STRÖMGREN RADII AS A FUNCTION OF SPECTRAL TYPE

Robert Rubin

National Radio Astronomy Observatory
Green Bank, West Virginia

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This report is concerned with the calculation of the size of a spherical H II region surrounding early-type stars. We assume that the nebula is ionization bounded and compute the Strömgren radius, R_S, from a relation obtained by equating the rate of absorption of ionizing photons in the nebula to the rate of emission of Lyman continuum photons from the star. We have from Rubin (1967),

$$R_S = \left[\frac{a^2 f \int_{\nu_1}^{\infty} F^s\ (r = a;\nu) d\nu}{\int_0^{R_S} N_e N_i (\beta - \beta_1)(r/R_S)^2\ dr/R_S} \right]^{1/3} \qquad (1)$$

In this relation, a is the stellar radius, $\int_{\nu_1}^{\infty} F(r=a;\nu)d\nu$ represents the total number of Lyman continuum photons/cm^2-sec emitted by **the** star, and β and β_1 are respectively the recombination rate coefficients to all levels and to the ground state of hydrogen, integrated over frequency. Equation (1) allows for an arbitrary spherically symmetric density distribution in the interstellar matter surrounding the star and an ideal model for density fluctuations. This model assumes that there are clumps of uniform density randomly distributed in an otherwise empty volume so that for any volume in the nebula that is significantly larger than the volume of a clump, the ratio of the latter to the former is statistically equal to f, the clumping factor. The local mean electron density and ion density are N_e and N_i respectively, and f is assumed constant throughout the nebula. Equation (1) allows for N_e, N_i, and $\beta-\beta_1$ varying with distance r from the star. The quantity $\beta-\beta_1$ is a function of electron temperature T and is obtained from the relation by

Seaton (1959) for a hydrogenic atom,

$$\beta-\beta_1 = 5.197 \times 10^{-14} \lambda^{1/2} [-0.3488 + 0.5 \ln \lambda + 0.469 \lambda^{-1/3} + 0.4896 \lambda^{-1} - 0.4796 \lambda^{-2} + 0.057 \lambda^{-3} + 0.405 \lambda^{-4}] \text{ cm}^3/\text{sec}, \quad (2)$$

where $\lambda = 157890/T$. By subtracting β_1 from β, we have in effect accounted for the ionizing radiation caused by recombinations to the ground state.

In this paper we attempt to improve on previous calculations of the Strömgren radius, and make the usual simplifying assumptions of constant N_e with no clumping and constant T. Equation (1) can then provide a guide to changes in R_S when the more general problem must be treated. In effect the calculation is similar to that of Gould, Gold, and Salpeter (1963) with the following changes:

1) We use improved values of effective temperature, surface gravity, and radii for early-type stars obtained from Hjellming and Jarecke (1967).

2) We use model stellar atmospheres from Mihalas (1965) instead of black body atmospheres.

Table 1 presents values for the effective temperature, surface gravity, and radius for early-type stars on the main sequence obtained from Hjellming and Jarecke (1967). These stellar parameters for various spectral types were compiled from a literature search that included both theoretical and observational data. Using the values of effective temperature and surface gravity and assuming the number abundance of helium to hydrogen in the atmosphere is 0.15, we interpolate in a grid of model atmospheres by Mihalas

(1965) to obtain the flux of Lyman continuum photons emerging from the star, which is entered in column 5 of Table 1. Columns 6-8 give Strömgren radii calculated for a constant electron temperature of 7000°K and values of the electron density of 1, 10 and 100 cm^{-3}, respectively. These entries may be readily scaled for different temperatures and densities by using equation (1). Figure 1 provides values of $[(\beta-\beta_1)(T = 7000°K)/(\beta-\beta_1)(T)]^{1/3}$ in order that the Strömgren radii in Table 1 may be scaled for different values of the electron temperature.

As long as we are dealing with ionization limited regions, the entry for $N_e = 1\ cm^{-3}$ is equivalent to the excitation parameter, u, defined as $R_S N_e^{2/3}$ when there is no clumping and T and N_e are constant. We have from equation (1),

$$u = R_S N_e^{2/3} = \left[\frac{3a^2 \int_{\nu_1}^{\infty} F^s(r = a;\nu)d\nu}{(\beta-\beta_1)} \right]^{1/3}$$

It is stressed that values of R_S in Table 1 are values for ionization bounded nebulae. If the nebula is density bounded allowing ionization radiation from the star to escape the nebula, these values are upper limits to the extent of the plasma. Furthermore if the matter is clumped ($f < 1$), recombinations are enhanced relative to ionizations, and as indicated by equation (1), the Strömgren radii in Table 1 must again be considered an upper limit.

Let us now discuss a method of determining the excitation parameter from radio measurements of the free-free emission from a nebula.

The flux density S_ν (ergs/cm^2-sec-Hz) is given by

$$S_\nu = \frac{3.180 \times 10^{-19}}{D^2 T^{0.35} \nu^{0.1} f} \int_0^{R_S} N_e(r) N_i(r) r^2 dr \quad , \tag{3}$$

where D, the distance to the nebula, and R_S are in pc and ν is in GHz. This relation assumes that the nebula is spherically symmetric, optically thin at the frequency ν, and isothermal. We have used the expression for the free-free emission given by Mezger and Henderson (1967) setting their quantity a equal to unity. In equation (3) we have allowed for density fluctuations. Since $N_e(r)$ and $N_i(r)$ are the local mean electron and ion densities, a factor f^{-2} enters the equation; but only a fraction f of the nebular volume is contributing to the emission. Hence the net effect of clumping is as indicated. Equation (1) for an isothermal nebula may be written as

$$\int_0^{R_S} N_e N_i r^2 dr = \frac{a^2 f}{\beta - \beta_1} \int_{\nu_1}^{\infty} F^S(r = a;\nu) d\nu .$$

When we combine this last relation with equation (3), we eliminate the clumping factor and the integral over the density, and obtain for the excitation parameter

$$u = (9.434 \times 10^{18} S_\nu D^2 \nu^{0.1} T^{0.35})^{1/3} . \tag{4}$$

For an assumed distance and electron temperature, the observed flux density yields the excitation parameter. The method could also be turned around to solve for the distance if the exciting stars and their radii and Lyman continuum fluxes are all known. If the nebula is density bounded, equation

(4) provides a lower limit for the excitation parameter or an upper limit for the distance.

References

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Table 1

Strömgren Radii for Various Values of N_e and T = 7000 °K

Spectral Type	Effective Temperature (°K)	log g (cm/sec^2)	$a/a_\odot$	$\int_{\nu_1}^{\infty} F^s(r=a;\nu)d\nu$ (photons/cm^{2}sec)	R_S(pc) for N_e(cm^{-3}) 1	10	100
04	50 000	4.4	8.0	5.4 (24)	79	17	3.7
05	45 000	4.3	6.8	3.1 (24)	59	13	2.8
06	40 000	4.3	5.8	1.5 (24)	41	8.9	1.9
07	38 000	4.3	5.5	1.0 (24)	35	7.6	1.6
08	36 000	4.3	5.2	5.4 (23)	28	6.0	1.3
09	35 000	4.3	5.0	3.6 (23)	23	5.0	1.1
09.5	33 000	4.3	4.7	1.3 (23)	16	3.5	0.75
BO	31 000	4.3	4.5	3.7 (22)	10	2.2	0.48
BO.5	29 000	4.3	4.4	9.7 (21)	6.5	1.4	0.30

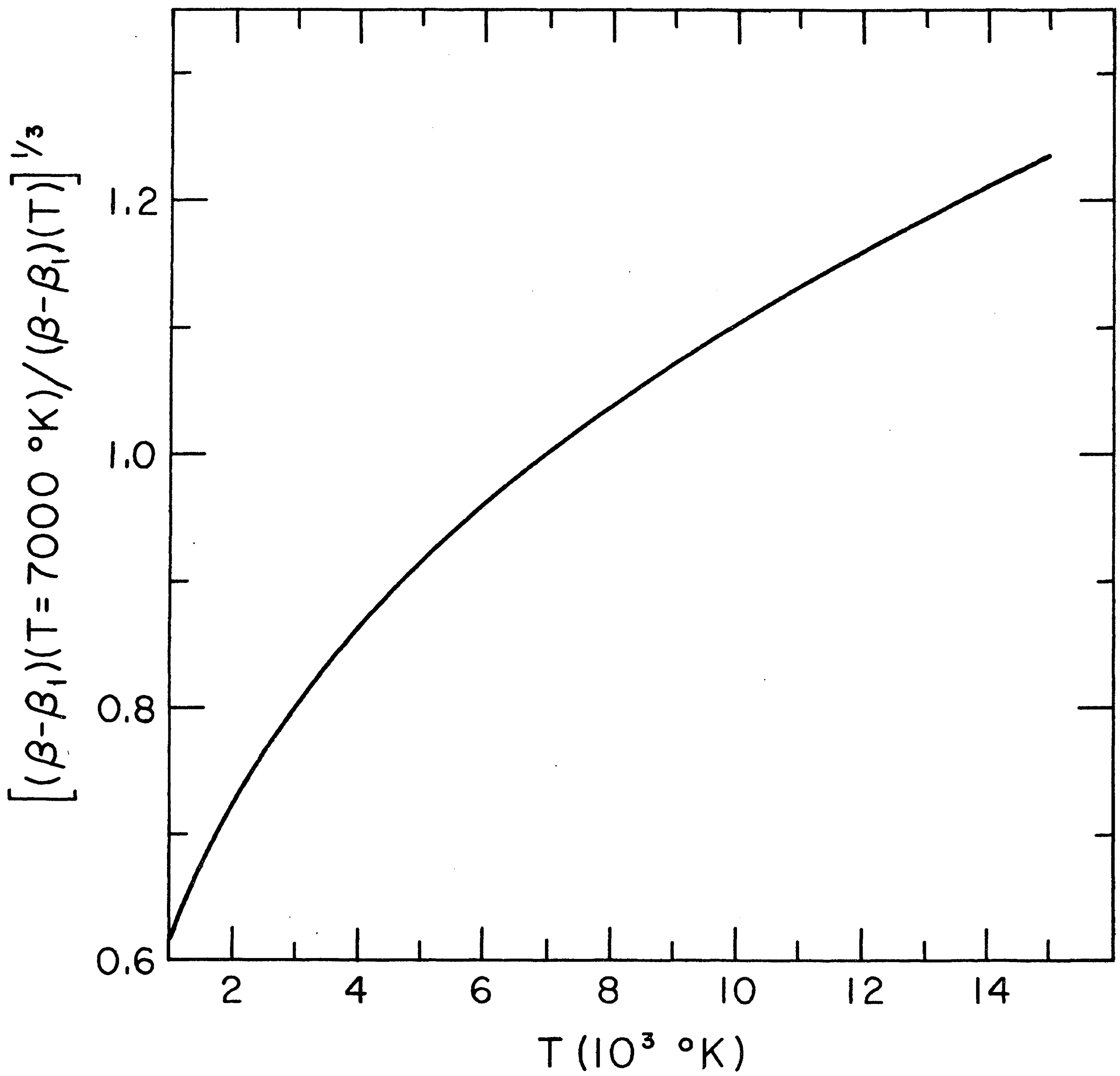

FIG. 1- Electron temperature dependent factor for scaling the Strömgren radii in Table 1.