Next Generation Very Large Array Memo No. 10

Considerations for a Water Vapor Radiometer System

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Introduction

The purpose of this memo is to consider using a stand-alone water vapor radiometer to calibrate the ngVLA interferometers at 3mm wavelength. For a 3mm interferometer to successfully operate, the atmospherically induced phase fluctuations on all baselines must be less than about one twelfth of a wavelength rms (about a 12% coherence loss). The principal source of atmospheric phase variations is water vapor inhomogeneities in the atmosphere above the interferometer elements. (There are also inhomogeneities in the dry part of the atmosphere, but, under New Mexico conditions, they are expected to be smaller than the wet term. They are not noticeable at one centimeter and longer wavelengths, so we have no experience seeing what they are like in New Mexico.) In the approximation that the atmospheres above the two antennas forming the baseline are independent, the estimation of the atmosphere above a single antenna needs to be estimated with an rms error better by square root two, about 180 microns. It will be shown below that an accuracy of 180 microns of phase path corresponds to an accuracy of about 50 mK in the peak brightness of the water line.

For maintenance reasons, it is very desirable that a water vapor radiometer system should not require a cryogenic receiver. On the face of it, this seems very reasonable. An ambient temperature receiver system can be constructed with a system temperature approaching 300 K. If a channel of a few hundred MHz width can be devoted to the measurement, the noise fluctuations are about 20 mK with one second averaging. More detailed arguments confirming the feasibility are given below.

Theory

The starting point of the calculation is the Van Vleck-Weisskopf line shape, as given, for instance in *Thompson, Moran, and Swenson* (2^{nd} edition, equation 13.42). The units used there, and here, are frequencies in GHz, pressure in millibars, temperature in Kelvin, and water vapor density in grams per cubic centimeter (the numerical coefficients have been altered from those used by TMS, who quoted water vapor density in grams per cubic meter). The equation is reproduced below.

$$\alpha = [c_0 \ \exp(-c_1/T)\, \nu^2\, P\, T^{-3.125}\, (1 + c_2\, \rho\, T/P)$$

$$(((\nu - \nu_0)^2 + \Delta^2)^{-1} + ((\nu + \nu_0)^2 + \Delta^2)^{-1}))$$

$$+ c_3\, \nu^2\, \Delta\, T^{-3/2}\,]\, \rho$$

$$\Delta = c_4\, (1 + c_2\, \rho\, T\, P^{-1})\, P\, T^{-0.625}$$

α is the absorption coefficient, T is the temperature in Kelvin, P is the pressure in millibars, ν is the frequency in GHz, and ρ is the water vapor density in grams per cubic centimeter. Δ is approximately the line half width at half maximum. In these units, the constants are

$c_0 = 324$	$c_1 = 644$	$c_2 = 1.47e4$
$c_3 = 2.55e\text{-}8$	$c_4 = 0.0945$	$\nu_0 = 22.235$

The absorption and the line width vary with location in the atmosphere, so the line profile depends somewhat on the distribution of water vapor with elevation, which is, of course unknown. The sky brightness is the integral of α T along the line of sight. If the weather is reasonably good, it does not introduce appreciable errors to consider P, T, and Δ to be the water vapor density weighted means, rather than as functions of height.

$$B = [c_0 \ \exp(-c_1/T)\, \nu^2\, P\, T^{-2.125}\, (1 + c_2\, \rho\, T/P)$$

$$(((\nu - \nu_0)^2 + \Delta^2)^{-1} + ((\nu + \nu_0)^2 + \Delta^2)^{-1}))$$

$$+ c_3\, \nu^2\, \Delta\, T^{-1/2}\,]\, w$$

Where w is the total water content, and P, T and Δ are now mean values of the quantities, weighted by the water content of the air. If we look only at the part of the brightness due to the Lorentzian line profile,

$$B_L = a_0\, \Delta^2\, ((\nu - \nu_0)^2 + \Delta^2)^{-1}\, w$$

$a_s = c_2\, \rho_0\, T\, P^{-1}$ (usually < 0.1 in practical cases)

$$\Delta = c_4\ (1 + a_s)\, P\, T^{-0.625}$$

$a_0 = c_0 \exp(-c_1/T)\, \nu^2 P^{-1} T^{-0.875} (1 + a_s)^{-1}\; c_4^{-2}$

Let us consider a specific case.

$P_0 = 700 \qquad T_0 = 280 \qquad \rho_0 = 10$

whence

$a_s = 0.059 \qquad \Delta = 2.07$ GHz $\qquad a_0 = 20$ K per cm

At this point we should note that the conversion between a centimeter of precipitable water and a centimeter of path length is a factor of 6.3. So the desired accuracy of 180 microns of phase path, 0.0029 cm of water, is about 60 mK of brightness at the line peak.

A Simple System

The phase deviations caused by water vapor vary over a few tens of seconds (ngVLA memo 1 says a 30 second cycle suffices at the VLA site about 50% of the time at night.). Here we consider a system generating corrections every ten seconds. The expected mode of operation is that we will depend on finding a reasonably nearby calibrator, and use the water vapor radiometer system to interpolate between calibrator observations. That is, the device would record a spectrum every ten seconds. After observing a complete calibration cycle, these spectrometer observations would be averaged, including the observations from the beginning and ending calibrator. This mean spectrum would then be subtracted from the individual integrations leaving a set of residual spectra. This eliminates many slowly varying terms, and partially controls the effects of the receiver bandpass. We then attribute deviations from the mean as being due to atmospheric water vapor.

In the simplest possible system, one would assume the water vapor line width, Δ, is known, perhaps as an estimate from surface conditions. The residual spectra are then fit with a set of functions representing things that might change on a timescale shorter than an hour. I suggest:

$\Delta^2 ((\nu-\nu_0)^2 + \Delta^2)^{-1}$	The Lorentzian water line
$\nu^2 \nu_0^{-2}$	Hydrosols
1	Constant – drift in digitizer levels, for instance
B_r	Receiver bandpass – small gain changes

I have picked the functional form for convenience, and such that they all peak at an amplitude of one, so that the fitted value fairly directly relates to physical properties.

I assume that bandpass is sufficiently near a constant that its presence will not much influence the error analysis, and I have considered only the first three of these terms. A bandpass of 18 to 26 GHz is sufficient to reasonably well separate the terms. With that bandwidth, the diagonal element of the covariance matrix for the first comes to 2 GHz^{-1}; that is, it would have the variance of a 0.5 GHz channel. That is, for a 300 K system temperature and ten seconds integration, the noise on the flux of the line is about 10 mK, or 5 microns of water content, or about 30 microns of phase path.

The flaw in this very simple system is the necessity to use an estimated value for Δ. The seriousness of this flaw is illustrated by the table below. To make the table, I have taken an assumed Δ calculated for 283 K, 700 mB pressure, and 10 grams per cubic meter water vapor, which comes out to be 2.10 GHz. I then calculate the effect on the spectrum of an additional one millimeter of water at a height above the surface given by the first column. I assume a scale height for pressure of 8 km, and a temperature lapse rate of -7 K per kilometer. Temperature and pressure are given in the second and third columns of the table. I then fit this spectral increment with the fixed 2.1 GHz water line, and give the resulting calculated addition of water in the fourth column.

Height (km)	Temperature (K)	Pressure (mB)	Solved water
0	290	800	0.87
1	283	706	1.00
2	276	623	1.13
3	269	550	1.26
4	262	485	1.39
5	255	428	1.50
6	248	378	1.61

Therefore, the simple system described here is likely to reduce fluctuations only by a factor of order three, and is adequate for operational use only when the phase fluctuations are already less than 0.5 mm. These conditions are not unusual at the VLA site (“Ka band weather”), but are likely to be unacceptably rare at the various sites of the ngVLA.

Solving for the Height Dependence

Solving for the line width is a non-linear process, which is most conveniently done by iteration. However, the signal-to-noise properties of the process are equivalent to adding a term to the solution of the second derivative of the Lorentzian line shape. That is:

$$2\,((\nu - \nu_0)^2 + \Delta^2)^{-3}\,(3\,(\nu - \nu_0)^2 - \Delta^2)$$

This term is highly correlated with the desired water vapor term, and including it increases the term in the covariance matrix from about 2 GHz^{-1} to about 26 GHz^{-1}.

In addition to this severe penalty in signal-to-noise ratio, properly determining the water contribution requires a fairly good absolute measure of the sky temperature of the line. It is possible that gain and bandpass of the amplifier can be absolutely calibrated, and the calibration maintained by, for instance, observations of the sun. It is also possible that they cannot. A more conservative approach is to trust the stability of a noise source to be added to the input of the receiver, the approach of VLA Memo 177. (The system described in that memo was an add-on to the VLA K band receiver, and thus inherited the switched noise calibration of that receiver, a somewhat unfortunate feature. Examining the equations therein for signal-to-noise ratio, one sees that they are dominated by the noise induced by determining the switched noise level. Having a stronger switched noise source could have given a noise performance about five times better.) For a stand-alone system, the appropriate noise injection is roughly equal to the system temperature. This increases noise by approximately sqrt(12). The net result of this is that the noise on a ten second integration is about 6e-5 times system, so to attain the desired accuracy in the water line intensity requires a system temperature less than about 600 K.

Although an uncooled system of this system temperature can be built, it may be desirable for greater stability of the differential measurements to solve for the line width on a scan average basis, and to use that line width for the points within a scan, which results in returning the differential measurements nearly to the very high signal-to-noise ratio of the simple system. This procedure basically assumes only that the height distribution of water vapor variations is the same as the height distribution of water vapor itself. This procedure may also be invoked in integrations shorter than ten seconds turn out to be profitable. This might arise if one tries to use the WVR to correct phases in weather appreciably worse than the fiftieth percentile cited in ngVLA memo 1 (probably inadvisable) or if other antenna locations in the ngVLA have systematically much worse weather than the Plains of San Augustine. Picking the right strategy or strategies and even defining the conditions under which 3mm observing can be done at all will depend on knowing the characteristics of the instrument and the variations of weather across the array.

Some remarks on hardware

A quick look at the internet for low noise amplifiers did not yield a perfect match for the desired 18-26 GHz band, however, there was a 20-27 GHz amplifier with an excess noise temperature below 300 K. A suitable room temperature amplifier should not be a problem.

The Hitite HMC 5381 used in the VLA samplers still seems to be the item of choice for the digitizer. With its 26 GHz maximum clock rate, it might appear capable of digitizing the desired 18 to 26 GHz band directly, with no frequency conversion. However, the analog portion of the device starts to roll off at about 20 GHz, so a frequency conversion appears necessary. A local oscillator at 27 GHz or 37 GHz, and a digitizer clock at 20 GHz seem appropriate.

The digital processing implementing the spectrometer may be compared with the FPGA on the Widar correlator station board. The input clock is about five times higher, suggesting five times the parallelism if clock speeds remain the same. On the other hand, implementation of a simple spectrometer looks like it takes about three times fewer arithmetic elements than the sixteen subband channels of the station board. So an FPGA that is a little bigger or a little faster may be required. Such devices are now available.

There is a considerable commercial market for K band antennas and waveguide (not least for police radar, which uses this band). The radiometer needs to have a sufficiently small beam to confine the sampled volume to a diameter of perhaps 300 meters at the top of the troposphere, about a 1.5 degree beam, or an antenna diameter of about 50 cm.

Perhaps the most difficult part of the design is getting stable operation of the components: the receiver, the anti-aliasing filters, the local oscillator and mixer, the noise source and the digitizer. The requirements do not seem too extreme, and should be met with appropriate care in the thermal control of the components and the voltages supplied.

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