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Relative Integration Times for the ALMA Cycle 1 12-m, 7-m, and Total Power Arrays

NAASC Memo #113/ALMA Memo 598

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ABSTRACT

We derive some basic considerations relating combining synthesis observations made with different arrays, and their relation to single dish imaging. The goal is to make clear and explicit a method by which ALMA7-meter array and the ALMA total power array integration times can be chosen for a given ALMA 12-meterarray integration time. We posit two complementary matching criteria, apply them to the M100 Science Verification (SV) data as a sanity check, and use them to calculate the needed 7m-array integration times for the four relevant ALMA Cycle 1 configurations (assuming 32 12-m antennas, 9 7-m antennas, and 2 total power 12-m antennas). Adopting the least stringent of the criteria, we find required 7m-array integration times of 1.3 (configuration 4) to 6.3 times (configuration 1, most compact) the 12m-array integration time. We extend this to the single-dish/total power case, finding cycle 1 12m total power array integration times that are 2.0 times the 7m-array integration times, or 2.6 to 13 times the 12m interferometric array integration time. Full results are in Table 1.

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1 Executive Summary

We derive some basic considerations relating combining synthesis observations made with different arrays, and their relation to single dish imaging. The goal is to make clear and explicit a method by which ALMA 7-meter array and the ALMA total power array integration times can be chosen for a given ALMA 12-meter array integration time. We posit two complementary matching criteria, apply them to the M100 Science Verification (SV) data as a sanity check, and use them to calculate the needed 7m-array integration times for the four relevant ALMA Cycle 1 configurations (assuming 32 12-m antennas, 9 7-m antennas, and 2 total power 12-m antennas). Adopting the least stringent of the criteria, we find required 7m-array integration times of 1.3 (configuration 4) to 6.3 times (configuration 1, most compact) the 12m-array integration time. We extend this to the single-dish/total power case, finding cycle 1 12m total power array integration times that are 2.0 times the 7m-array integration times, or 2.6 to 13 times the 12m interferometric array integration time. Full results are in Table 1.

Config.	t_{7m}/t_{12m}	t_{tp}/t_{12m}
1	6.3	13
2	2.8	5.6
3	1.8	3.6
4	1.3	2.6

Table 1: Minimum required 7m-array total integration times, and 12m single dish ("total power") integration times, both relative to 12-m interferometric total integration time, for ALMA Cycle 1. Number of assumed antennas: 32× 12m; 9× 7m; and 2× single dishes.

If the number of antennas differs from what was assumed in these calculations, then the nominal integration time ratios given in Table 1 can be approximately rescaled as

$$\frac{t_{7m}}{t_{12m}} \propto \left(\frac{N_{12m}}{32}\right)^2 \times \left(\frac{9}{N_{7m}}\right)^2 \tag{1}$$

and

$$\frac{t_{tp}}{t_{12m}} \propto \left(\frac{N_{12m}}{32}\right)^2 \times \left(\frac{2}{N_{SD}}\right) \tag{2}$$

The single dish integration times do not include an allowance for frequency or position switching, which would further increase the required single dish integration times.

2 Relative Mosaic Sensitivities for Different Interferometer Arrays

2.1 Basic Relationships

Suppose we image a given area of sky (total area Ω Sr) with two independent synthesis arrays labeled 1 and 2. Assume that an equal integration time τ_1 is spent on each pointing in the mosaic. As derived in Thompson, Moran & Swenson (2001, eq. 6.56) — and also in Taylor, Carilli & Perley (1999, Chapter 9) whose notation we roughly follow— the single-field sensitivity at the center of each pointing of array 1 is

$$\Delta I_{m,1} = \frac{\sqrt{\sum \mathscr{T}_{1,k}^2 \mathscr{W}_{1,k}^2 \omega_{1,k}^2 \Delta S_{1,k}^2}}{\sum \mathscr{T}_{1,k} \mathscr{W}_{1,k} \omega_{1,k}} \tag{3}$$

$$= \frac{\sqrt{\sum \mathscr{T}_{1,k}^2 \mathscr{W}_{1,k}^2 \omega_{1,k}}}{\sum \mathscr{T}_{1,k} \mathscr{W}_{1,k} \omega_{1,k}} \tag{4}$$

where the second step assumes that the data "signal to noise" weight $\omega_{1,k} = 1/(\Delta S_{1,k}^2)$. Table 2 summarizes our notation. For natural weighting ($\mathscr{W}_{1,k} = 1$) and no uv taper ($\mathscr{T}_{1,k} = 1$), this becomes

$$\Delta I_{m,1} = \frac{1}{\sqrt{\sum \omega_{1,k}}} \tag{5}$$

The noise $\Delta S_{1,k}$ on a measurement on a single baseline k involving antennas i and j, is:

$$\Delta S_{1,k\rightarrow(i,j)} = \frac{2k_B}{A_1 \eta_{Q,1}} \sqrt{\frac{T_{sys,1,i} T_{sys,1,j}}{\eta_{a,1,i} \eta_{a,1,j} \Delta\nu \tau_1}} \tag{6}$$

assuming all antennas, receivers and measurements in array 1 are identical, the image noise (Eq. 5) becomes

$$\Delta I_{m,1} = \frac{2k_B}{\eta_{Q,1} A_1 \eta_{a,1}} \frac{T_{sys,1}}{\sqrt{2N_{bas,1} \Delta\nu \tau_1}} \tag{7}$$

Let the area Ω be covered by array 1 in some number of pointings, $N_{ptg,1}$, which for a Gaussian primary beam is given by:

$$N_{ptg,1} = \alpha \frac{\Omega}{\Omega_{beam,1}} = \alpha \frac{\Omega}{2\pi\sigma_{beam,1}^2} \tag{8}$$

Here Ω_{beam} is the primary beam volume, which for a Gaussian primary beam is $\Omega_{beam} = 2\pi\sigma_{beam}^2$ (with $\sigma_{beam} = \theta_{FWHM}/\sqrt{8\,ln(2)}$). α is a number of order unity that depends on the mosaicking strategy. For instance, for a hexagonal mosaic with spacing between pointings of $0.5085\times\theta_{fwhm}$ we can show that $\alpha \sim 4.18$. This yields an approximately uniform image domain sensitivity over the mosaic of

$$\Delta I_{tot,1} = \beta \Delta I_{m,1} \tag{9}$$

where β is also a factor of order unity that depends on the mosaicking strategy (for the same hexagonal mosaic, $\beta \sim 1/1.59$). Assuming the same strategy is used for both arrays, the numerical values of α and β do not matter for determining the ratio of integration time which is needed. The total time used in array 1 is then

$$t_{tot,1} = N_{ptg,1} \times \tau_1 \tag{10}$$

and the total weight of the mosaic dataset is

$$\omega_{tot,1} = N_{ptg,1} \sum_k \omega_{1,k} \tag{11}$$

assuming, as we do throughout, that the weights for each mosaic pointing are the same. In practice this may not be the case since weather conditions can vary through the course of a project. Our analysis also assumes in effect that each pointing is a "snapshot" observation of arbitrary depth, a single long integration of duration τ. Although the effects of the projected baselines changing over a track are of great practical importance they should be a minor effect in this sensitivity calculation. In the context of this approximation,

Symbol	Explanation
A_1	geometric area, array 1
D_1	dish diameter, array 1
$\sigma_{beam,1}$	Gaussian σ of primary beam, array 1
$\eta_{a,1,i}$	aperture efficiency, antenna i, array 1
$T_{sys,1,i}$	system temperature, atenna i, array 1
$\eta_{Q,1}$	array 1 sampling or quantization efficiency
$t_{tot,1}$	total mosaic integration time, array 1
$N_{bas,1}$	# complex baselines measured per ptg, array 1
τ_1	integration time per pointing, array 1
$N_{ptg,1}$	Number of pointings in mosaic, array 1
$\omega_{1,k}$	per baseline noise variance weight, array 1, baseline k
$\omega_{tot,1}$	total mosaic dataset weight, array 1
$\mathscr{T}_{1,k}$	taper weight, array 1, baseline k
$\mathscr{W}_{1,k}$	uv weight, array 1, baseline k
$\Delta I_{m,1}$	array 1 single-field sensitivity (field center)
$\Delta I_{tot,1}$	array 1 mosaicked sensitivity
$\Delta S_{1,k}$	noise in baseline k, array 1
Ω	solid angle of area to be mosaicked

Table 2: Notation.

the number of complex visibilities associated with one array's observation of a single pointing in the mosaic is equal to the number of baselines.

In order to combine observations from the two arrays, we require the total mosaic image noise from each, on similar or identical spatial scales, to be comparable, such that the noise from neither array is dominant. Assuming identical mosaicking strategies, this in turn implies

$$\sum_k \omega_{1,k} = \sum_k \omega_{2,k} \tag{12}$$

i.e. the *single pointing* total weights must be equal for the two arrays. This summation of weights holds only over some subset of baselines which sample common spatial scales between the two interferometers. For the simple case (identical measurements within an array; no taper; etc) this implies

$$\frac{\tau_2}{\tau_1} = \left(\frac{T_2 A_1 \eta_{a,1}}{T_1 A_2 \eta_{a,2}}\right)^2 \frac{N_{bas,1}}{N_{bas,2}} \tag{13}$$

Note that, again, the N_{bas} factors refer to the number of baselines (visibilities) in the overlap region of uv space under consideration here. If all of the system temperatures and aperture efficiencies are equal, then the ratio (one array to the other) of the integration time per pointing in the mosaic becomes:

$$\boxed{\frac{\tau_2}{\tau_1} = \left(\frac{D_1}{D_2}\right)^4 \frac{N_{bas,1}}{N_{bas,2}}} \tag{14}$$

The assumptions made here of equal system temperatures and aperture efficiencies are not unreasonable since ALMA specifications call for the system temperatures and aperture efficiencies of the 7m and 12m antennas to be similar to within a nominal tolerance. Multiplying by the number of pointings in the mosaic, the ratio of *total* integration time is

$$\frac{t_{tot,2}}{t_{tot,1}} = \frac{N_{ptg,2}\,\tau_2}{N_{ptg,1}\,\tau_1} \tag{15}$$

If we assume identical mosaicking strategies, identical areas covered Ω, and Gaussian beams with widths proportional to the dish diameter, then

$$\frac{N_{ptg,2}}{N_{ptg,1}} = \left(\frac{D_2}{D_1}\right)^2 \tag{16}$$

and it follows that the ratio of *total* integration times for two arrays' coverage of the mosaic is:

$$\boxed{\frac{t_{tot,2}}{t_{tot,1}} = \left(\frac{D_1}{D_2}\right)^2 \frac{N_{bas,1}}{N_{bas,2}}} \tag{17}$$

Note that the D^4 factors in Eq. 14 for the per-pointing integration time ratio have become D^2 factors in this equation for the total mosaic integration time ratio.

Referring back to Equations 11 and 12, we find that the ratio of the *total weights* for the datasets will be

$$\frac{\omega_{tot,2}}{\omega_{tot,1}} = \frac{N_{ptg,2}}{N_{ptg,1}}. \tag{18}$$

Although the per-pointing weights for array 1 and array 2 must be equal to achieve equal noise in the map, the total weight for the array with the larger aperture will be higher since it will have done more pointings to cover the same sky area. Again, for this equality to hold assuming the matching criteria we posit here, these weight sums should be done only over the "matched" baseline ranges.

2.2 Application to ALMA

If the number of visibilities is equal, then the ratio of total time required for the 7m-array and the 12m-array

$$\frac{t_{tot,7m-array}}{t_{tot,12m-array}} = \left(\frac{D_{12m}}{D_{7m}}\right)^2 = (12/7)^2 \sim 2.93 \tag{19}$$

In fact the number of baselines measuring similar angular scales will rarely be equal for 12m and 7m arrays—Figure 1 shows the distribution of baseline lengths for ALMA Cycle 1, configurations 1 through 4[1], and the 7m-array. We are faced with the task of counting how many baselines are comparable for each 12m-array configuration.

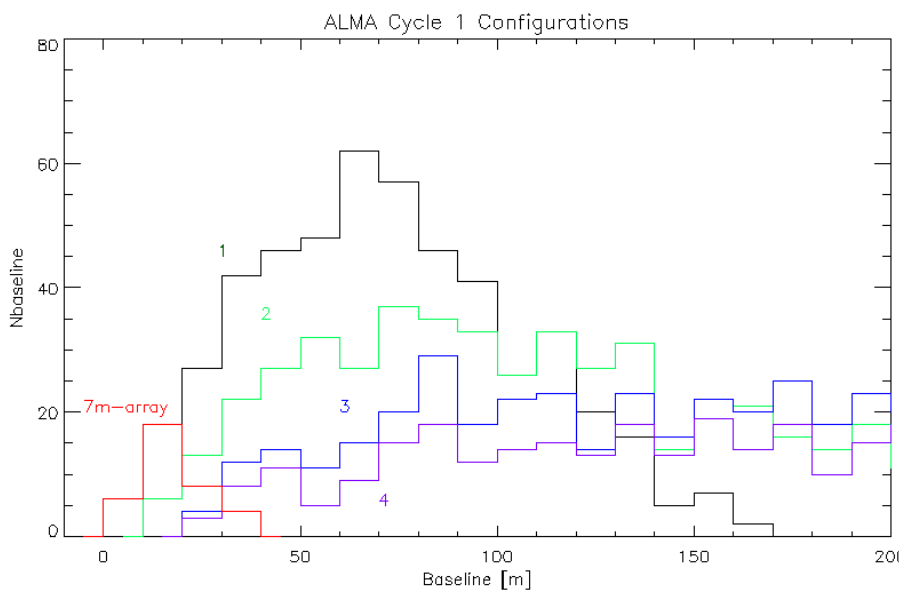

Figure 1: Baseline lengths for the ALMA Cycle 1 12m-array configurations 1 through 4 and the 7m-array.

In order to combine 12m-array and 7m-array data, one would ideally like equal noise in maps made with comparable uv ranges. Precisely how to count these "comparable" baselines is somewhat a matter of taste – similar to asking: what is the exactly optimal array configuration? Guiding principles are to avoid holes in the uv plane and to keep the (single field) weight distribution continuous. Two reasonable approaches

[1] 7m-array data are not acquired with Cycle 1 configurations 5 or 6.

to the counting problem would be: i) count the number of baselines in the range of uv space where both arrays have coverage; ii) to count the number of baselines in the larger array from the minimum baseline up to that plus Δq, where Δq is the total range in uv radius $q = \sqrt{u^2 + v^2}$ covered by the smaller array. We have applied these two criteria to calculate the ratio of 7m-array time to 12m-array time for each legal cycle 1 ALMA configuration using Eq. 17, with results shown in Table 3.

ALMA	Overlap Criterion			Δq Criterion		
config.	$N_{12m\,baselines}$	$N_{7m\,baselines}$	t_{7m}/t_{12m}	$N_{12m\,baselines}$	$N_{7m\,baselines}$	t_{7m}/t_{12m}
1	52	22	6.92	77	36	6.27
2	23	21	3.21	35	36	2.85
3	6	11	1.60	22	36	1.79
4	4	11	1.06	16	36	1.30

Table 3: Ratio of 7m-array total observing time to 12m-array total observing time required for the four different Cycle 1 configurations which include the 7m antennas, computed according to the two alternate criteria discussed in the text. These ratios use Eq. 17, which assumes equal system temperatures, aperture efficiencies, and identical areas covered by each interferometer. The Cycle 1 ALMA configurations used have 32× 12m antennas and 9× 7m antennas.

With 36× 12-m antennas and 12× 7-m antennas the Cycle 2 integration times for the 7-m array— relative to the 12-m integration times for Cycle 2, and assuming similar baseline distributions as for Cycle 1— will be a factor of ~ 0.7 lower. This should be more precisely calculated for each Cycle 2 configuration once the projected antenna locations are reasonably known.

2.3 Practical Aspects

Several practical considerations present themselves:

1. Since the telescope primary beams can be large compared to the size of the fields being covered, and they differ for 12m and 7m, we must take into account the fact that the two arrays may cover slightly different areas Ω. This turns out not to matter much, because:

2. What is of practical importance in planning an actual observation is the integration time required per pointing. The integration time required *per 7m pointing* can be calculated by multiplying the 12m integration time *per pointing* by the *total time* ratios in Table 3, and multiplying by an extra factor of $(D_{12m}/D_{7m})^2 = 2.93$ (c.f. eq. 14 vs . eq. 17). This does not depend on the areas covered being identical, and is equivalent to using Eq. 14 to determine the per pointing 7m-array integration time.

3. The number of pointings required for each array will have been predetermined by the PI's choice of mosaicking strategy and field size. This number (for each array) times the per field integration time calculated as above, determines the total observing time.

4. If tapering or weighting of the final map has been specified, the time requirements can be calculated from the requirement that the single-pointing image noises be equal, and the full forms given (i.e. Eqs. 3 and 6).

2.4 Trial Application: ALMA Science Verification M100 Observation

ALMA Science Verification data were obtained on M100, comprising 12m interferometric observations, 7m interferometric observations, and 12m total power observations. Basic properties of the interferometric data are:

- The 12m mosaic consists of 48 pointings, with 101.8 minutes total integration time, or about 2 minutes per pointing.

- The number of 12m antennas varied from 12 to 13– assume on average 12.5 antennas.

- The 7m mosaic consisted of 24 pointings, with 207.65 minutes of total integration time, or about 8.7 minutes per pointing.

- The number of 7m antennas varied between 7 and 8– assume on average 7.5 antennas.

It has been established that once the CASA weights are correctly[2] set, the combination of the 7m-array and 12m-array data give a qualitatively good image.

As a sanity check, we can see what we predict the 7m-array integration time *should* be to get a good result. The critical figure of merit is weight (therefore to zeroth order, integration time) per mosaic pointing for each array. From eq. 14

$$\frac{\tau_{7m}}{\tau_{12m}} = \left(\frac{12}{7}\right)^4 \times \frac{N_{12m\,baselines}}{N_{7m\,baselines}} \tag{20}$$

The 12m-array data for M100 were acquired in a compact configuration with a baseline distribution similar to Cycle 1 Configuration 1. However, Cycle 1 targets 32 12m antennas instead of the 12 or 13 present in the SV data. Similarly Cycle 1 targets 9 7m antennas, vs the 7 or 8 present in the SV data. We adopt the Δq criterion and refer to the baseline overlap counts in Table 3, correcting them by N_{ant}^2, a reasonable approximation since the ALMA *uv* coverage is fairly uniform and centrally condensed. This gives an estimate of the ratio of per mosaic pointing integration times:

$$\frac{\tau_{7m}}{\tau_{12m}} = \left(\frac{12}{7}\right)^4 \times \frac{77 \times (12.5/32)^2}{36 \times (7.5/9)^2} \sim 4.1 \tag{21}$$

So indeed the factor of four in integration time per pointing between 7m and 12m arrays is about right for these observations.

3 Total Power Single Dish Data

Suppose a single dish maps the same area Ω with an identical "mosaicking" strategy as a synthesis array, and we wish to combine these data together. The sensitivity at the center of each pointing of the single dish map is

$$\Delta I_{sd} = \frac{2k_B}{\eta_{Q,sd} A_{sd} \eta_{sd}} \frac{T_{sys,sd}}{\sqrt{\Delta\nu \tau_{sd}}} \tag{22}$$

for an integration of duration τ_{sd} on each pointing in the map. Suppose further that we have N_{sd} identical "single dishes" observing this area in parallel (2 for ALMA cycle 2, 4 for final ALMA). As before, the details of the mosaicking strategy do not matter if they are consistent. We require the Jy/beam noise for this single dish map to be equal to the noise in the synthesis map with which it is to be combined, when the synthesis map is made with a relevant selection of baselines N_{bas}. Equating Eq. 22 with Eq. 6 and assuming equal system temperatures etc., we derive the required ratio of single dish to interferometer integration time per pointing of

$$\frac{\tau_{sd}}{\tau_{interf.}} = \left(\frac{D_{interf.}}{D_{sd}}\right)^4 \frac{2N_{bas}}{N_{sd}} \tag{23}$$

The ratio of total integration times, by the same argument as in § 2.1 is

$$\frac{t_{tot,sd}}{t_{tot,interf.}} = \left(\frac{D_{interf.}}{D_{sd}}\right)^2 \frac{2N_{bas}}{N_{sd}} \tag{24}$$

We are again faced with the somewhat subjective choice of which baselines are relevant for comparison with the noise in the single dish map, and propose three criteria: i) select those baselines which fall within the range of spatial frequencies measured by the single dish; ii) as before, select the shortest baselines of the synthesis array that give the same range Δq as the range of spatial frequencies measured by the single dish; or iii) select baselines which have any spatial frequency overlap at all between interferometer and single dish, considering the full range of spatial frequencies measured by each. Criterion (iii) is similar to criterion (i), but also allows for the range in spatial frequencies due to the interferometer's primary beam. These criteria are illustrated for the Cycle 1 7m-array configuration in Figure 2. The ratios of single dish to 7m-array integration times for Cycle 1 and for the full ALMA Compact Array, given these criteria, are shown in Table 4.

[2] By default, the weights in CASA 4.0 only account for T_{sys}– they are not true data variance weights, accounting for antenna size, channel width, integration time, *etc.*

Config.	Criterion	N_{bas}	N_{sd}	τ_{tp}/τ_{7m} (per ptg.)	$t_{tot,tp}/t_{tot,7m}$ (total)
Cycle 1 (9x7m, 2x12m TP)	i: $q_{7m} < 12\,\mathrm{m}$	6	2	0.69	2.04
	ii: $\Delta q_{7m} < 12\,\mathrm{m}$	24	2	2.78	8.17
	iii: $q_{7m} < 19\,\mathrm{m}$	22	2	2.55	7.49
Full ACA (12x7m, 4x12m TP)	i: $q_{7m} < 12\,\mathrm{m}$	13	4	0.75	2.21
	ii: $\Delta q_{7m} < 12\,\mathrm{m}$	40	4	2.32	6.80
	iii: $q_{7m} < 19\,\mathrm{m}$	35	4	2.03	5.95

Table 4: Estimated ratio of required 12m total power array to 7m interferometeric array integration times for Cycle 1 and for the full ACA.

We have not considered position or frequency switching. If the single dishes are switching, then in the simplest instance this will increase the total power array integration time— the total time, including both ON and OFF/REFERENCE phases— by a factor of four.

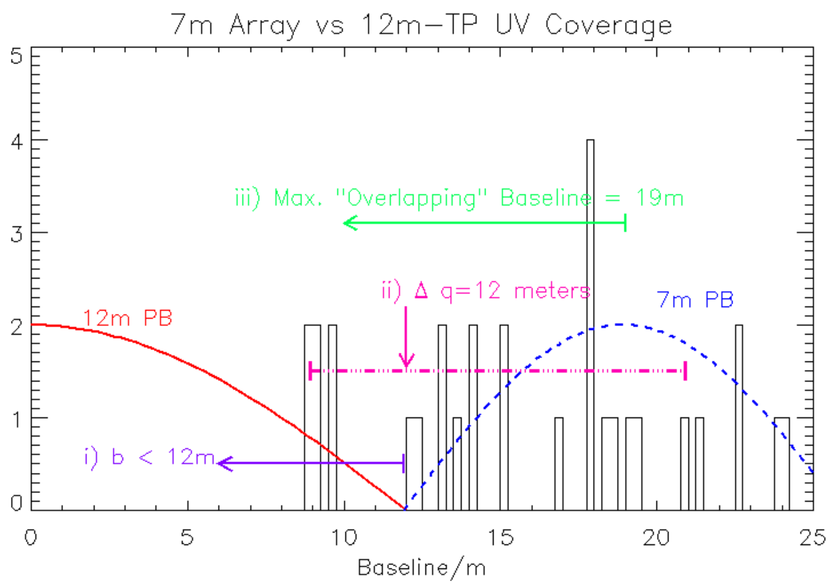

Figure 2: Cycle 1 7m-array baseline distribution. The red solid line schematically illustrates the range of spatial frequencies measured, in principle, by a 12m in total power mode— the "primary beam" in Fourier space, or more precisely, the autocorrelation of the aperture illumination. The blue dashed line illustrates the range of spatial frequencies measured in a single pointing of the 7m-array for the longest 7m-array baseline with any spatial frequency overlap with the 12m single dish. There are 9 elements in the 7m configuration used. Also illustrated are the three criteria, discussed in the text, for selecting synthesis baselines which "match" the total power map.

4 Previous Results and Alternative Approaches

For combining two synthesis arrays (§ 2), we have stipulated equal noise (in mJy/bm) on matched *uv* scales. The scales are intrinsically not precisely matchable when combining single dish data with synthesis data, so we have proposed to match noise on *similar* scales by several criteria (§ 3). An alternative approach in this case could be to match the signal-to-noise ratio (SNR). For extended sources, this would typically result in lower required single dish integration times. In practice, however, matching the SNR between a single dish and a synthesis array is a complicated parameter space— probably too complicated for a user tool— and requires detailed knowledge of the structure of the source. In most cases this information will not exist prior to collection of the data in question.

The question of 12m-array, 7m-array, and total power integration times for ALMA has been investigated previously by Y. Kurono ("Observing time ratios and imaging performance with the ACA 7-m array", February 28, 2012). The criteria in this analysis were image fidelity and total flux recovery for the specific case of a Gaussian source model with a FWHM roughly one third the 12m primary beam FWHM. For this case, reasonable results are obtained for $(12m : 7m : tp)$ total time ratios of $(1 : 2 : 3)$. Issues of signal to noise and relative noise were not specifically considered, although the last figure in this document ("Weight distribution in the u-v domain") suggests similar results would be obtained as we have found here.

Kurono et al. (2009) and Koda et al. (2011) present an alternative "pseudo-visibility" formalism for calculating the required relative weights for combining single dish and interferometric array data. We have not quantitatively compared the results of our calculation with theirs, but expect that more single dish integration time would be required since the noise in the single-dish pseudo-visilbities progressively increases away from $\sqrt{u^2 + v^2} = 0$. A quantitative application of this formalism to the observing cases considered here would be useful.

Rodríguez-Fernández, Pety, & Guth (2008) have also investigated the question of the required single-dish and interferometer integration times in the context of the pseudo-visibility approach. They generally find very long single-dish integration times are needed. We note that the criterion they adopt for matching the single dish and interferometer data— requiring an approximately smooth, Gaussian distribution of weights— is similar to our approach of requiring equal noise on comparable angular scales. Due to the long single dish integration times inferred, Rodríguez-Fernández et al. explore allowing the weights (i.e. noises) to be matched not only by the intrinsic data noise (set in part by the relative integration times), but also by manually "up-weighting" the single dish data after the fact. They add a cautionary note that better combined images are obtained with progressively deeper single dish datasets (less "up-weighting"). Here again, a detailed numerical application of their results to the observing cases we consider here would be useful.

A Revision History

- v.28jun2013 - first complete draft
- v.08jul2013 - fix error in single dish uv coverage definition, slight revision to single dish time reqmts; add third total power matching criterion
- v.08jul2013b - Add brief comparison to Kurono 2012 analysis.
- v.25jul2013 - incorporate referee comments, add to comparison with other results.

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