mmA Antenna Memo No. /0

DISCUSSION OF SLANT AXIS ANTENNARGONGERE U.S. GOVERNMENT RADIO ASTRONOMY OBSERVATORY

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Abstract

A slant axis antenna concept differs from a conventional antenna design in many aspects. The slant axis antenna involves a different coordinate system and has a different structural arrangement. Compared with conventional antennas with similar backup structure supporting diameter, it may result a higher natural frequency telescope structure. This is important to allow switching rapidly from source to source. However, the slant axis antenna has a changeable force condition on its two main bearings, which is undesirable to its pointing performance. In this memo, the coordinate transformation formulas for slant axis antenna are worked out, the changeable force condition is analyzed and the effect of changeable force condition to the antenna pointing performance discussed. Its moment of inertia is compared with that of a conventional antenna for which the location of the elevation axis is similar to the location of slant bearing. Ways of improving the structural performance of the slant axis antenna are discussed and different arrangements of offset version of slant axis antennas are provided. Finally the polarization and sweeping diameter problems of the slant axis antenna are also examined. For an antenna structure, the wind and temperature effects on the reflector surface and on the pointing accuracy are always serious issues. In this memo, this part of work is not included.

1. Introduction

Recently, Lamb and Payne had suggested that a slant axis antenna concept might be suitable for the proposed MMA antenna design. The slant axis antenna concept is an unconventional one, which differs in many aspects from a conventional antenna structure. In this memo, the slant antenna structure and its related problems are discussed.

Very few records ^{[1] [2] [3]} have been found on the slant axis antenna structure. However, an early slant axis antenna had been built at the University of Birmingham, England, a few decade ago (Fig. 1). That antenna has an offset Cassegrain optical configuration. The slant axis design nicely fits the tilted main reflector and greatly helps the support structure of its comparatively large offset secondary mirror. With its slant axis directly under its primary mirror backup structure, the connection between antenna support structure and reflector dish is compact. The antenna is rigid. In 1991, a recent slant axis antenna was built at the south pole by MAN (Fig. 2). The design of this antenna is very similar to the proposed concept for MMA by Lamb (Fig. 3). However, it has a third bearing ring which allows the antenna to reach the zenith and horizon pointing. Another example of applying a slant axis design is the Large European Solar Telescope (LEST) project dome, which is separated from the telescope. Compared to the MMA antenna, the specification of these three examples are less demanding.

2. Structural arrangement of slant axis antennas

A slant axis MMA antenna arrangement is shown in Fig. 3. The whole antenna can be divided into four parts: (1) the base, (2) the azimuth turntable and receiver cabin, (3) the slant axis turntable and reflector support structure, and (4) the backup structure with secondary mirror. The radio wave received by the antenna is reflected from primary and secondary mirrors to a third mirror. The third mirror is located at the center of the slant bearing and rotates around the slant axis with the primary mirror. A fourth mirror is also required to direct radio wave towards the receiver. The third mirror has a larger incident angle since the optical axis and the slant axis has an angle of 135 degrees. For the antenna structure, two bearings are the main components. Both bearings have a larger diameter. However, they may have the same dimension and requirements to reduce antenna cost. The antenna backup structure is directly jointed with the main structure of the antenna. The connection is a continuous ring and the main structure has a closed tubular shape. The dimension of this tubular structure is usually larger than the elevation shaft diameter or the dimension of the supporting structure of backup structure. This ensures a higher stiffness of the slant axis antenna. Therefore, it will have higher natural frequency as:

$f \sim [K / M]^{1/2}$

(1)

where f is the natural frequency;

K is the stiffness coefficient. K is directly related to the structure dimension and support distance;

M is a generalized mass.

However, from this conceptual arrangement, one can find that the reflector supporting structure is less compact than that of the Birmingham offset antenna as the bottom of the backup structure is symmetrical about the optical axis and the slant bearing has an angle of 45 degrees about the same axis. The distance between the backup structure and the slant bearing is almost half of the supporting diameter of the backup structure. The larger the supporting diameter of the backup structure, the bigger the distance form the slant bearing. Therefore the center of gravity of the rotating part above the slant bearing will be at a distance to the bearing center. This will cause complicated force conditions when the antenna is rotating about the slant axis, as discussed later.

The MMA telescope is designed as a complete imaging instrument^[4]. The MMA control system has to calibrate the data continuously throughout its observing period. Therefore, the antenna needs to allow a rapid switching performance. Brown^[5] had suggested that the primary mirror of the MMA antenna will switch between source and a nearby calibrator once every 6 seconds and the mean separation of source and calibrator is 1.5 degrees or less. This requirement demands a rigid structure design for the antenna. For this reason, the slant axis concept is chosen as a candidate for the MMA antenna structure.

3. Antenna movement and coordinate transformation

Differing from conventional telescopes, the slant axis antenna has a rotating axis which is inclined to the horizon. Taking the inclined angle as 45 degrees (which may be varied to suit different requirements of the antenna), the slant axis movement will involve not only the change in the elevation direction, but also in azimuth direction. Understanding the movement and the coordinate system is the first thing in the antenna structure design.

In Fig. 4, OXYZ is a rectangular coordinate system, OA is a unit vector along the X axis. By rotating the system OXYZ -45 degrees about Z axis, a new coordinate system $OX_1Y_1Z_1$ system is formed. Relative to the new coordinate system, rotating unit vector OA an angle θ about the OY₁ axis, the coordinate of point A₁ in the OX₁Y₁Z₁ system are:

	$ \left[\begin{array}{ccc} \cos \theta & 0 & \sin \theta \end{array} \right] \left[\begin{array}{c} \cos \left(-45^{\circ}\right) \\ 0 & 1 & 0 \end{array} \right] \left[\begin{array}{c} -\sin \left(-45^{\circ}\right) \end{array} \right] $	sin (-45°) cos (-45°)	0 [1]	
	$\lfloor -\sin \theta \ 0 \ \cos \theta \rfloor \lfloor 0$	0	1][0]	
2	[cos (45°) cos θ] sin (45°) L -cos (45°) sin θ]			

Where θ is the slant axis angle. By transferring back to the original OXYZ coordinate system,

(2)

the coordinates of the point A_1 will be:

 $\begin{bmatrix} \cos (45^{\circ}) \sin (45^{\circ}) & 0 \end{bmatrix} \begin{bmatrix} \cos (45^{\circ}) \cos \theta \\ -\sin (45^{\circ}) \cos (45^{\circ}) & 0 \end{bmatrix} \begin{bmatrix} \sin (45^{\circ}) \\ \sin (45^{\circ}) \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\cos (45^{\circ}) \sin \theta \end{bmatrix}$ $\begin{bmatrix} \cos (45^{\circ}) \cos (45^{\circ}) \cos \theta + \sin (45^{\circ}) \sin (45^{\circ}) \\ -\cos (45^{\circ}) \sin (45^{\circ}) \cos \theta + \sin (45^{\circ}) \cos (45^{\circ}) \end{bmatrix}$ $\begin{bmatrix} 0.5 (1 + \cos \theta) \\ \cos (45^{\circ}) \sin \theta \end{bmatrix}$ $\begin{bmatrix} 0.5 (1 + \cos \theta) \\ 0.707 \sin \theta \end{bmatrix}$

If the elevation angle is E, and the azimuth angle is A, then the relationships between slant axis angle θ and E and A are as:

$$\sin E = 0.5 (1 - \cos \theta)$$

 $(0.707 \sin \theta)$

 $\tan A =$

 $[0.5(1 + \cos \theta)]$

The relationship between θ and E is plotted in Fig. 5 and that between θ and A is in Fig. 6. From Fig. 5, one can find that the elevation angle increase very slowly at the horizon position until the elevation angle reaches 5 degrees. Afterwards, the slope of the curve increases to 1/2and most parts of the curve have a slope of 1/1.5. This means that if the antenna is required to reach horizon pointing, the slant axis should rotate very fast in order to raise the antenna in the elevation direction. Also from Fig. 6, the azimuth angle increases rapidly when the slant axis angle increases near horizon. This means that the azimuth axis of the slant axis antenna has to move fast as well when the antenna is pointed near horizon. This is an undesirable feature of this configuration if the antenna is required to reach very bottom of horizon. But above 5 degrees from horizon, the situation will be improved, the slant axis rotates 2 or 1.5 degrees while the elevation angle changes 1 degree. For azimuth motion, a larger slope occurs at the beginning and reaches 1/3 near zenith. Since the antenna has an azimuth axis as well, slow movement at azimuth direction caused by slant axis rotation will not affect the antenna position response. From the figures, it can be found that the elevation angle will be 30 degrees and the azimuth angle will be 54.5 degrees when the slant angle is 90 degrees. Lamb^{[4][5]} has calculated detailed curves for both axis when the antenna is tracking to different celestial

(3)

(4)

(5)

objects. No discontinuous point has been found for the slant axis antenna.

4. Force condition and bearing requirement

For the slant axis antenna, the center of gravity of the rotating part above the slant axis is away from the slant axis. If no counterweight is applied, the center of gravity of the rotating part C passes through the azimuth axis when the antenna is pointed to zenith. However, when it is away from zenith, the center of gravity above slant bearing will sweep through an inclined circle. Its lowest point is C_0 as shown in Fig. 7. This circle has its center O_1 on the slant axis and has one edge point C passing through the azimuth axis. When slant axis angle is θ , the center of gravity will be at point C_{θ} . Due to the rotation of the center of gravity, all the bearings of the antenna will suffer from a gradually changed force condition.

For the slant bearing, the forces applied include radial force, axial force, and bending moments in two directions. Fig. 7 shows the force condition of slant bearing at different orientations. Fig. 7 (a) shows the condition when the antenna is pointed to zenith, Fig. 7 (b) is the condition when the antenna is in horizon pointing and (c) shows the condition when the antenna has a slant angle of θ . If the gravitational force of the rotating part above slant bearing is W and the distance from the center of gravity to the slant bearing center is D, then the forces applied on the slant bearing when the antenna is pointed to zenith are:

 $W_r = 0.707 W$ $W_a = -0.707 W$ $M_x = 0$ $M_y = 0$

Where W_r is the radial force on slant bearing;

W_a is the axial force on slant bearing;

 M_x is the moment about X axis;

 M_y is the moment about Y axis.

When the antenna is pointed to horizon, the center of gravity will move to the lowest point and the forces on the slant bearing will be:

$$W_r = 0.707 W$$

 $W_a = -0.707 W$
 $M_x = 0$
 $M_y = DW$

(7)

(6)

In the general case, when the slant axis angle is θ , the forces applied on the bearing will be:

$$\begin{split} W_{r} &= 0.707 \ W \\ W_{a} &= -0.707 \ W \\ M_{x} &= 0.707 \ DW \sin \theta \\ M_{y} &= 0.5 \ DW \ (1 - \cos \theta \) \end{split}$$

One can find that all the axial and radial forces are unchanged when the bearing is rotating. However the moments applied on the slant bearing do change both in orientation and magnitude. These changeable moments are also applied to the azimuth bearing (the azimuth bearing only has an axial force which equals W and no radial force). Therefore the azimuth bearing also tilts when the antenna is away from zenith pointing.

Taking an estimated value, if the slant bearing has following stiffness coefficients:

Axial stiffness	15730 N/µm,	
Radial stiffness	8247 N/µm,	
Moment stiffness	34050 N-m/arc sec	(Quoted data from Rotec co. ^[6])

and, for an 8 m antenna dish, the estimated weight and the distance between the center of gravity and the slant bearing center as:

W = 5500 kgD = 1.2 m

(9)

(8)

then the radial displacement of slant bearing will be $5.5 \mu m$, the axial displacement will be 2.4 μm , the tilt angle in Y direction will be 1.9 arc sec and in X direction will be 1.3 arc sec. For the azimuth bearing, the axial displacement will be 4.8 μm , the tilt angle in Y direction is 1.9 arc sec and in X direction is 1.3 arc sec. The axial and radial displacements will not affect the pointing of the antenna, but the tilt, although mostly (about 80 %) repeatable, will certainly cause some pointing error. Furthermore the tilt error caused by two bearings will have the same orientation and their effects will add together. This results in a total error in the Y direction of 3.8 arc sec and 2.7 arc sec in the X direction.

The MMA specification requires high pointing performance of antenna structure. The total allowed pointing error is 1 arc sec.. This is a tight error requirement for an open air antenna since the wind and temperature effects usually make the pointing error near or larger than this error budget already. For a slant axis antenna, since the poor force conditions are applied at both bearings and this additional errors from bearing will added to those from wind and tem-

perature effect, the bearings used for the MMA antenna therefore should be critical components. The basic requirements for the bearings are: high stiffness, high precision and allowance for axial preload. These requirements may result in heavier and costly bearing design. For improving this situation, some structure modifications of the slant axis antenna may be necessary.

5. Moment of inertia of the slant axis antenna structure

Moment of inertia is another important parameter for antenna structure, especially when the antenna is requested to switch rapidly back and forth. Larger moment of inertia will not only cause slower response during switching process, but also demand higher drive power. These are both undesirable.

If the moment of inertia of the backup structure about its symmetrical axis is I_o , then the moment of inertia about the slant axis will be:

$$I_{ss} = I_o + W D^2/2$$

For a conventional antenna structure, if the elevation axis is located at the same place as the slant bearing center, then the moment of initia of the backup structure about the elevation axis is:

$$I_{co} = I_0 + WD^2$$

From Eq. 10 and 11, it can be found that the slant axis antenna has a slightly smaller moment of inertia about the first axis compared to a conventional one with a similar axis location. For a drive axis near to the center of gravity, the moment of inertia will be smaller. It should be mentioned that Eq. 11 does not include the effect of the counterweight which is always necessary for a conventional arrangement. The moments of inertia of the antenna about azimuth axis are very much the same for both conventiional and slant axis antennas.

6. Offset versions of slant axis design

For improving the force condition of the slant axis design, an offset version of the antenna may be worth considering. The idea of this modification is to intersect the bearing axis and the axis of the backup structure far ahead of the bearing plane. In this way the center of gravity of the backup structure will be near to the slant axis. However, this version of antenna will have its reflector axis away from the azimuth axis when the antenna is pointed to zenith. Two advantages will be gained from this modification. First, the turning moment applied on both bearings will be greatly reduced. The pointing error of the antenna will be improved. Secondly, the

(11)

(10)

moment of inertia about either axis will be reduced as the center of gravity of the backup structure moves closer to the rotating axes of the antenna (please note the case when the antenna is pointed to horizon.). This will assure that the antenna will have a better dynamic performance. The conceptual drawing of an offset version antenna is shown in Fig. 8.

By applying an offset arrangement, counterweights may be applied to the bottom of the antenna to put the center of gravity of the rotating part above slant axis on the slant axis. In this way, the bearing will have a stationary force condition although a moment is still unavoidable. Also it is possible to add weight to the receiver cabin to make the azimuth bearing free from bending moments. In this way the pointing error will be reduced to less than half of that in the symmetrical design.

In fact, the changeable force condition on the bearings can be totally removed by placing the center of gravity of the moving part on the slant axis as shown in configurations in Fig. 9. All these designs will have a stationary force condition about the slant axis. At the same time, since the center of gravity passes through the rotating axis, the second term in the moment of inertia expression will be vanished as well. Therefore these designs will have smallest moment of initia about the first axis. By adjusting the receiver cabin size and weight, the center of gravity above azimuth bearing may be adjusted to pass through the azimuth axis. Therefore the azimuth bearings will have a symmetrical force condition. No tilting error will appear at this axis. Incorporating the design of an offset structure with the backup structure, a homologous design such as the Bonn 100 m^[9] dish might possible be applied as shown in the bottom of Fig. 9.

7. Sweeping diameter of slant axis antenna

For an array instrument, the sweeping diameter of the antenna determines the minimum spacing that can be achieved for the array. This in turn determines the u-v coverage of the MMA array^[10]. For this purpose, a smaller sweeping diameter is desirable for MMA antenna. For conventional antennas, the sweeping diameter is determined by the distance between the elevation axis and the primary focus, if it is larger than that between elevation axis and the edge of the dish. For a slant antenna structure, the sweeping volume can be easily determined as shown in Fig. 10. In Fig. 10 (a), a symmetrical slant axis design is illustrated. The dotted line shows the outline of the structure rotated about the slant axis. By rotating this shape about the azimuth axis, the final sweeping volume will be generated. The resulting sweep diameter is 10.4 m which is 1.3 times the aperture. This is almost the same as that of a conventional antenna structure. However, for an offset version of slant axis design, the sweeping diameter is smaller as shown in Fig. 10 (b). The resulting sweeping diameter is only 9.6 m which is 1.2 time of the antenna aperture. This is because that the secondary mirror is closer to the azimuth axis when the antenna is in horizon pointing.

8.Polarization effect of reflector mirror

One slant axis design involves a third mirror for directing radio wave to the receiver cabin where the mirror has larger incident angle of 67.5 degrees. At this angle, polarization effect is serious in the optical region^[11]. For assuring the design concept, a calculation has been performed in the millimeter region. The results calculated show that the larger incident angle here will not cause serious effect on the polarization of the radiation received in the millimeter region. The detailed calculation is attached in the Appendix of this memo.

9. Other ways to improve slant axis antenna pointing performance

9.1 Movable weight on the receiver platform

When the antenna rotates about its slant axis, the force condition changes even for the azimuth bearing, that condition can be removed by adding a movable weight on the receiver platform. In this way force condition will be improved on the azimuth bearing and the pointing performance of the antenna will be improved as well. Since the projection of the locus of the gravitation center is an ellipse, so the counterweight applied should have a moving rail shaped as an ellipse.

9.2 Offset optics

From the experience of Birmingham antenna, the slant axis design may be more suitable for the offset optics design. For offset antenna, the primary mirror has an angle relative to the incoming radiation. If offset optics are used, the backup structure of the primary mirror will be closely fitted with the angle of the slant bearing. The structure is much compact. The center of gravity above slant axis will be near to the bearing center. The moment applied at the bearing will be smaller. Then the tilt of bearings will be smaller as well. But offset optics will certainly bring difficulties in the structure design.

9.3 Heavier ball design

In Fig. 3, under the backup structure, there is a complete ball shaped structure. If the ball is large and comparably heavier, the force conditional change when the antenna is rotating will be less significant. The tilt of the bearings will be small as well. This may be a slightly more costly design.

10. Computer model of a slant axis design

Fig. 11, 12 and 13 show a computer model of a slant axis design. The model is used for NASTRAN analysis. This model is a simplified one. Its backup structure is similar to that of the JCMT design, which involves a three dimensional truss structure. The advantage of this structure is its high stiffness. However, only three supporting points are provided for each surface panel which is different from other antenna designs. The drawing shown in Fig. 11 is at zenith pointing position. That in Fig. 12 is at horizon position and Fig.13 shows the slant axis has rotated 90 degrees from previous positions. At this time the elevation angle is 30 degrees and the azimuth angle is 54.5 degrees. The changes of force applied on the bearing can be easily realized. Both static and dynamic analysis have been performed by the NASTRAN program.

REFERENCES

[1] Cook, J. S. et. al., The open cassegrain antenna: part I. electromagnetic design and analysis, *the bell system technical journal*, 1255-1300, Sept., 1965.

[2] Denkmann, W. J. et. al., The open cassegrain antenna: part II. structural and mechanical evaluation, *the bell system technical journal*, 1301-1319, Sept., 1965.

[3] Nelson, W. L. and Cole, W. J., Autotrack control systems for antenna mounts with nonorthogonal axes, *the bell system technical journal*, 1367-1430, Sept., 1965.

[4] Natinal Radio Astronomy Observatory, Millimeter Array Design and Development Plan, Sept., 1992.

[5] Brown, R., E-mail messege to Lamb, Sept., 1992.

[6] Lamb, J., Coordinate transformations for slanted-bearing antenna, Sept., 1992.

[7] Lamb, J., Pointing equations for slanted-bearing antenna, Oct., 1992.

[8] Sable, H. J., Fax to Cheng, Nov., 1992.

[9] Mar and Liebowitz, Structures technology for large radio and radar telescope systems, The MIT press, Cambidge, MA, 1969.

[10] Lamb, J., Minimum spacing constraints for MMA antennas, MMA antenna memo No. 2, 1992.

[11] Born and Wolf, Principles of optics, Pergamon Press, Oxford, 1970.

APPENDIX POLARIZATION EFFECT AT 1 MM WAVELENGTH OF REFLECTING MIRRORS

1. Incident wave with its E vector normal to the plane of incidence.

From Paul Lorrain and Dale Corson[1], the ratio of reflected E $_{or}$ vector and incident E_{oi} vector is:

 $\mathbf{E}_{or} \qquad n_1 \mu_{r2} \cos \theta_i - \lambda_o \left(1 - j\right) / \delta$

 E_{oi} $n_1 \mu_{r2} \cos \theta_i + \lambda_o (1-j)/\delta$

where E_{or} is reflected E vector,

E_{0i} is incident E vector, n₁ is index of refraction, here n₁ = 1 μ_{r2} is relative permeability, here μ_{r2} = 1 (non-magnetic) θ_i theta is incident angle $\lambda_o = \lambda/2 \pi$, for millimeter wave $\lambda_o = 0.159$ mm δ is skin depth, $\delta \sim [1/f]^{1/2}$, for aluminum, when f = 3 GH, δ = 1.6 µm; when f = 300 GH, δ = 0.16 µm.

$$E_{or} = \frac{\cos \theta_i - 1000 (1-j)}{\cos \theta_i + 1000 (1-j)}$$

$$= \frac{[\cos^2 \theta_i - 2^* 1000^2] + 2000 \cos \theta_i j}{\cos^2 \theta_i + 2^* 1000^2 + 2000 \cos \theta_i}$$

 $\cos^4 \theta_i + 4^* 1000^4$

≈ 1 + 2.5 e -13

Phase = arc tan (
$$-\frac{2000 \cos \theta_i}{\cos^2 \theta_i - 2*1000^2}$$

≈ 0

2. Incident wave with its E vector parallel to the plane of incidence

 $-n_2 \cos \theta_i / \mu_{r2} + n_1 \cos \theta_t / \mu_{rI}$ Eor $n_2 \cos \theta_i / \mu_{r2} + n_1 \cos \theta_t / \mu_{r1}$ Eoi (Equ. 12-102 of [1]) where $n_2 = (\lambda_o / \delta)(1-j)$ $\mu_{r2} = 1$ $\mu_{rl}=1$ θ_t is transmission angle, $\theta_t = 0$ - 1000 (1 - j) cos θ_i + 1 Eor E_{oi} $1000(1-j)\cos \theta_i + 1$ $-2^*(1000\cos \theta_i)^2 + 1 + 2000\cos \theta_i j$ $2^{*}(1000\cos\theta_{i})^{2}+2000\cos\theta_{i}+1$ $1 + 4 * (1000 \cos \theta_i)^4$ Amplitude = $1 + 4*(1000\cos \theta_i)^4 + 4(1000\cos \theta_i)^2$ $(1000 \cos \theta_i)^2$ (curve as in Fig. 14) $1 + (1000 \cos \theta_i)^2$

Phase = arc tan ($-\frac{2000 \cos \theta_i}{1 - 2^*(1000 \cos^2 \theta_i)}$)

Note, this appendix is calculated with help from J. Lamb.

References

[1] Lorrain, P. and Corson, D., Electromagnetic fields and waves, W. H. Freeman and Company, San Francisco, 1962.







Figure 2 A south pole 9 m slant axis antenna built by MAN co.



Figure 3 A concept design of MMA antenna by Lamb







Figure 5 Relationship between slant angle and elevation angle



Figure 6 Relationship between slant angle and azimuth angle



Figure 7 Force condition of slant bearing



Figure 8 Offset version of slant axis design







Figure 10 Sweeping diameter of non-offset and offset versions of slant axis antenna structure

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Figure 11 Computer model of a slant axis design at zenith pointing





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Amplitude ratio



Figure 14 Amplitude ratio of reflected wave with its E vector parallelto the plane of incidence