ngVLA Electronics Memo # 9

Design Considerations for Band 1 Receiver of ngVLA

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Summary

Band 1 of ngVLA will incorporate direct-RF-sampling receiver module. Analysis of receiver's performance, in the presence of multitone interference (RFI) within Band 1 frequency limits, and the design methodology for retaining dynamic range of an A/D Converter is outlined here. Crowded spectrum of RF interference, covering parts of Band 1 frequency range, will resemble a "noise-like loading" on the A/D Converter of the Digitizer. Impact of such "noise loading" on dynamic range of the receiver, and RF system level design methodology to prevent any further dynamic range degradation (due to RF amplification chain's imperfection) is described.

1. A/D Converter – *RFI "loading"*

Fig.1 depicts a spectrum of multitone RF interference (**RFI** – shown in purple, on the left side of Fig.1) at the input of Analog to Digital Converter (ADC), as compared to scenario of a "noise load" on the ADC (as shown in red – on the right side of Fig.1). Case of unequally-spaced and strong RFI tones is shown, in order to illustrate the possibility of filling entirely all the gaps between RFI tones by the intermodulation products (**IMD** – shown in light blue), and causing a noticeable increase of ADC noise floor above level of its quantization noise (shown in blue). Vertical scale represents power at the input to ADC, and horizontal scale represents frequency. As the number of RFI tones increases (while combined power of all tones is kept unchanged), spectrum of RFI will start resembling the noise (as shown in red – on the left side of Fig. 1).

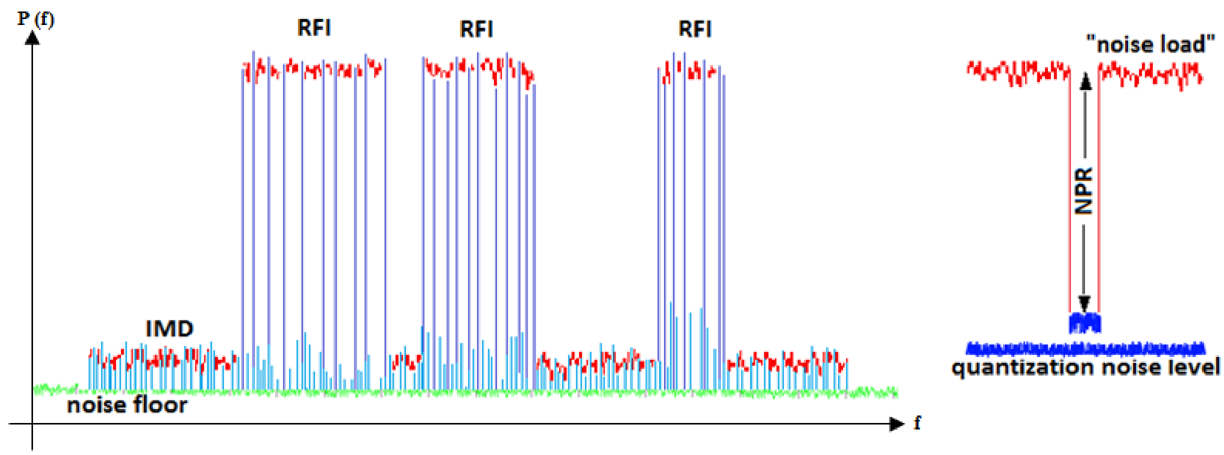

Fig. 1 Unequally-spaced multitone RF interference frequency spectrum at the input to A/D Converter (**left**) v/s "noise load" (of Gaussian distribution) on the A/D Converter (**right**).

Cluster of RFI tightly-spaced, and mostly uncorrelated, interference tones at ADC input will resemble noise-like "loading signal" with Gaussian-like distribution (PDF) of time waveform. Noise Power Ratio (**NPR**) is a convenient measure of ADC noise-free dynamic range for the case of multitone excitation [1], which can be verified in the lab (either by analog test method - using noise source followed by a notch filter, or a digital method - using an arbitrary waveform generator, to synthesize a baseband I and Q signals, and a vector signal generator to synthesize notched multitone RF signal spectrum).

Graphical representation of relationship between noise "loading factor" (shown on horizontal scale, and defined as a ratio of rms voltage of a random input signal to one half of the full-scale differential input voltage range of ADC; i.e. basically describing the back-off distance from the clipping level, parameter also closely related to a crest factor of the noise signal) v/s the NPR value (shown on vertical scale) is displayed in Fig. 2 (graph, reproduced from reference [1], represents the case of 8-bit A/D Converter).

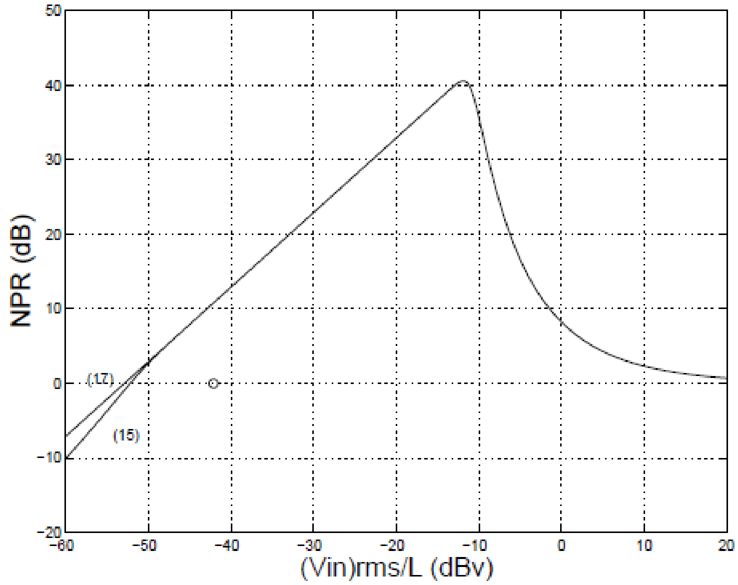

Fig. 2 NPR for Gaussian-PDF "noise loading" - 8-bit ADC (**Irons et al., 2000 [1]**)

At low values of "loading factor" only the quantization noise degrades the ADC noise floor, and NPR rises dB-per-dB to a **maximum NPR** value (which defines an optimal A/D noise-free dynamic range of approximately **41dB** for the case of a 8**-bit ADC**), while **rms** level of "noise loading" is reaching approx. **12dB** below the clipping threshold (FS) of A/D Converter. At such level **quantization efficiency** of **0.999912** is achieved [2]. System requirements for ngVLA allow for a **degradation** of quantization efficiency down **to 0.96** [3]. Therefore, available **"linear" range** (i.e. dB-per-dB) for the 8-bit ADC is approximately from level of **-39dBFS** (which corresponds to NPR of 13.8dB) **to -12dBFS**, giving **27dB** for the signal range/ gain flatness. Allocating 6dB for gain flatness, and 26dB for RFI headroom [3], would require a **32dB range** (which can be met - **correction of** approximately **5dB** is applied **to quantization noise** since **RFI does not occupy the entire Nyquist band**, as it will be derived in this study).

Increase of the "noise loading" past -12dBFS will result in an abrupt change to hard limiting (clipping) of input signal, causing a "clipping noise" to appear (in addition to, already present, quantization noise). It will also rapidly increase the ADC noise floor. It is possible to operate A /D Converter up to approximately **-4dBFS** in this " non-linear" mode while still achieving an acceptable quantization efficiency, thus extending the signal range by an **additional 8dB**.

*) Operation in this "non-linear" regime will not guarantee the correctness of representation of RFI peak values at the output of ADC, thus may influence the ability to remove RFI completely. Under the "load" of multitone RFI signal, ADC will also produce own discrete intermodulation products, which are device specific and must be verified (when A/D Converter will become available for testing) – it is now assumed that such spurious products fall below the ADC noise.

Taggart, et al. [5] arrived at similar results, while analyzing an equally-spaced multitone signal at the input to the ADC (equally-spaced nine tones, each 8-PSK modulated) instead of Gaussian-distribution random noise. Such multitone excitation is a typical RFI scenario within Band 1 of ngVLA due to a fleet of commercial L-band satellites using some portions of Band 1 frequencies. Analysis from the reference [5] further proves the equivalence between multitone and Gaussian-distribution noise excitations of ADC, and also reveals an A/D Converter behavior as a function of its resolution. Results, from reference [5], are reproduced in Fig. 3.

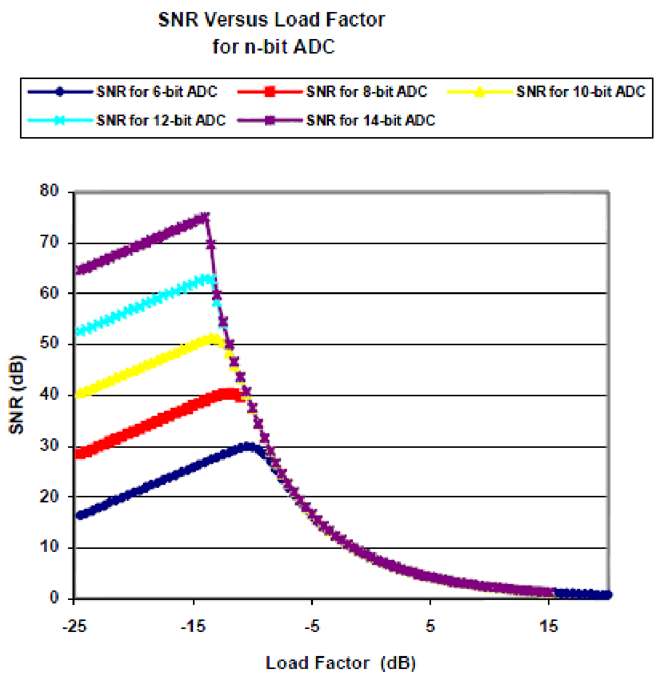

Fig. 3 SNR versus Load Factor for 6-bit to 14-bit A/D Converters (**Taggart et al., 2007 [5]**).

ADC having larger number of bits per sample would require lower value of "load factor", but will achieve larger optimal Signal to Noise Ratio (SNR – as displayed on the vertical scale). Despite different naming used: NPR in the reference [1], and SNR (as used in reference [5]); both refer to a noise-free dynamic range. Reference [5] suggests that the optimal SNR value (in dB) could be estimated according to the following formula:

$$\boldsymbol{SNR_{[dB]} \approx -4 + (5.65b)} \quad (1)$$

valid for a random (multitone) input to the ADC, and for the case of $6 < \boldsymbol{b} < 14$ (where $\boldsymbol{b}$ is the ADC resolution = number of bits).

*) $\boldsymbol{SNR} = \frac{3}{2}\left(2^{2b}\right)$ or $\boldsymbol{SNR_{[dB]} \approx 1.76 + (6.02b)}$ is valid for all values of $\boldsymbol{b}$ (for a sinewave input).

Multiple techniques have been proposed and implemented to combat the increasing number of RFI interfering signals, both of satellite and terrestrial origin. While these techniques intend to flag and excise the particular RFI signal frequencies, using a post-digitizer processing; the present study concerns with an intermodulation products and distortions (**IMD**) resulting from RFI, being produced inside the RF chain, and the distortions in the ADC under "RFI loading". Nonlinearities in the RF amplification chain, preceding the input to the ADC, will in presence of multitone RFI, result in multitude of intermodulation products. Such spectrum "re-growth", combined with noise contributions from the RF amplifiers and the A/D Converter (quantization noise, and eventually also clipping noise - for the case of extreme RFI "load" when limiting occurs), will effectively degrade achievable sensitivity of the direct-RF-sampling receiver.

Power level of the ADC quantization noise can be estimated as below (assuming no-dead-zone and uniform-step-size quantizer) [6]:

$$\boldsymbol{N_{q\,[W]}} = \frac{\boldsymbol{\Delta^2}}{\mathbf{12}\boldsymbol{R}} = \frac{\left(\frac{\boldsymbol{V_{pp}}}{\mathbf{2}^{\boldsymbol{b}}}\right)^{\mathbf{2}}}{\mathbf{12}\boldsymbol{R}} \qquad (2)$$

where: $\boldsymbol{N_q}$ is a quantization noise mean power (in Watts) integrated over Nyquist band,
$\boldsymbol{V_{pp}}$ is a full-scale differential signal peak-to-peak voltage (at the ADC input),
$\boldsymbol{R}$ is an equivalent resistance value of parallel load impedance (at the ADC input),
$\boldsymbol{\Delta}$ is a quantization step voltage.

Assuming $\boldsymbol{V_{pp}} = \mathbf{0.8V}$, $\boldsymbol{R} = \mathbf{100\ Ohm}$, and **8-bit A/D**,
quantization noise power level will be equal to:

$$\boldsymbol{N_q} \approx \mathbf{8.1nW} \ \textbf{(approx. -51dBm)} \qquad (3)$$

"Clipping noise" threshold will have input power level (resistive load of 100 Ohm) equal to:

$$\boldsymbol{P_{max}} = \mathbf{1.6mW} \ \textbf{(approx. +2dBm).} \qquad (4)$$

To achieve an optimal NPR of:

$$\boldsymbol{NPR_{opt\,[dB]}} \approx \mathbf{41dB} \qquad (5)$$

power level of RFI (having Gaussian distribution) at the input of ADC should not exceed:

$$\boldsymbol{P_{RFI\_load}}_{\,[\mathbf{dBm}]} = \boldsymbol{P_{max}}_{\,[\mathbf{dBm}]} - \mathbf{12dB} \approx -\mathbf{10dBm} \qquad (6)$$

Since the RFI will (hopefully) never occupy the entire Nyquist bandwidth of the ADC, it will be convenient to introduce the spectrum availability factor χ_{BW} defined as:

$$\chi_{BW} = \frac{\boldsymbol{BW_{total} - BW_{RFI}}}{\boldsymbol{BW_{total}}} \qquad (7)$$

$$1 > \chi_{\boldsymbol{BW}} > \mathbf{0}$$

where**:** $\boldsymbol{BW_{total}}$ is a total number of frequency bins covering the entire Band 1 frequency range,
$\boldsymbol{BW_{RFI}}$ is a number of frequency bins flagged as RFI.

At the beginning of year 2021, at the VLA location, availability of spectrum within the Band 1 frequency limits was about 70% [3]:

$$\chi_{\boldsymbol{BW}} \approx \mathbf{0.7} \qquad (8)$$

For the case when RFI does not occupy the entire Nyquist bandwidth of ADC, NPR increases by a correction factor $\boldsymbol{NPR_c}$ (similar to a process gain when oversampling), which is equal to:

$$\boldsymbol{NPR_c} = \frac{\mathbf{1}}{\mathbf{1}-\chi_{\boldsymbol{BW}}} = \frac{\boldsymbol{BW_{total}}}{\boldsymbol{BW_{RFI}}} \qquad (9)$$

$$\boldsymbol{NPR_{c\,[dB]}} = \mathbf{10\log} \boldsymbol{NPR_c} \qquad (10)$$

Therefore, at present:

$$\boldsymbol{NPR_{c\,[dB]}} \approx \mathbf{5dB} \quad \text{(assuming } \chi_{\boldsymbol{BW}} \approx \mathbf{0.7}\text{)} \qquad (11)$$

which will yield (after applying this correction):

$$\textbf{corrected} \qquad \boldsymbol{NPR}_{\,[\boldsymbol{dB}\}} \approx \mathbf{46dB} \quad \text{(assuming } \chi_{\boldsymbol{BW}} \approx \mathbf{0.7}\text{)} \qquad (12)$$

and consequently, equivalent quantization noise level of:

$$\textbf{corrected} \qquad \boldsymbol{N_{q\,[dBm]}} \approx \mathbf{-56dBm} \quad \text{(assuming } \chi_{\boldsymbol{BW}} \approx \mathbf{0.7}\text{)} \qquad (13)$$

Corrected NPR (under the assumption of 30% RFI "pollution") fulfills system allocations [3].

*) There will be an extra margin (of approx. **1.8dB**, thus NPR will increase to **47.8dB**) due to band roll-off below **1.2GHz** (i.e. highpass filtering) – this margin will not apply if highpass filter is removed for the Low-Frequency Astronomy observations. Therefore, the rest of analysis is made under the assumption that there is only lowpass-filtering (in order to establish the same requirement for both configurations) .

Spectral purity of sampling clock will have an impact on the level of ADC noise floor. Phase jitter of the sampling clock will be transferred to the output of A/D Converter, and will appear on each of the input RFI tones causing noise sidebands - "noise skirts", as depicted in Figure 4.

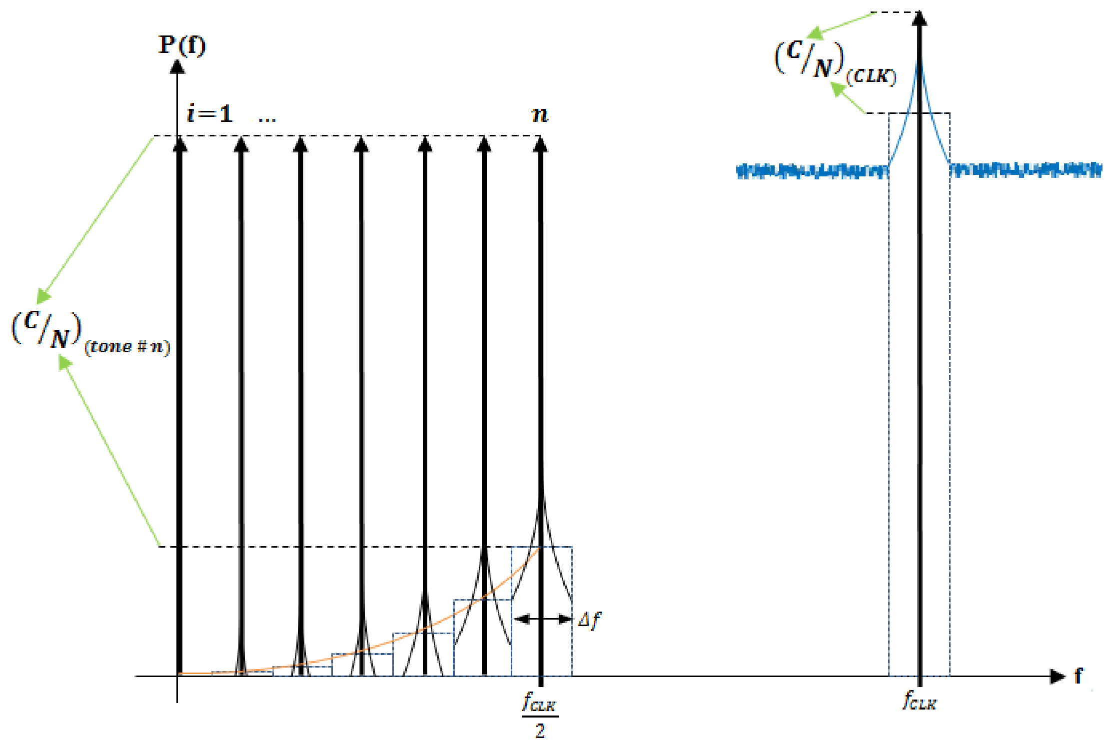

Fig. 4 Transfer of the phase noise from sampling clock to each of RFI tones (across the Nyquist band).

Phase jitter transferred from the sampling clock to the output of ADC will be modified by a factor equal to the ratio of input tone frequency to clock frequency [7]. Considering the case of direct-RF-sampling A/D Converter, this ratio will be equal to **(½)** max., at 1st Nyquist band's upper edge, and will roll-off down linearly for the rest of 1st Nyquist band. **Carrier-to-Noise ratio** (C/N) (linear value) is equivalent to inverse of (relative to carrier) **phase noise power** $(\Theta_{rms})^2$ (under small phase deviation approximation) [8]:

$$\left(C/N\right) \equiv \frac{1}{(\Theta_{rms})^2} \tag{14}$$

Therefore:

$$(\Theta_{rms})^2 = \frac{1}{(C/N)} = \frac{P_\Theta}{P_c} \tag{15}$$

where: P_Θ is a double-sideband (**DSB**) integrated phase noise power,
and P_c is a carrier power.

Equivalent double-sideband (**DSB**) phase noise power for each of tones ($P_{\phi_{(i)\,ADC}}$) is calculated by integrating noise power spectral densities (**PSD**) over finite bandwidth of Δf (as is illustrated in Fig. 4 by the width of the rectangles around each of tone's frequencies). Thus, if expressing **DSB** phase noise power in terms of integrated single-sideband (**SSB**), relative to carrier, phase noise density $\mathcal{L}(f_m)$:

$$(\Theta_{rms})^2 = 2\int_0^{\left(\frac{\Delta f}{2}\right)} \mathcal{L}(f_m)\; d f_m \qquad (16)$$

Single-sideband (**SSB**) phase noise spectral densities (relative to carrier) for each of tones at the output of ADC $\mathcal{L}_{(i)ADC}(f_m)$, can each be separately expressed in terms of sampling clock's **SSB** relative phase noise spectral density $\mathcal{L}_{CLK}(f_m)$ [7]:

$$\mathcal{L}_{(i)ADC}(f_m) = \left(\frac{f_{(i)}}{f_{CLK}}\right)^2 \mathcal{L}_{CLK}(f_m) \qquad (17)$$

where: $f_{(i)}$ is the frequency of i^{th} RFI tone, and
f_{CLK} is the frequency of sampling clock.

Fig. 5 displays the above relation for RFI tone frequencies within 1st Nyquist band ($\frac{f}{f_{CLK}} \leq$ ½).

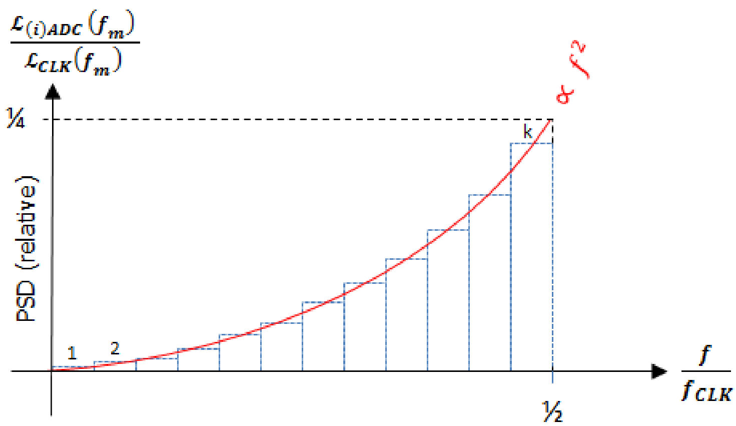

Fig. 5 Phase noise density transfer ratios for each of RFI tones across the 1st Nyquist band – ratios of single-sideband (SSB) phase noise spectral densities (relative to carrier) at the output of ADC v/s SSB phase noise spectral density of the sampling clock.

*) Despite the fact that phase noise sidebands of each tone originate from the same source (i.e. sampling clock's noise), phase of each of the tones will jitter around different initial phase; and the initial phase will jump in a pseudo-random fashion between a discrete phase states, due to an independent digital phase modulations imposed on each of RFI tones (the cluster of RFI tones will likely include several independently modulated satellite channels). Therefore, phase noise powers for each of tones will be considered uncorrelated, one to another.

For a number of tones equal to $\boldsymbol{k}$, such as all DSB integrated noise powers will side-by-side occupy entire Nyquist band, the total phase noise power $\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\ BW\ DSB}]} \right)^{\mathbf{2}}$ transferred from the sampling clock to the output of ADC, (integrated over the entire Nyquist bandwidth) can be estimated based on transfer ratios' curve shown in Fig. 5 (intuitive reasoning will be provided below, while systematic calculations can be found in the Appendix). An area underneath the red parabolic curve (in Fig. 5) is a Riemann integral equivalent to sum of $\boldsymbol{k}$ Riemann (midpoint) intervals (rectangles in blue). When $\boldsymbol{k} \rightarrow \infty$ (which implies that width of each interval $\boldsymbol{\Delta f}$ is minimized - representing narrowing integration bandwidths down to unit lines of **1Hz** while calculating the equivalent phase noise powers).

$$\int_{\mathbf{0}}^{(1/2)} \boldsymbol{f}^{\mathbf{2}}\ \boldsymbol{df} = \frac{\left(\mathbf{1}/\mathbf{2}\right)^{3}}{\mathbf{3}} = \frac{\mathbf{1}}{\mathbf{24}} \qquad (18)$$

Therefore, the total phase noise power transferred from the sampling clock to the output of ADC is equal to ($\mathbf{1}/\mathbf{24}$) of the sampling clock's integrated phase noise power (if both powers are expressed in terms of **1Hz** noise bandwidth - **DSB**).

To account for the integration across the entire DSB Nyquist zone bandwidth, the phase noise power (relative) transferred to the output of ADC (from the sampling clock) can be derived as:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\ BW\ DSB}]} \right)^{\mathbf{2}} = \left(\frac{\mathbf{1}}{\mathbf{24}}\right)\ (\boldsymbol{f_{CLK}})\ \mathbf{2}\int_{\mathbf{0}}^{\left(\frac{\mathbf{1Hz\ DSB}}{2}\right)} \boldsymbol{\mathcal{L}_{CLK}}\ (\boldsymbol{f_m})\ \boldsymbol{d f_m} \qquad (19)$$

Total integrated phase noise power of the sampling clock $\left(\Theta_{rms\,CLK_{total}\,[\mathrm{f_{CLK}\ BW\ DSB}]}\right)^{2}$ will include: an excess phase noise power $\left(\Theta_{rms\,CLK_{excess}\,[\mathrm{f_{CLK}\ BW\ DSB}]}\right)^{2}$ – due to phase jitter around the carrier, and the ultimate wideband "flat" noise power $\left(\Theta_{rms\,CLK_{ultimate}\,[\mathrm{f_{CLK}\ BW\ DSB}]}\right)^{2}$ - due to the phase component of the sampling clock's noise pedestal (also accounting for the noise contributions from the buffer stages or multipliers, if used). All of these noise powers are DSB integrated over the same bandwidth – the entire DSB Nyquist zone width around sampling clock carrier

$$\left(\Theta_{rms\,CLK_{total}\,[\mathrm{f_{CLK}\ BW\ DSB}]}\right)^{2} = \left(\Theta_{rms\,CLK_{excess}\,[\mathrm{f_{CLK}\ BW\ DSB}]}\right)^{2} + \left(\Theta_{rms\,CLK_{ultimate}\,[\mathrm{f_{CLK}\ BW\ DSB}]}\right)^{2} \qquad (20)$$

To account for the fact that the spectrum of noise due to phase jitter will dominate only a portion of the entire DSB Nyquist zone around clock carrier, the **DSB noise power** $\left(\Theta_{rms\,[\mathbf{2f_c\ BW\ DSB}]}\right)^{\mathbf{2}}$ due to **phase jitter (rms)** Θ_{rms} (which is integrated over $\mathbf{2f_c}$ bandwidth **DSB** around carrier) is converted to equivalent noise power DSB integrated over the entire Nyquist zone width:

$$\left(\Theta_{rms\ CLK_{excess}\ [f_{CLK}\ BW\ DSB]}\right)^2 = \left(\frac{2\,f_c}{f_{CLK}}\right)\left(\Theta_{rms\ [2f_c\ BW\ DSB]}\right)^2 \qquad (21)$$

where: f_c is the offset frequency to which single-sideband (**SSB**) excess noise (due to phase jitter) dominates over noise pedestal of the sampling clock.

Scaling the sampling clock noise power to an integration over **1Hz** noise bandwidth (**DSB**):

$$2\int_0^{\left(\frac{1Hz\ DSB}{2}\right)} \mathcal{L}_{CLK}(f_m)\ d\,f_m = \left(\frac{1}{f_{CLK}}\right)\left(\Theta_{rms\ CLK_{total}\ [f_{CLK}\ BW\ DSB]}\right)^2 \qquad (22)$$

where: ratio $\left(\frac{1}{f_{CLK}}\right)$ accounts for conversion down to **1Hz** bandwidth (**DSB**).

Using equations (19) and (22):

$$\left(\Theta_{rms\ ADC_{total}\ [f_{CLK}\ BW\ DSB]}\right)^2 = \left(\frac{1}{24}\right)\left(\Theta_{rms\ CLK_{total}\ [f_{CLK}\ BW\ DSB]}\right)^2 \qquad (23)$$

Carrier-to-Noise ratio (due to phase noise transferred from sampling clock to the output of ADC) $\left(C/N\right)_{P_\Theta\ ADC}$ could be expressed as:

$$\left(C/N\right)_{P_\Theta\ ADC} \equiv \frac{1}{\left(\Theta_{rms\ ADC_{total}\ [f_{CLK}\ BW\ DSB]}\right)^2} = \frac{24}{\left(\Theta_{rms\ CLK_{total}\ [f_{CLK}\ BW\ DSB]}\right)^2} \qquad (24)$$

Accordingly, in a logarithmic scale:

$$\left(C/N\right)_{P_\Theta\ ADC\ [dB]} = 10\log\left[\frac{24}{\left(\Theta_{rms\ CLK_{total}\ [f_{CLK}\ BW\ DSB]}\right)^2}\right] \qquad (25)$$

*) Let's assume that, in order to avoid the folding of far-from-carrier noise back to the A/D Converter's 1st Nyquist zone, double-sideband (DSB) wideband noise of sampling clock has its spectrum limited to twice the SSB Nyquist bandwidth (around sampling clock carrier frequency): **7GHz ± 3.5GHz** (i.e. from 3.5GHz to 10.5GHz).

Since the **rms time jitter** of LO sources will not exceed **76fsec** at 112GHz (thus less than **1.2psec** at the sampling clock frequency of **7GHz)**, **DSB rms phase jitter** of the sampling clock can be estimated as:

$$\Theta_{rms} \approx \mathbf{0.0535rad} \tag{26}$$

*) For a practical reasons, phase jitter is measured (and integrated) starting from small SSB frequency offset (of 1Hz to 10Hz) rather than from 0Hz; thus, it is assumed this will not introduce a significant discrepancy into the calculations, and therefore can be neglected.

DSB phase noise power $\left(\Theta_{rms\,[\mathbf{2f_c\,BW\,DSB}]}\right)^2$ due to **phase jitter** Θ_{rms} (integrated over $\mathbf{2f_c}$ bandwidth **DSB** around sampling clock carrier frequency) is:

$$\left(\Theta_{rms\,[\mathbf{2f_c\,BW\,DSB}]}\right)^2 \approx \mathbf{2.86mrad^2} \tag{27}$$

Estimating the corner frequency $\boldsymbol{f_c}$ as equal to **1MHz**, and using equation (21) will yield:

$$\left(\Theta_{rms\,CLK_{excess}\,[\mathbf{f_{CLK}\,BW\,DSB}]}\right)^2 \approx \mathbf{0.82\mu rad^2} \tag{28}$$

Estimating **SSB phase noise spectral density (**relative to carrier) for the ultimate (“flat”) noise pedestal as less than **-136dBc/Hz**, and **integrating** it over SSB offsets **from 1MHz to 3.5GHz** will yield **phase noise to carrier (DSB) power ratio** of approx. **-37.6dB**. Using equation (15), equivalent phase noise power can be calculated as:

$$\left(\Theta_{rms\,CLK_{ultimate}\,[\mathbf{f_{CLK}\,BW\,DSB}]}\right)^2 \approx \mathbf{0.175mrad^2} \tag{29}$$

which will dominate over excess noise contribution, thus using equation (20):

$$\left(\Theta_{rms\,CLK_{total}\,[\mathbf{f_{CLK}\,BW\,DSB}]}\right)^2 \approx \mathbf{0.176mrad^2} \tag{30}$$

Total phase noise power transferred to the output of A/D Converter, based on equation (23):

$$\left(\Theta_{rms\,ADC_{total}\,[\mathbf{f_{CLK}\,BW\,DSB}]}\right)^2 \approx \mathbf{7\mu rad^2} \tag{31}$$

Using equation (25) will yield:

$$\left(C/N\right)_{P_\Theta\,ADC\ [dB]} \approx \mathbf{51dB} \tag{32}$$

Since RFI tones do not occupy the entire Nyquist bandwidth of ADC (at present, 30% RFI "pollution" is estimated as mentioned earlier), $\left({}^{C}/_{N}\right)_{P_{\Theta}\,ADC\ [dB]}$ should be increased by approx. **5dB** (same amount as for the case of calculating NPR); thus, with a "process gain":

$$\left({}^{C}/_{N}\right)_{c\ P_{\Theta}\,ADC\ [dB]} \approx \mathbf{56dB} \quad (\text{assuming } \chi_{BW} \approx \mathbf{0.7}) \qquad (33)$$

where: $\left({}^{C}/_{N}\right)_{c\ P_{\Theta}\,ADC\ [dB]}$ is a new carrier-to-noise ratio after applying **5dB** correction (similar as when calculating the NPR).

Therefore, double-sideband (**DSB**) integrated equivalent phase noise power (due to phase jitter transferred from the sampling clock) at the output of A/D Converter $P_{\Theta\,ADC}$ will be about **56dB** below the power level of RFI "load" (i.e. RFI "carrier" level at the input to ADC), which is not to exceed **-10dBm**, as defined by the equation (6); thus:

$$P_{\Theta\,ADC} \approx -\mathbf{66dBm} \qquad (34)$$

which is **10dB below** the equivalent quantization noise level of **-56dBm**, per equation (13), thus causing only negligible degradation of the ADC available dynamic range (**~0.4dB degradation**).

*) If ultimate ("flat") noise of the sampling clock is limited to narrower offsets than +/-3.5GHz from the carrier frequency (i.e. band-limited by means of filtering), -136dBc/Hz requirement for SSB phase noise spectral density can be further relaxed due to the "process gain" (as in the case of an input signal not occupying the entire Nyquist bandwidth).

2. RF Amplification Chain – *RF system requirements*

The main purpose of this study is to develop a requirement for an acceptable level of distortions (**IMD**) due to intermodulation within the RF amplification chain, such as these will be matched to the NPR performance of particular ADC being used. Since high-resolution, high-sampling-rate ADC will not be a cost-effective solution at present (not that easily available for GHz range sampling clock rates), 8-bit A/D is the feasible option for Band 1 of ngVLA Integrated Receiver and Digitizer [4]. On contrary, cost-effective RF components for the amplification chain are already commercially available (for the frequency range of Band 1), having a decent noise figure, IP3, IP2 and compression levels, and allowing to minimize the RF chain's own IMD products.

A/D Converter will ultimately limit the achievable dynamic range of the entire direct-RF-sampling Integrated Receiver for Band 1 of ngVLA. Analysis of ADC dynamic range based on noise excitation (NPR) is adequate for strongly nonlinear behavior modeling (such as clipping behavior of A/D converter). On contrary, RF amplification chain is expected to display only a weak nonlinear behavior (having operating point well into the linear region). Pedro, et al. [9] argue that even the NPR test, in the case of weakly nonlinear component, will give an optimistic result when modeling a "co-channel" intermodulation products (i.e. falling on the frequencies being inside the RFI "cluster"). Fortunately, in the case of RFI on Band 1 of ngVLA, the "co-channel" IMD products will likely align with the RFI tones' frequencies inside the RFI "cluster" (RFI from a satellite communication systems will have channels placed on a uniform frequency raster). Since RFI tones will be flagged for excision, "co-channel" distortions will be eliminated, as well.

Reference [9] discusses RF memoryless nonlinear third-order system, while applying the Wiener-Volterra series expansion (up to 3rd order term) to analyze its intermodulation products. Fig. 6 illustrates several cases of the third-order intermodulation products (relevant to Band 1 of ngVLA RFI scenario). For clarity of the picture, not all of the intermodulation products are shown (for the case of third-order products - being combination of three equally-spaced tones, there will also be "co-channel" product aligned with the frequency of the middle tone, and one more product at its 3rd harmonic, away from RFI "cluster"). Third-order products which result from interaction between all three tones will appear above third-order products involving only two tones, thus such products will set the worst-case scenario for the intermodulation distortions. Therefore, Pedro, et al. [9] argue that performance test using three tones will already provide sufficient information for the assessment of the worst-case IMD. Multitone excitation , which eventually leads to the case of noise excitation (for the large number of tones), would give more complete picture of all intermodulation products (approximating expected for Band 1of ngVLA intermodulation RFI scenario). Based on calculations in reference [9], the worst-case level of noise intermodulation sidebands can be compared to the level of (more commonly used as a measure of nonlinearity) two-tone third-order products. Intermodulation Ratio (**IMR** - defined as a **ratio of output power per tone to** an "adjacent channel" **IMD product power**) for both cases can be compared as shown below:

$$\boldsymbol{IMR_n = \frac{1}{4} IMR_2} \qquad \text{or} \qquad \boldsymbol{\Delta_{IMR_{n\,[dB]}} \approx \Delta_{IMR_{2\,[dB]}} - 6\mathrm{dB}} \qquad (35)$$

where: $\boldsymbol{IMR_n}$ is an IMR ratio (linear) for noise excitation of RF system, and
$\boldsymbol{IMR_2}$ is an IMR ratio (linear) for two-tone excitation of same RF system.

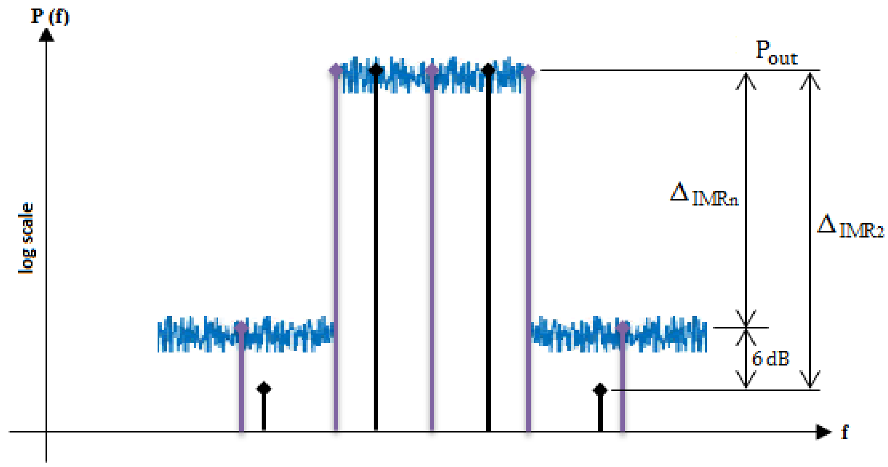

Fig. 6   Third-order intermodulation products for two-tone (**black**), three-tone (**purple** - for picture clarity, not all of 3rd order intermodulation products are shown), and the noise (**blue**) excitations of a weakly nonlinear RF system.

Reference [10] describes a load-pull measurement system employing an unequally-spaced multitone excitation signal to test nonlinearity of RF power amplifier. Such test signal exhibit a Gaussian probability density function (PDF), thus is closely resembling a random noise. Eight input tones create abundance of intermodulation products at the output of memoryless nonlinear circuit. Example of spectrum (at IF output of a vector network analyzer's receiver) is reproduced here in Fig. 7, from reference [10]. "Triple-beat" 3rd order products are 6dB above two-tone 3rd order intermodulation products, confirming the theoretical prediction from reference [9].

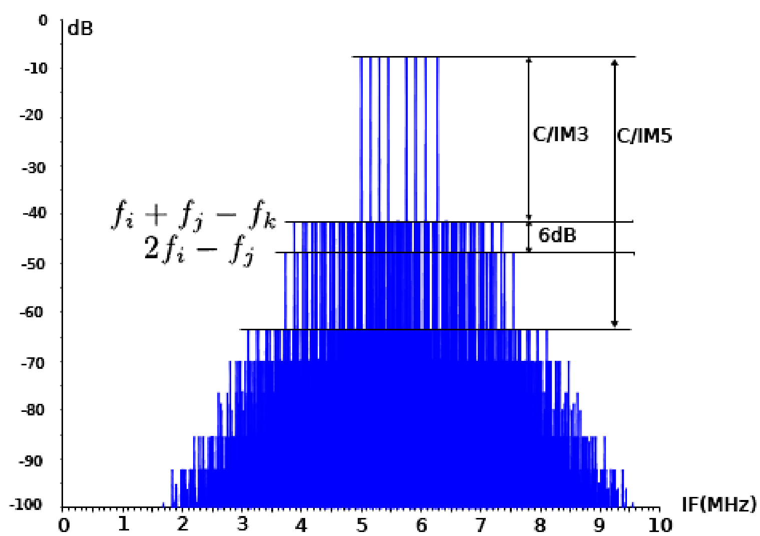

Fig. 7   "Triple-beat" v/s two-tone 3rd order IMD products (**Laurent et al., 2016 [10]**).

From reference [11], a **Two-Tone Output Intercept Point** (of $\boldsymbol{m}$ order) can be expressed as a function of its respective **Intermodulation Ratio** (in logarithmic scale):

$$\boldsymbol{OIP}_{\boldsymbol{m}\,[\boldsymbol{dBm}]} = \boldsymbol{P}_{\boldsymbol{out}\,[\boldsymbol{dBm}]} + \left(\frac{\boldsymbol{\Delta}_{\boldsymbol{IMR}_{2}\ \boldsymbol{m}\,[\boldsymbol{dB}]}}{\boldsymbol{m}-\mathbf{1}}\right) \qquad (36)$$

where: $\boldsymbol{OIP}_{\boldsymbol{m}\,[\boldsymbol{dBm}]}$ is a $\boldsymbol{m}^{\text{th}}$ order Output Intercept Point (**in dBm**),
$\boldsymbol{m}$ is an order of intermodulation distortion,
$\boldsymbol{\Delta}_{\boldsymbol{IMR}_{2}\ \boldsymbol{m}\,[\boldsymbol{dB}]}$ is a two-tone IMR ratio (**in dB**) for the order of $\boldsymbol{m}$,
and $\boldsymbol{P}_{\boldsymbol{out}\,[\boldsymbol{dBm}]}$ is an output power of each of two tones (**in dBm**).

Intermodulation products of the 3rd order needs to be at (or below) the noise level of ADC, in order to match linearity of RF analog amplification chain with linearity of the digitizer system. Therefore, power level of the "triple-beat" 3rd order products should not exceed the equivalent quantization noise level from equation (13): $\boldsymbol{N}_{\boldsymbol{q}\,[\boldsymbol{dBm}]} \approx -\mathbf{56dBm}$ (assuming $\boldsymbol{\chi}_{\boldsymbol{BW}} \approx \mathbf{0.7}$). Accordingly, the two-tone 3rd order products have to be 6dB below this level. Referencing to the **power level of RFI**, which is, from equation (6), equal to $-\mathbf{10dBm}$:

(for **3rd order products**) $$\boldsymbol{\Delta}_{\boldsymbol{IMR}_{2}\ \boldsymbol{3rd}\,[\boldsymbol{dB}]} \approx \mathbf{46dB} + \mathbf{6dB} = \mathbf{52dB} \qquad (37)$$

Using equation (36), one can conclude that the **3rd order two-tone output intercept point of RF amplification chain** (for the whole analog system) should be better than:

$$\boldsymbol{OIP}_{\mathbf{3}\,[\boldsymbol{dBm}]} = +\mathbf{16dBm} \qquad (38)$$

Noise sidebands resulting from RF chain's intermodulation products (spectrum re-growth) will not occupy the entire Nyquist bandwidth of ADC. Analysis can be simplified to only include 3rd order products since the levels of 5th order (and higher) are by more than 10dB below the level of the 3rd order ones. Therefore, **spectrum re-growth** will occupy **three times** wider bandwidth than the **total bandwidth occupied by the RFI** noise-like "cluster" of tones. Under the assumption of 30% RFI "pollution", noise due to 3rd order intermodulation will extend (in worst case) for 90% of Nyquist band, thus requiring correction to the noise power (similar to process gain when oversampling) of 0.9 (**approx. -0.5dB)**. After applying this correction, noise due to 3rd order ("triple-beat") intermodulation distortion in RF amplification chain will be equal to approximately **-56.5dBm**. **Combining** that noise power **with** noise power due to phase jitter transferred from the sampling clock, from the equation (34) level of **-66dBm**, **and** quantization noise (level of **-56dBm**), (assuming that all are uncorrelated) the **total equivalent noise power at the output of A/D Converter** will rise to $\boldsymbol{N}_{\boldsymbol{total}\,[\boldsymbol{dBm}]}$:

$$\boldsymbol{N}_{\boldsymbol{total}\,[\boldsymbol{dBm}]} \approx -\mathbf{53dBm} \quad \text{(assuming } \boldsymbol{\chi}_{\boldsymbol{BW}} \approx \mathbf{0.7}\text{)} \qquad (39)$$

Loss of 3dB for the equivalent (after combining all of noises) dynamic range of A/D Converter has to be traded for **3dB tighter allocation for the gain slope/ ripple** (i.e. 3dB max. for Band 1).

Band 1 frequency limits are relatively wide, thus allowing the 2nd order products to fall within it. Fig. 8 displays a "real-life" scenario - signal trace (shown in black) is from VLA recordings in L Band, and noticeably has the signatures of Inmarsat (GBAN by GroundControl), GPS ch. L1, GLONASS ch. L1, and Iridium satellite channels.

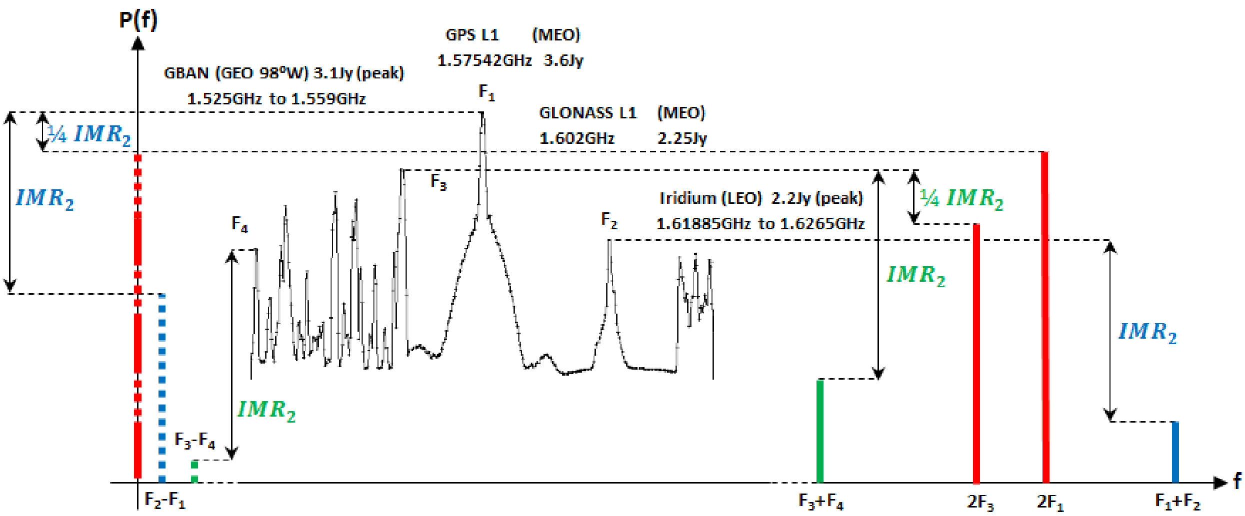

Fig. 8 Second-order intermodulation - shown are products of two-tone (**blue** and green), and single-tone (red). For picture clarity, not all of intermodulation products are shown (only products originating from two pairs of signals: F_1 and F_2, F_3 and F_4). Horizontal axis is not to scale – it is a composite of three separate spans of frequencies; vertical axis is scaled in linear power (flux numbers, corresponding to signal trace in black, are given only for reference – source: VLA recordings from Jan. 26, 2021; config. A, RR pol., only the IF5 sub-band of L Band).

In that portion of ngVLA Band 1 being shown, some of the two-tone 2nd order products, which are at frequency of the difference between tone's frequencies, fall below 1.2GHz. Tones spaced by at least 1.2GHz (and wider, up to 2.3GHz spaced-apart tones at the edges) will produce the frequency-difference products located within Band 1 limits. Therefore, 2nd order two-tone frequency-difference products will be of concern, especially if option for the Low-Frequency Astronomy will be actively pursued (i.e. Band 1 extended below 1.2GHz). In order to derive one common set of requirements, including option for Low-Frequency Astronomy, it will be assumed that Band 1 is only limited on the upper side.

Single-tone 2nd order products (consisting of two kinds; one at second harmonic of the tone, and the second at DC) will be **6dB higher than** corresponding **two-tone 2nd order products**, thus these will set the limit for the worst case scenario. In this study, products located at DC (i.e. for modulated tones, there will also be DSB sidebands representing signal modulation, centered around 0 Hz) are intentionally not addressed here (these may, worst case, upset the DC bias for the gain blocks of RF amplification chain).

Having, in the worst case, **power level of 2nd order intermodulation products** at least **10dB below** the **total equivalent noise power at the output of ADC**, of **-53dBm** per equation (39), (i.e. single-tone 2nd order products have to be at **-63dBm**), it will limit the degradation of dynamic range of A/D Converter (due to 2nd order intermodulation) to a negligible amount (**~0.4dB degradation**). Since a **two-tone 2nd order products** will be **6dB below** the **single-tone 2nd order products**, referencing this level to the worst case **power level of RFI**, which per equation (6) is equal to $-\mathbf{10dBm}$:

(for **2nd order products**) $$\Delta_{IMR_2\ 2nd\ [dB]} \approx \mathbf{53dB + 6dB = 59dB} \qquad (40)$$

Using equation (36), the **2nd order two-tone output intercept point of RF amplification chain** (for the whole analog system) should be better than:

$$\boldsymbol{OIP_{2\ [dBm]}} = \mathbf{+49dBm} \qquad (41)$$

These requirements could be relaxed in the case when **each of two tones** (contributing to generation of two-tone products) **comes from a different satellite** (rather than, in a general case, would originate from different channel of the very same satellite); therefore each would be received by different sidelobe of the antenna. In worst alignment condition, **one tone** could come **via main beam**, and **second would align with peak of adjacent sidelobe**, which would **lower power** of the sidelobe tone **by approx. 18dB** (from expected antenna pattern at 2.4GHz).

To accommodate unequal levels of tones, equation (36) could be rearranged as below:

$$\boldsymbol{OIP_{m\ [dBm]}} = \frac{\mathbf{2}\left(\boldsymbol{P_{out\ [dBm]}}\right)}{\mathbf{2}} + \left(\frac{\boldsymbol{\Delta_{IMR_2\ m\ [dB]}}}{\boldsymbol{m-1}}\right) = \frac{\left(\boldsymbol{P_{1\ out\ [dBm]}} + \boldsymbol{P_{2\ out\ [dBm]}}\right)}{\mathbf{2}} + \left(\frac{\boldsymbol{\Delta_{IMR_2\ m\ [dB]}}}{\boldsymbol{m-1}}\right) \qquad (42)$$

where: $\boldsymbol{P_{1\ out\ [dBm]}}$ is an output power of the first of two tones (**in dBm**)
and $\boldsymbol{P_{2\ out\ [dBm]}}$ is an output power of the second tone (**in dBm**).

Accordingly, **power level difference of -18dB**, between each of tones, would **translate to** a **correction of -9dB** for both $\boldsymbol{OIP_{m\ [dBm]}}$ requirements, to account for spatial separation of tones' sources (i.e. same amount of correction for the 2nd and for the 3rd order intermodulation).

Applying such correction, will yield a new values of the **minimum required two-tone** 3rd order ($\boldsymbol{OIP_{3s\ [dBm]}}$) and 2nd order ($\boldsymbol{OIP_{2s\ [dBm]}}$) **output intercept points for the particular case of spatial separation of tones' sources (which will happen in portions of ngVLA Band 1)**:

$$\boldsymbol{OIP_{3s\ [dBm]}} = \mathbf{+7dBm} \qquad (43)$$

and

$$\boldsymbol{OIP_{2s\ [dBm]}} = \mathbf{+40dBm} \qquad (44)$$

3. Conclusion

Crowded spectrum of RF interference (within the ngVLA Band 1 frequency limits) will produce a "noise load" on the A/D Converter. To avoid further degradation of performance, linearity of RF amplification chain (analog system - which delivers input signal to the A/D Converter) must be as specified by minimum levels of **two-tone output intercept points** (listed in the Table 1).

Table 1
Required two-tone intercept points for Band 1 of ngVLA
(referenced to the output of RF amplification chain).

	Spatial Separation of RFI Sources	Low-Frequency Astronomy Option
	(within 1.2GHz to 3.5GHz)	(below 1.2GHz)
OIP_3	+7dBm	+16dBm
OIP_2	+40dBm	+49dBm

*) It is likely that RFI signals, contributing to a two-tone excitation of RF amplification chain (the analog system), will originate from different L-band satellites, and thus will be spatially separated (giving unequal levels of both tones - due to the advantage of narrow antenna beam and relative sidelobe level). If **Low-Frequency Astronomy** is considered, **antenna beam** will likely be **wider** for the frequency range **below 1.2GHz**. Therefore, the required two-tone output intercept points have to be higher (attenuation due to spatial separation will not be as significant).

Appendix

A systematic derivation of phase noise power transferred from clock jitter to the output of ADC (driven by a "cluster" of tightly-spaced RFI tones) is provided here. Since phase jitters of each of ***k*** output tones can be considered totally uncorrelated to each other, total phase jitter at the output of ADC (due to clock's phase noise) will be a root-sum-square of RFI tone's jitters. Accordingly, phase noise powers can be combined into total equivalent integrated noise power:

$$\left(\Theta_{rms\,ADC_{total}\,[f_{CLK}\,BW\,DSB]}\right)^2 = \sum_{i=1}^{k}\left(\emptyset_{rms_{(i)}\,ADC}\right)^2 \qquad \text{(A-1)}$$

Using equation (16):

$$\left(\Theta_{rms\,ADC_{total}\,[f_{CLK}\,BW\,DSB]}\right)^2 = \sum_{i=1}^{k}\left[2\int_0^{\left(\frac{\Delta f}{2}\right)} \mathcal{L}_{(i)ADC}\left(f_m\right)\, d\,f_m\right] \qquad \text{(A-2)}$$

Taking into account equation (17) yields:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\,BW\,DSB}]}\right)^2 = \sum_{i=1}^{k}\left[2\int_0^{\left(\frac{\Delta f}{2}\right)}\left(\frac{f_{(i)}}{f_{CLK}}\right)^2 \mathcal{L}_{CLK}\,(f_m)\;d\,f_m\right] \tag{A-3}$$

Since the ratio $\left(\frac{f_{(i)}}{f_{CLK}}\right)$ is fixed for each of the tones, equation (A-3) can be rearranged:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\,BW\,DSB}]}\right)^2 = \sum_{i=1}^{k}\left[\left(\frac{f_{(i)}}{f_{CLK}}\right)^2 2\int_0^{\left(\frac{\Delta f}{2}\right)}\mathcal{L}_{CLK}(f_m)\;df_m\right] \tag{A-4}$$

Splitting the integration of sampling clock's SSB phase noise relative power density $\mathcal{L}_{CLK}(f_m)$ into sum of partial integrations over unit bandwidth **1Hz** (**DSB**) would yield Δf partial integrations:

$$\int_0^{\left(\frac{\Delta f}{2}\right)}\mathcal{L}_{CLK}(f_m)\;df_m = (\Delta f)\int_0^{\left(\frac{\mathrm{1Hz\,DSB}}{2}\right)}\mathcal{L}_{CLK}\,(f_m)\;d\,f_m \tag{A-5}$$

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\,BW\,DSB}]}\right)^2 = \sum_{i=1}^{k}\left[\left(\frac{f_{(i)}}{f_{CLK}}\right)^2 (\Delta f)\;2\int_0^{\left(\frac{\mathrm{1Hz\,DSB}}{2}\right)}\mathcal{L}_{CLK}\,(f_m)\;d\,f_m\right] \tag{A-6}$$

Using the following notation:

$$2\int_0^{\left(\frac{\mathrm{1Hz\,DSB}}{2}\right)}\mathcal{L}_{CLK}\,(f_m)\;d\,f_m = \left(\Theta_{rms\;CLK\;[\mathrm{1Hz\,BW\,DSB}]}\right)^2 \tag{A-7}$$

where: $\left(\Theta_{rms\;CLK\;[\mathrm{1Hz\,BW\,DSB}]}\right)^2$ is the equivalent sampling clock's phase noise power in **1Hz** bandwidth (**DSB**).

equation (A-6) is rewritten as:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\,BW\,DSB}]}\right)^2 = \sum_{i=1}^{k}\left[\left(\frac{f_{(i)}}{f_{CLK}}\right)^2 (\Delta f)\;\left(\Theta_{rms\;CLK\;[\mathrm{1Hz\,BW\,DSB}]}\right)^2\right] \tag{A-8}$$

Thus:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\,BW\,DSB}]}\right)^2 = \left[\frac{\left(\Theta_{rms\;CLK\;[\mathrm{1Hz\,BW\,DSB}]}\right)^2}{f_{CLK}{}^2}\right]\sum_{i=1}^{k}\left[f_{(i)}{}^2\,(\Delta f)\;\right] \tag{A-9}$$

If number of intervals (rectangle bars of equal width in Fig. 5) $k \to \infty$, Riemann sum of intervals can be substituted by integral:

$$\lim_{k\to\infty}\sum_{i=1}^{k}\left[f_{(i)}{}^2\,(\Delta f)\;\right] = \int_0^{\left(\frac{f_{CLK}}{2}\right)} f^2\,df \tag{A-10}$$

Taking equations (A-10) into account, equation (A-9) could be rearranged:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\;BW\;DSB}]}\right)^2 = \left[\frac{\left(\Theta_{rms\;CLK\;[\mathrm{1Hz\;BW\;DSB}]}\right)^2}{f_{CLK}{}^2}\right] \int_0^{\left(\frac{f_{CLK}}{2}\right)} f^2\, df \qquad \text{(A-11)}$$

Solving the integral will yield:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\;BW\;DSB}]}\right)^2 = \left[\frac{\left(\Theta_{rms\;CLK\;[\mathrm{1Hz\;BW\;DSB}]}\right)^2}{f_{CLK}{}^2}\right] \frac{f_{CLK}{}^3}{24} \qquad \text{(A-12)}$$

Therefore:

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\;BW\;DSB}]}\right)^2 = f_{CLK} \left[\frac{\left(\Theta_{rms\;CLK\;[\mathrm{1Hz\;BW\;DSB}]}\right)^2}{24}\right] \qquad \text{(A-13)}$$

By rearranging equation (A-13) using (A-7), it becomes identical to equation (19):

$$\left(\Theta_{rms\,ADC_{total}\,[\mathrm{f_{CLK}\;BW\;DSB}]}\right)^2 = \left(f_{CLK}\right) \left(\frac{1}{24}\right) 2 \int_0^{\left(\frac{\mathrm{1Hz\;DSB}}{2}\right)} \mathcal{L}_{CLK}\left(f_m\right)\, d f_m \qquad \text{(A-14)}$$

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