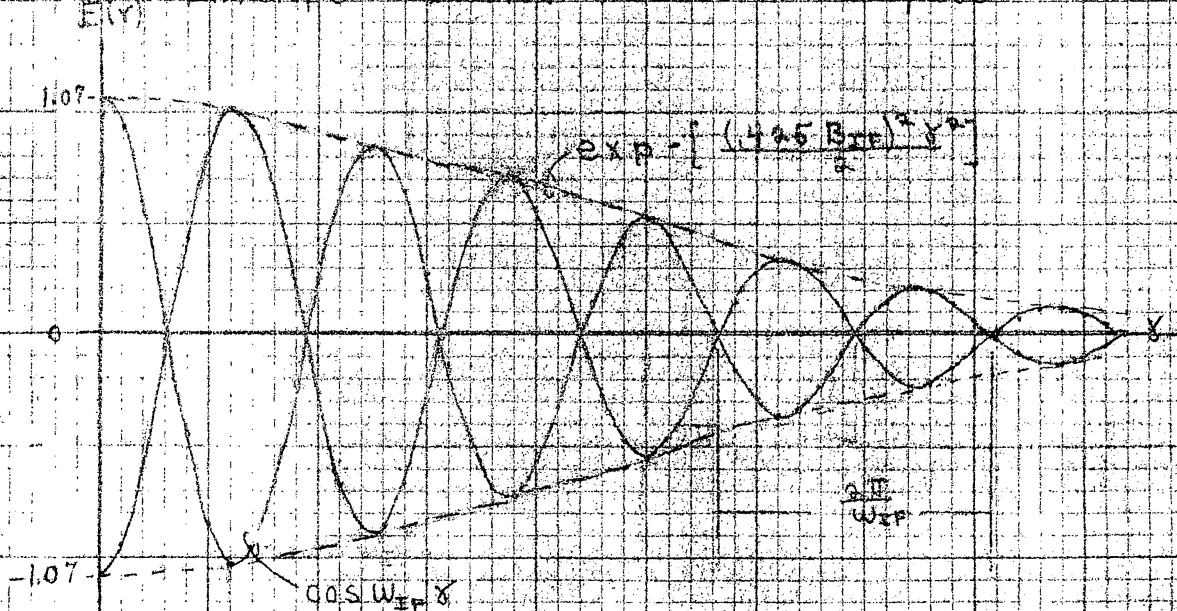
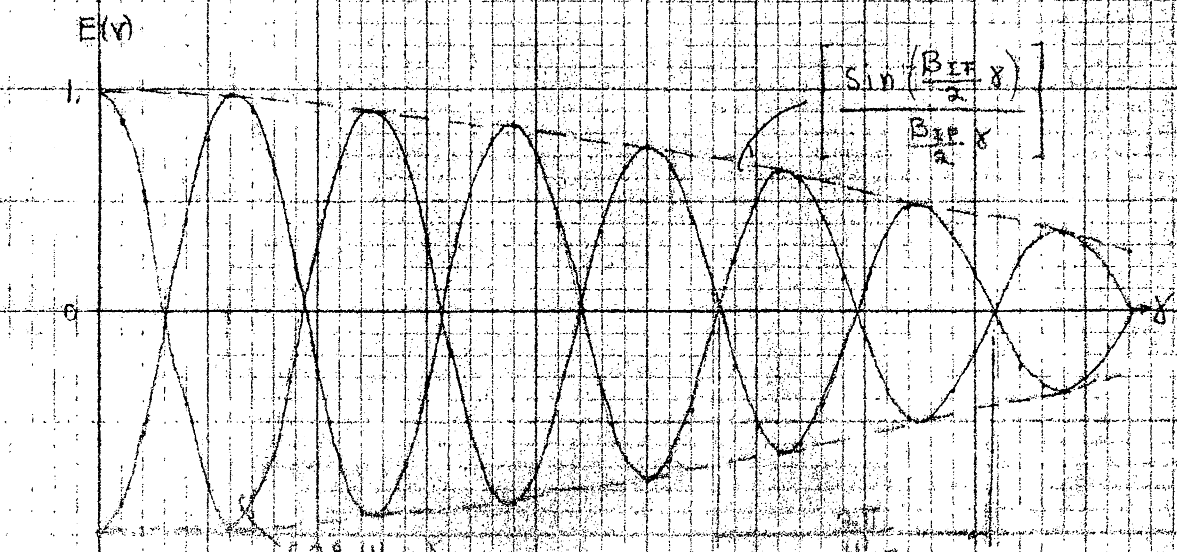


Fig (A 7)

Envelope Function - Gaussian Bandpass



Envelope Function - Rectangular Bandpass



From "System Description", page 6, equation ____, one has"

$$\rho_{uv}(\tau) = \rho_{ss}(\tau + \gamma) \qquad \Phi_{uv}(w) = e^{j\gamma w} A(w)$$

$$h(t) = u(t) \sin w_{1o} t = [s(t) + s_1(t)] \sin w_{1o} t$$

$$l(t) = -v(t) \cos w_{1o} t = -[s(t - \gamma) + s_2(t)] \cos w_{1o} t$$

$$\rho_{hl}(\tau) = \overline{\left\{[s(t) + s_1(t)] \sin w_{1o} t\right\} \left\{-[s(t - \gamma - \tau) + s_2(t - \tau)] \cos w_{1o}(t - \tau)\right.}$$

$$\rho_{hl}(\tau) = \left(\frac{-1}{2}\right) \rho_{ss}(\tau + \gamma) \sin w_{1o} \tau$$

$$\Phi_{hl}(w) = \frac{1}{2\pi} \int \rho_{hl}(\tau) e^{-jw\tau} d\tau = \frac{1}{4\pi} \int -\rho_{ss}(\tau + \gamma) \sin w_{1o} \tau e^{-jw\tau} d\tau$$

$$= +\frac{1}{4} e^{j\gamma w} \left\{\sin w_{1o}\gamma [A(w - w_{1o}) + A(w + w_{1o})]\right.$$

$$\left. + \cos w_{1o}\gamma [A(w - w_{1o}) - A(w + w_{1o})]\right\}$$

$$\Phi_{xy}(w) = |F(w)|^2 \Phi_{hl}(w)$$

$$\Phi_{xy}(w) = +\frac{T_s}{2} e^{j\gamma w} |F(w)|^2 \sin w_{1o}\gamma \text{ if } B_{RF} > B_{IF}$$

$$\rho_{xy}(\tau) = \int \Phi_{xy}(w) e^{jw\tau} d\tau$$

$$r(\gamma) = \rho_{xy}(0) = + T_s \sin w_{1o}\gamma \int_0^{\infty} |F(w)|^2 \cos \gamma w dw$$

Rectangular bandpass

$$r(\Upsilon) = + T_a B_{IF} \left(\frac{\sin \frac{B_{IF} \Upsilon}{2}}{\frac{B_{IF} \Upsilon}{2}} \right) \cos w_{IF} \Upsilon \quad \sin w_{lo} \Upsilon$$

Gaussian

$$r(\Upsilon) = + \sqrt{2\pi}\, T_a\, (.425\ B_{IF}) \exp - \left[\frac{(.425\ B_{IF})^2 \Upsilon^2}{2} \right] \cos w_{IF} \Upsilon \sin w_{IF} \Upsilon$$

Using the envelope function defined in the last section

$$r(\Upsilon) = +E(\Upsilon)\ T_a B \sin w_{lo} \Upsilon$$

The effect of the 90° oscillator shift then can be seen to be a 90° shift in the output fringe, the envelope function remaining unchanged. If this result is combined with the unshifted pattern obtained in the last section, equation ___, a measurement of the in phase and quadrature components of the correlated radiation between antennas is obtained.

IF Phase Delay

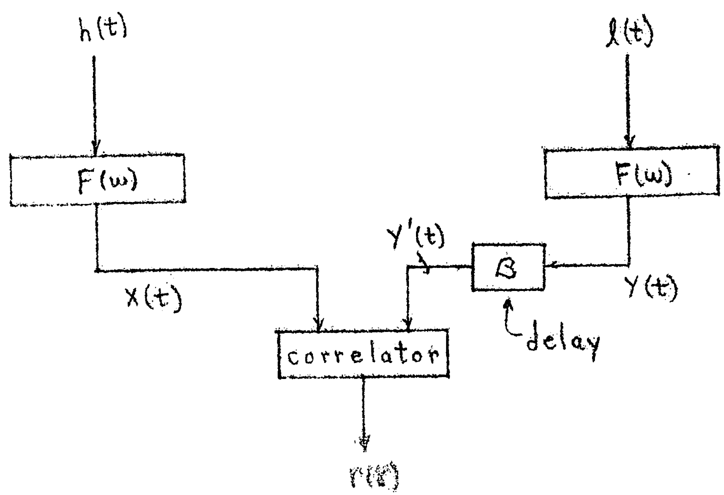


Fig. A(8)

Fig. A(6) is a portion of the normal system shown in Fig. A(1), but with the addition of a time delay (β seconds) in the right channel.

From equation ___, page 8, $\Theta_{xy}(w) = \left(\frac{T_a}{2}\right) e^{j\gamma w} |F(w)|^2 \cos w_{1o} \gamma$

$$\Theta_{xy^1}(w) = \Theta_{xy}(w) e^{-j\beta w}$$

$$\Theta_{xy^1}(w) = \left(\frac{T_a}{2}\right) e^{j(\gamma-\beta)w} |F(w)|^2 \cos w_{1o} \gamma$$

$$\rho_{xy^1}(\tau) = \int_{-\infty}^{\infty} \Theta_{xy^1}(w) e^{jw\tau} dw$$

$$r(\gamma) = \rho_{xy^1}(0) = \int_{-\infty}^{\infty} \Theta_{xy^1}(w)\, dw = T_a \cos w_{1o} \gamma \int_{-\infty}^{\infty} |F(w)|^2 \cos (\gamma - B)\, w\, dw$$

Rectangular bandpass

$$r(\gamma) = T_a B_{IF} \left[\frac{\sin \frac{B_{IF}(\gamma - \beta)}{2}}{\frac{B_{IF}(\gamma \beta)}{2}}\right] \cos w_{IF}(\gamma - \beta) \cos w_{1o} \gamma$$

$$r(\gamma) = E(\gamma - \beta)\ T_a B_{IF} \cos w_{1o} \gamma$$

Gaussian bandpass

$$r(\gamma) = \sqrt{2\pi}\, T_a (.425\, B_{IF}) \exp - \left[\frac{(.425\, B_{IF})^2(\gamma - \beta)^2}{2}\right] \cos w_{IF}(\gamma - \beta) \cos w_{1o} \gamma$$

$$r(\gamma) = E(\gamma - \beta)\ T_a B_{IF} \cos w_{1o} \gamma$$

The time delay β is thus seen to linearly shift the envelope function by an amount β. The fringe information itself, $\cos w_{1o} \gamma$, remains undisturbed.