



ngVLA Antenna Memo # 9

## Practical Limits to Axis Offsets

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### Abstract

We provide a rough constraint on the allowable axis offsets accounting for thermal deformations of the antenna and pointing offsets, while supporting the higher-level delay calibration requirements. We conclude that, given modern calibration strategies, the most critical offsets are the Elevation-Boresight and Azimuth-Boresight offsets. Azimuth-Elevation offsets are also important as documented by others, but the path length errors due to offsets to the optical boresight dominate during a typical astronomical gain calibration time period.

## I Introduction

The ngVLA project is preparing to advance an antenna concept towards a preliminary design. The offset Gregorian antenna optics introduce practical constraints on the mount structural design. In an effort to have a balanced elevation structure, and avoid interference between the backup structure and the pedestal, all concepts developed to date include offsets between the Azimuth and Elevation and/or Optical Boresight ( $Z_{MR}$ ) axes, as defined in the ngVLA Antenna Coordinate Systems [3]. The allowable offsets between the various axes have not been constrained in this phase of the design, but we aim to assess the impact of these offsets and provide a constraint before selecting an antenna design to advance towards a preliminary design and prototype.

In parallel to the antenna development, a preliminary set of Calibration Requirements [1] have been developed. A relevant requirement is CAL0313, which limits the post-astronomical calibration delay drift residual from the antenna structure to 60 fsec, for an on-axis source, when bracketed by calibration observations at a 5 minute cadence.

We explore constraints to key dimensions of the antenna in this memo, aiming to satisfy requirement CAL0313, while applying lessons from the VLA and ALMA.

## 2 Delay Changes in a Common Alt-Az Frame

Wade [4] explored the effects of axis non-intersection in the VLA antennas when using a common Alt-Az frame (i.e., all pointed in the same direction, in Alt-Az coordinates). The non-intersection offsets form a three-dimensional vector from  $P$  to  $P'$ , with constituent components  $a$ ,  $b$ , and  $c$ . (Figure 1)

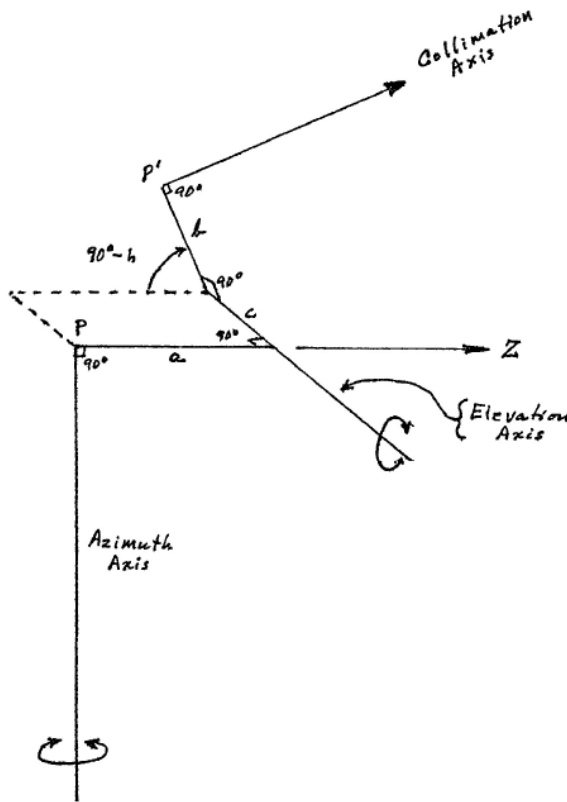


Figure 1 – Axis non-intersection offsets as defined by Wade [4]. The collimation axis is the optical boresight (ZMR axis) in the ngVLA coordinate system.  $h=0$  at the local horizon.

Wade has three key findings relevant to the ngVLA antenna design:

- 1) The magnitude of  $a$ ,  $b$ , and  $c$  is not important, but rather differences between the values on a baseline antenna pair. Differences are denoted:  $\alpha = a_2 - a_1$ ;  $\beta = b_2 - b_1$ ;  $\gamma = c_2 - c_1$ .
- 2) The difference in the  $a$  term dominates the delay error due to axis non-intersection, with a relationships of  $\alpha \cos h$ , where  $h$  is the elevation angle.
- 3) Offsets  $b$  and  $c$  in Figure 1 only contribute to delay error for off-axis sources within the primary beam.

At present we are only concerned with on-axis errors, so we will not constrain  $b$  or  $c$  based on this analysis (the off-axis response should be analyzed in a separate future memo).

In order to avoid calibration ambiguities, Wade constrains the phase effect due to the offset  $a$  term to not exceed 90 degrees. In the worst case elevation, this limits  $\alpha$  to  $\lambda/4$ , or contributions from each antenna to  $\lambda/8$ .

Wade focused on absolute terms since (1) the VLA axes nominally intersected and (2) the calibration approach aimed to ignore these terms. For our purposes, we can treat them as a measurement error or change across antenna pairs. At the ngVLA's upper operating limit of 116 GHz (2.6mm), the most stringent interpretation would be to constrain changes in  $\alpha$  to 0.65mm between delay model parameter updates.

Focusing on thermal effects, each antenna is in a different environment that may have cooled or heated since  $a$  was last measured. Common-mode diurnal and seasonal effects would not contribute to  $\alpha$ , so it is only the variation across sites that matters (e.g., a relatively cold day at one site and a warm day at another). Assuming a steel structure (CTE =  $12E-6/^\circ\text{C}$ ), and  $20^\circ\text{C}$  thermal deviation across sites, this would constrain  $a$  to approximately 2.7m. A more extended analysis might consider geometric effects from gravitational sag, bearing run out, and other effects, but thermal effects are anticipated to dominate.

### 3 Delay Changes with Pointing Errors

Using the geometry as defined in Figure 1, we will extend this analysis to consider small changes in the elevation angle ( $h$ ) and azimuth ( $Z$ ) for a single antenna. Terms  $a$  and  $c$  introduce path length ( $L$ ) changes with changes in azimuth, while term  $b$  introduces a change with elevation. The relationships are:

$$\Delta L_a = [a - a * \cos(\Delta Z)] * \cos(h) \tag{1}$$

$$\Delta L_b = b * \tan(\Delta h) \tag{2}$$

$$\Delta L_c = c * \tan(\Delta Z) * \cos(h) \tag{3}$$

The path length changes associated with an azimuth error contribute most at lower elevations due to the cosine elevation relationship. The  $a$  term being parallel to the pointing vector means its influence is diminished, while the normal orientation of  $b$  and  $c$  to the pointing vector amplifies their effects on the path length. The expected values of  $c$  are small. It is always zero by design, and real values are due to manufacturing and assembly tolerances. The  $b$  term can be large, varying from 0.5m to 9m in the design concepts explored to date. It is therefore the  $b$  term that produces the most significant path length changes due to pointing errors.

### 4 Example Case

In order to explore the feasibility of the path length stability required in CAL0313, and consider practical design constraints, we consider the impacts of thermal changes and pointing errors in a simplified path length error budget for a representative antenna design.

CAL0313 is specified as an rms drift residual over 5 minutes. Drift is used to denote changes on a 1 sec timescale or longer, while it is a residual since we assume linear drift subtraction between two astronomical calibrations.

We will assume the precision operating conditions defined in the Antenna Technical Requirements [2], which result in temperature changes of no more than 3.6°C/hr. The specified referenced pointing error is 3 arcsec rms in this environment.

We will assume uniform positive temperature change for 2.5 minutes, followed by uniform negative temperature change for 2.5 minutes, with no net change in temperature over the 5 minute period, negating the effect of astronomical calibration. This is equivalent to a 0.15°C rise and fall in temperature. The corresponding rms path length change over 5 minutes is  $\Delta L_{max}/\sqrt{3}$ .

The resulting path length changes from a representative antenna design can be seen in

Table 1 (Appendix). The effects of temperature change are small, accounting for less than 30% of the available path length error budget with an all steel structure. The thermal effects are most pronounced at low elevation angles, as would be expected given the conclusions of Section 2 and the importance of  $\alpha \cos h$ .

The effect of pointing errors dominates the budget, especially the impact of the  $b$  term. The influence of pointing errors is approximately 3-15x larger than thermal changes over the short 5-min calibration cycle, depending on choices in geometry. Longer calibration cycles would lead to a convergence of the impact of the two factors.

The impact of the Elevation-Boresight offset ( $b$ ) is so large that an offset of 1.25m would consume the entire delay drift error budget. A key finding from this exercise is that the combined length of  $b$  and  $c$  should not exceed 1.25m, and would ideally be significantly smaller.

## 5 Conclusions

We have evaluated the first-order influence of axis non-intersection errors on the on-axis delay error. We find that pointing errors can contribute significantly to the path length error, and constrain the offsets from the Elevation Axis and Azimuth Axis to the Optical Boresight to no more than 1.25m combined. Reducing these offsets, and estimating the path length stability more rigorously, should be a priority in the preliminary design phase. The offset from the Elevation Axis to the Azimuth Axis is the third most important term, and is unlikely to present calibration issues if constrained to 2.7m or less. Given these limits, we find that the path length delay error requirement (CAL0313) is likely feasible, since the implied offsets appear realizable for a practical, balanced antenna design.

We have only considered the impact of axis offsets on the delay stability of the antenna for a source that is on-axis. Further consideration of the off-axis case is necessary to understand the impact on wide-field imaging cases. Consideration should also be given to the special use cases associated with astrometry and geodesy.

## 6 References

[1] Hales, C. “ngVLA Calibration Requirements” ngVLA Doc # 020.22.00.00.00-0001-REQ, rev B, May 2020.

[2] Selina, R., Dunbar, D. “ngVLA Antenna Technical Requirements” ngVLA Doc #020.25.00.00.00-0001-REQ, rev A, July 2019.

[3] Selina, R., Sturgis, S. “ngVLA Antenna Coordinate Systems” ngVLA Doc #020.10.30.00.00-0001-SPE, rev A, July 2020.

[4] Wade, C. “Tolerances for the Intersection of the Azimuth, Elevation, and Collimation Axes of the VLA Antennas” VLA Test Memorandum #104, April, 1974.

Table 1 – Simplified Path Length Error Budget for a practical antenna design. The most significant contributor is highlighted.

Dimension	Segment Length (mm)	CTE	$\Delta T_{\max}$	$\Delta L_{\max}$	$\Delta$ Path Length (Max) @ $h =$		
					12	45	88
<b>Impact of Temperature Change</b>							
Pedestal Gnd - El. Axis	11100	1.20E-05	0.15	2.00E-02	-4.15E-03	-1.41E-02	-2.00E-02
Az. to El. axis ( $a$ )	2100	1.20E-05	0.15	3.78E-03	-3.70E-03	-2.67E-03	-1.32E-04
Sub to Main Reflector	9000	1.20E-05	0.15	1.62E-02	1.62E-02	1.62E-02	1.62E-02
El. Axis to Main Reflector	3000	1.20E-05	0.15	5.40E-03	-5.40E-03	-5.40E-03	-5.40E-03
<b>Total <math>\Delta</math> Path (max)</b>					<b>2.95E-03</b>	<b>-6.00E-03</b>	<b>-9.30E-03</b>
<b><math>\Delta</math> Path, rms (5 min)</b>					<b>1.70E-03</b>	<b>3.46E-03</b>	<b>5.37E-03</b>
<b>Impact of Pointing Error</b>							
Az. to El. Axis ( $a$ , Az. Error, rms)	2100				2.17E-07	1.57E-07	7.75E-09
El. to $Z_{MR}$ Axis ( $b$ , El. Error, rms)	1000				1.45E-02	1.45E-02	1.45E-02
Az. to $Z_{MR}$ Axis ( $c$ , Az. Error, rms)	50				7.11E-04	5.14E-04	2.54E-05
<b><math>\Delta</math> Path, rms</b>					<b>1.46E-02</b>	<b>1.46E-02</b>	<b>1.45E-02</b>
<b>Budget</b>							
Temperature Change					1.70E-03	3.46E-03	5.37E-03
Pointing Error Influence					1.46E-02	1.46E-02	1.45E-02
Wind					5.00E-03	5.00E-03	5.00E-03
Other					5.00E-03	5.00E-03	5.00E-03
<b>Estimated Total Path Error, rms</b>					<b>1.63E-02</b>	<b>1.65E-02</b>	<b>1.70E-02</b>
<b>Estimated Total Delay Drift, rms</b>					<b>5.43E-14</b>	<b>5.52E-14</b>	<b>5.68E-14</b>
Requirement: Max Total Path Error, rms					1.80E-02	1.80E-02	1.80E-02
Requirement: Max Toal Delay Drift, rms					6.00E-14	6.00E-14	6.00E-14
<b>% Under/Over Requirement</b>					<b>-10%</b>	<b>-8%</b>	<b>-5%</b>