ngVLA Memo #96
ngVLA Antenna Noise Temperature Calculation

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Abstract

We describe the calculation of antenna noise temperature for ngVLA antennas. This memo follows closely the derivation in Cortes Medellin (2007) (hereafter referred to as CM07), with many common elements with Butler et al. (2019).

1. Introduction

In order to calculate noise flux density on the ngVLA, the value of the system temperature at each antenna, T_{sys}, must be known. We define T_{sys} as a combination of inputs from antenna noise temperature (T_{ant}), ambient temperature (T_P), and receiver temperature (T_{rx}), following CM07:

$$T_{sys} = \eta_L T_{ant} + (1 - \eta_L) T_P + T_{rx}$$

Where η_L is the antenna radiation efficiency. For the ngVLA antennas, $\eta_L \sim 1$, so this reduces to:

$$T_{sys} = T_{ant} + T_{rx}$$

2. Antenna Noise Temperature

The antenna noise temperature, T_{ant}, is the 2-D integral of the product of the brightness temperature of the surroundings of the antenna (T_b) and the antenna radiation pattern, P_n, over all directions θ, ϕ (see Figure 1 and Equation 2 of CM07, and Figure 1 below):

$$T_{ant} = \frac{\iint T_b(\theta,\phi)\, P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi}{P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi}$$

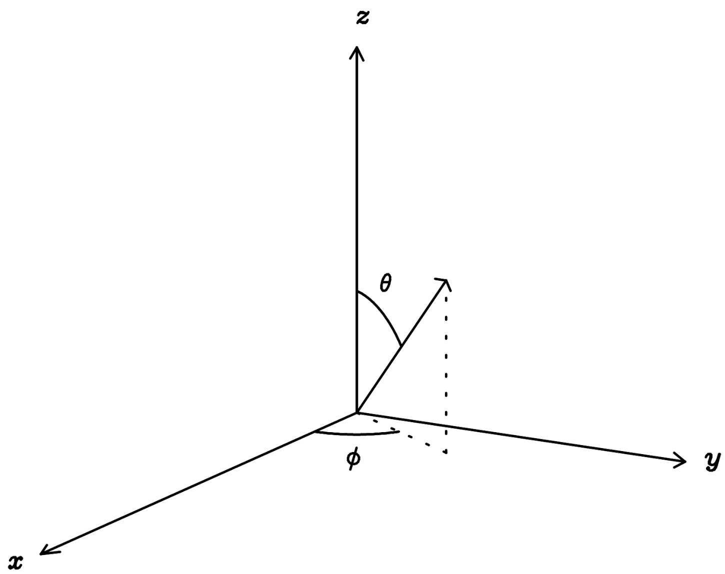

Figure 1. Geometry for calculating antenna noise contributions from the surroundings (sky and ground).

The brightness temperature in the full 4π steradians that radiates into the antenna can be broken into two elements: 1 – radiation from the sky, including background radiation, i.e., any direction θ, ϕ that does not intersect the ground ($z > 0$ in Figure 1); and 2 – radiation from the ground, i.e., any direction θ, ϕ that intersects the ground, which includes both emission from the ground, and emission from the sky that is scattered from the ground ($y < 0$ in Figure 1). With this decomposition we can rewrite the above equation:

$$T_{ant} = \frac{\iint [T_{sky}(\theta,\phi) + T_{ground}(\theta,\phi)]\, P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi}{P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi}$$

$$T_{ant} = \frac{\iint T_{sky}(\theta,\phi)\, P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi}{P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi} + \frac{\iint T_{ground}(\theta,\phi)\, P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi}{P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi}$$

Which is often simplified to:

$$T_{ant} = T_{atm} + T_{spill}$$

with the first term the contribution of the atmosphere to T_{ant}, and the second term (the "spillover") the contribution of the ground to T_{ant} (though there is also a contribution of the reflected atmospheric emission to this second term).

2.1 Atmospheric Temperature, T_{atm}

As defined above, the atmospheric contribution to the antenna temperature is:

$$T_{atm} = \frac{\iint T_{sky}(\theta,\phi)\, P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi}{P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi}$$

If we assume that most of the emission comes from the direction of the boresight (that the beam is narrow, compared to the change in T_{sky} with elevation; concomitantly, that the beam sidelobes are much lower than the peak of the main beam), then this is often simplified to:

$$T_{atm} = \eta_f \, T_{sky}(\theta)$$

with η_f the forward efficiency of the antenna and θ representing elevation or zenith angle. In this way the efficiency term is simply serving as a proxy for doing the full integral, which is usually acceptable given the level to which we know the antenna radiation pattern. Typical values for this efficiency are 0.95 or higher for precision-built antennas, and we expect that value to be 0.97 for ngVLA antennas.

The sky temperature (T_{sky}) in some direction (θ) is really the effective brightness temperature that produces the radiation brightness (B_{sky}) in that direction. If we consider that radiation intensity to come from three sources: 1 – the cosmic microwave background (B_{cmb}); 2 – galactic emission (mostly diffuse synchrotron) (B_{gal}); 3 – atmospheric emission (B_{atm}), and we further assume that the galactic emission is from material that is optically thin (so does not attenuate the emission from the CMB), then:

$$B_{sky} = \left(B_{cmb} + B_{gal}\right) e^{-\tau_{atm}} + B_{atm}$$

where τ_{atm} is the absorption through the entire atmosphere. The relationship between brightness and temperature is given by Planck's emission law:

$$B(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

for frequency ν, speed of light c, Planck's constant h, and Boltzmann's constant k. If $h\nu \ll kT$ then we are in the Rayleigh-Jeans limit of Planck's law, and the above can be simplified to:

$$B(T) = \frac{2kT}{\lambda^2}$$

and all brightnesses can be generalized directly to temperature, making all of the calculations much simpler since everything is linear in temperature. This was done in CM07. However, we are not afforded that luxury with ngVLA, since frequencies are high enough (and temperatures low enough) that that assumption can break down. We must therefore stick to the full Planckian expression to avoid errors from that assumption.

2.1.1 Background Emission, B_{cmb} and B_{gal}

We assume a CMB temperature of 2.725 K, and a galactic temperature of: $T_{gal} = 25.2(0.408/\nu_{GHz})^{2.75}$ K, for frequency ν_{GHz} in GHz. We then calculate the intensities using these temperatures.

2.1.2 Atmospheric Emission, B_{atm}

The atmospheric emission is an integral of the radiative transfer equation through the atmosphere:

$$B_{atm} = \int_0^{s_o} \kappa_a(s)\, B(T)\, e^{-\tau(s)}\, ds$$

With s_o representing the "top" of the atmosphere (the altitude at which the contribution to the total intensity becomes negligible), κ_a the absorption coefficient, and $B(T)$ the brightness at location s in the atmosphere.

We calculate B_{atm} by constructing a synthetic atmosphere, defined in discrete layers, then doing a full ray-traced radiative transfer calculation through this atmosphere, using the parameters in each layer (pressure, temperature, and water vapor pressure), and using the model of Liebe (1989) to calculate the absorption coefficient. This is described in full detail in Butler (1999) with the difference that we do not assume Rayleigh-Jeans in this application (we did in that previous application). This calculation also provides the opacity through the entire atmosphere, τ_{atm}. The inputs required for the creation of the synthetic atmosphere and the subsequent calculation of B_{atm} are: surface temperature, surface pressure, precipitable water vapor, scale height of water vapor, elevation (or zenith angle), and frequency.

2.1.3 Inverting For T_{sky}

Once the sky brightness, B_{sky}, is determined, the Planck function can be inverted to find the equivalent brightness temperature:

$$T_{sky} = \frac{h\nu}{k\,\ln\left(1 + \frac{2h\nu^3}{c^2\, B_{sky}}\right)}$$

2.1.4 Examples

Figures 2, 3, and 4 show plots of the value of T_{sky} for 3 values of precipitable water vapor (PWV): 1 mm, 6 mm, and 13 mm. See the discussion in Butler et al. (2019) for why those values were chosen.

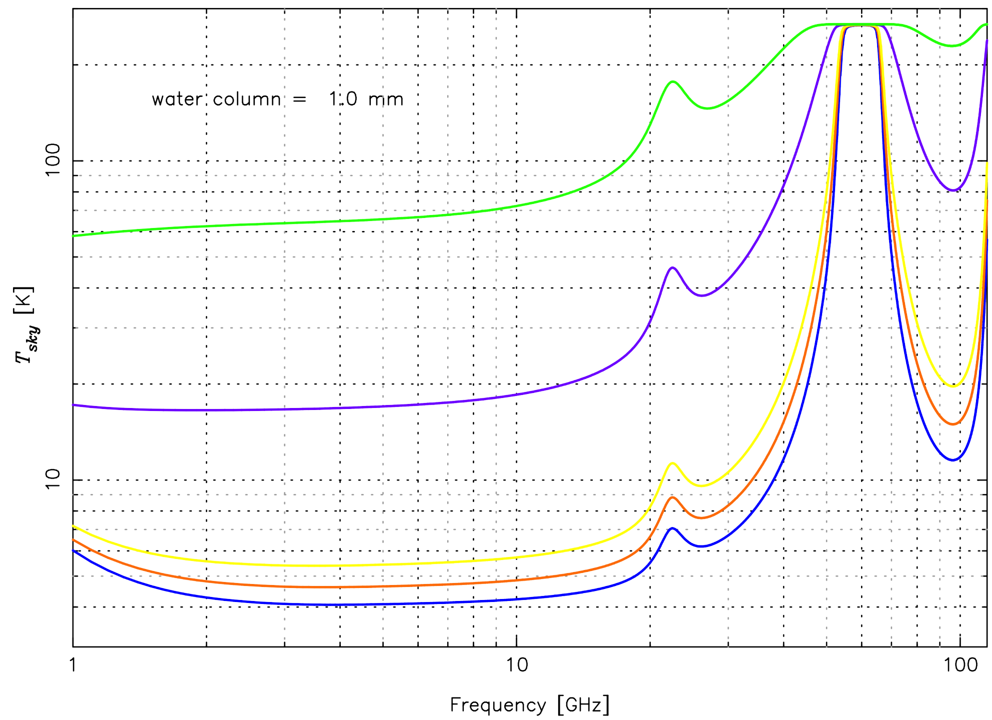

Figure 2. Contribution to the antenna noise temperature from the atmosphere (T_{sky}) for PWV = 1 mm. Colors are different zenith angles: blue = 0º; orange = 45º; yellow = 60º; purple = 85º; green = 90º.

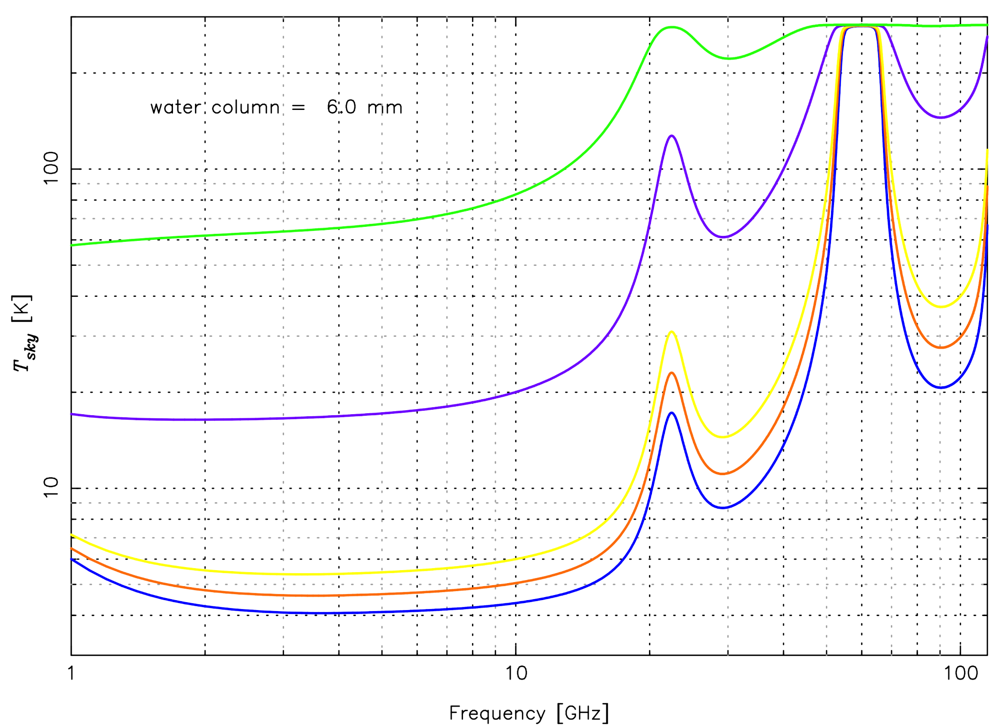

Figure 3. As Figure 2 but with PWV = 6 mm.

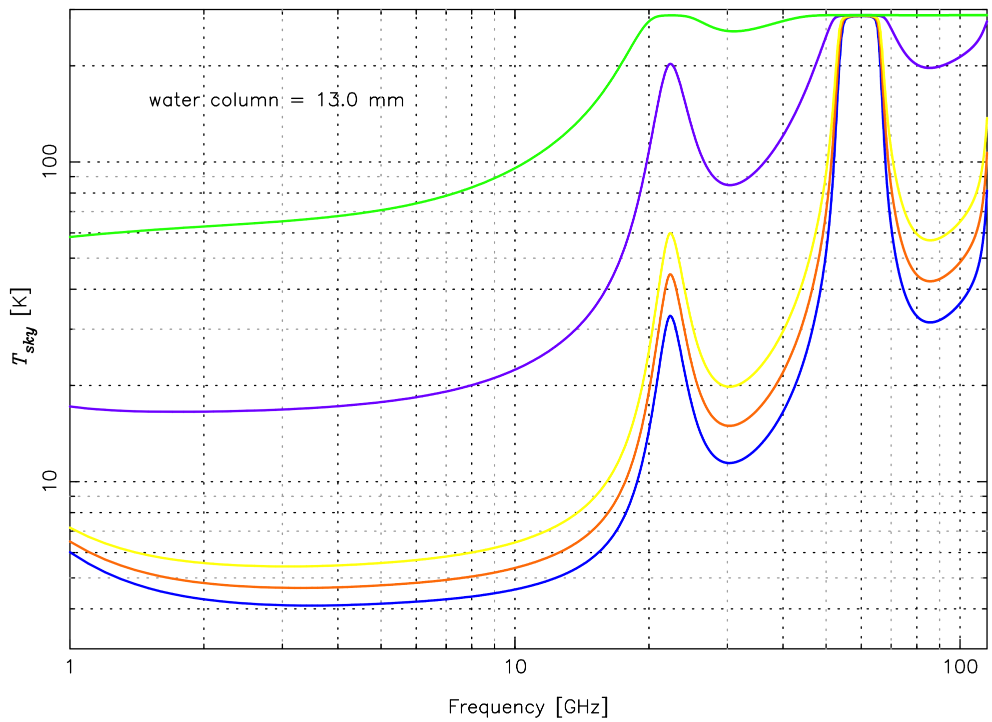

Figure 4. As Figures 2 and 3 but with PWV = 13 mm.

2.2 Spillover Temperature, T_{spill}

The spillover temperature, T_{spill}, is a combination of radiation that enters directly into the feed from any part of the subreflector illumination pattern that "spills over" onto the sky or ground, any ground emission that is scattered off of antenna surfaces into the feed, and any atmospheric emission that is reflected from the ground and then makes it into the feed (directly or via antenna surface scattering). To calculate it, one must know the properties of the feed illumination pattern, the ground around the antenna (temperature and scattering properties), the sky temperatures as a function of elevation (as derived above), and the details of the antenna structure (physical geometry, scattering properties, etc.). In principle, the integral must be done over the 2π steradians of "sky direction" that the ground covers, and that 2-D integral has as an integrand a quantity (T_{ground}) that must itself be integrated over the 2π steradians of the sky, twice (once for emission, once for atmospheric reflection), making it extremely difficult to calculate even in the best of cases. Calling the emission portion of T_{ground} T_{ground_e}, and the sky reflection portion T_{ground_r}, then $T_{ground} = T_{ground_e} + T_{ground_r}$.

For each ground area $dA = dxdy$ on the ground, the emission portion of T_{ground} is:

$$T_{ground_e} = \int\limits_{\substack{intersecting \\ rays}} \mathcal{T}(\theta')\, T_{surf}(x,y) \sin\theta' d\theta' d\phi'$$

(see Figure 5). The integral is done over all directions for which an outgoing ray strikes any part of the antenna structure (in principle, one might also need to include surrounding structures, trees, other nearby antennas, etc., but this is typically a small contribution to the total so ignored). The quantity $\mathcal{T}(\theta')$ is the surface Fresnel *transmittivity*, which can be calculated given the emission angle and properties of the surface (bulk dielectric constant, mostly). The reflected sky portion of T_{ground} is:

$$T_{ground_r} = 2\pi \int\limits_{\substack{intersecting \\ rays}} \int\limits_{\theta''<\pi/2} \mathcal{R}(\theta'',\theta')\, p(\theta'',\theta')\, T_{sky}(\theta'') \sin\theta'' d\theta'' \sin\theta' d\theta' d\phi'$$

(see Figure 6). As for the direct emission portion, the integral is done over all directions for which an outgoing ray strikes any part of the antenna structure. The quantity $\mathcal{R}(\theta'',\theta')$ is the surface Fresnel *reflectivity*, which can be calculated given the two angles and properties of the surface (bulk dielectric constant, mostly). The quantity $p(\theta'',\theta')$ is the phase function for reflection from the surface, and depends mostly on surface roughness (it is sometimes called the *directivity* function, notably for radar applications).

If one can assume that the ground emission and scattering is negligible for all but, say, some particular direction (to the subreflector, for example), then the calculation is greatly simplified (this is what was done in CM07, section 4). Without knowing antenna structure details, however, it is unclear if such an assumption is safe. Note that in detail, this contribution can be polarization dependent, because the emission and scattering from the ground is polarization dependent, but in practice, except for unusual terrains, the averaging over many kinds of surfaces and many multiple scatterings in the antenna structure almost always destroys this polarization dependence.

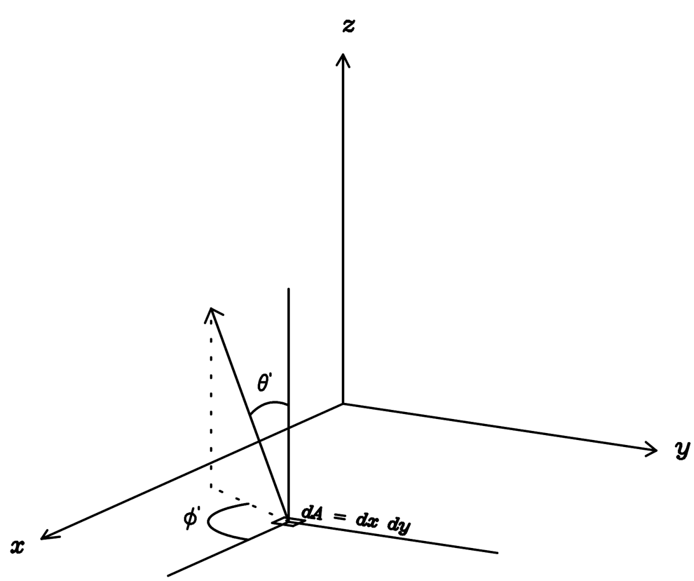

Figure 5. Geometry for direct ground emission contribution to T_{spill}.

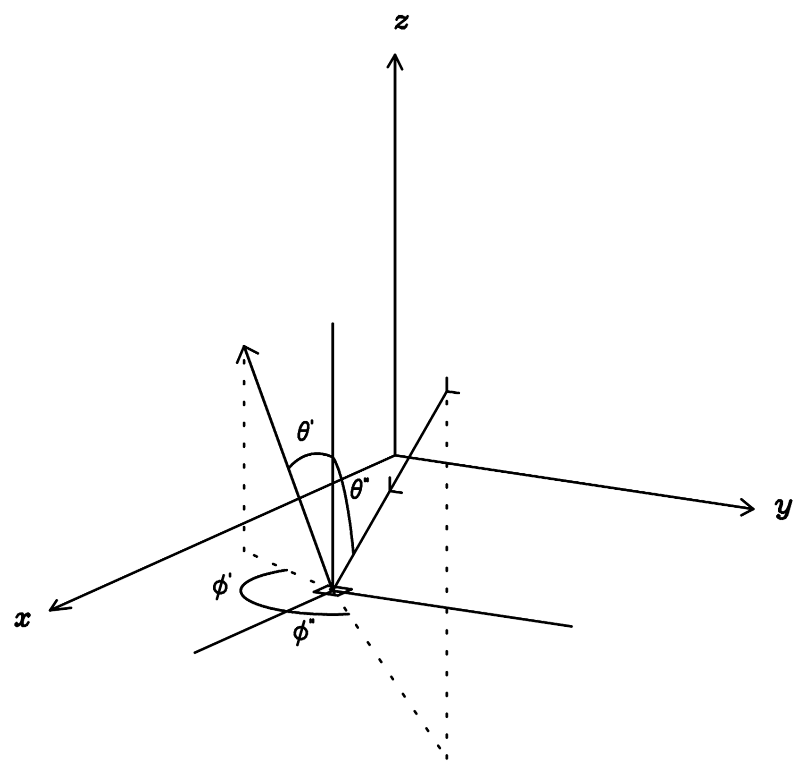

Figure 6. Geometry for reflected sky emission contribution to T_{spill}.

Fortunately, we have estimates of T_{spill} as a function of elevation and frequency, from calculations done by one of us (R.L.). Because a detailed model of the antenna structure is not presently available, only models of the main reflector, subreflector, and subreflector "ground shield" are included in these calculations, but that will account for the bulk of the spillover – what will be missing is mostly contributions from multiple scattering within the antenna structure. This model is shown in Figure 7 (after Lehmensiek, Grammer, and Sturgis 2020). The results of that calculation are in Grammer (2021), and we use those values. Note that the integrals are done in terms of brightness temperature, and then a correction for not being in the Rayleigh-Jeans regime of Planck's emission law is made to those values. This correction is the "Callen and Welton" (C&W) correction (Thompson, Moran, and Swenson 2017; Kerr, Feldman, and Pan 1997; Callen and Welton 1951):

$$T_x' = \frac{h\nu/k}{e^{h\nu/kT_x} - 1} + \frac{h\nu}{2k}$$

which is applied to the values in Grammer (2021).

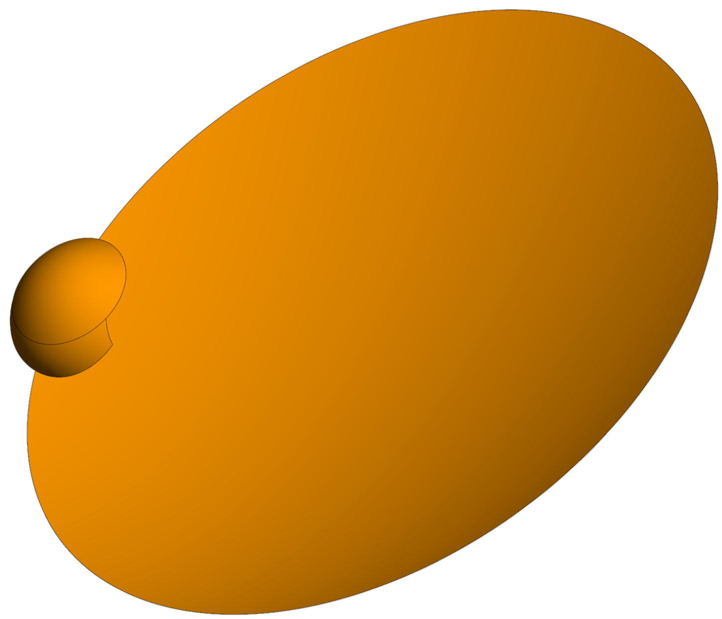

Figure 7. Model of main reflector, subreflector, and subreflector ground shield used for calculations of T_{spill}.

These values of T'_{spill} are per-band, with ten values as a function of frequency in each band, evaluated at elevations from 15° to 90°. Figure 8 shows the values for the "average" elevation (values at 15° are about half this; values at 90° about double). Also shown in that figure is a polynomial fit to enable calculation at any frequency, as in Butler et al. (2019).

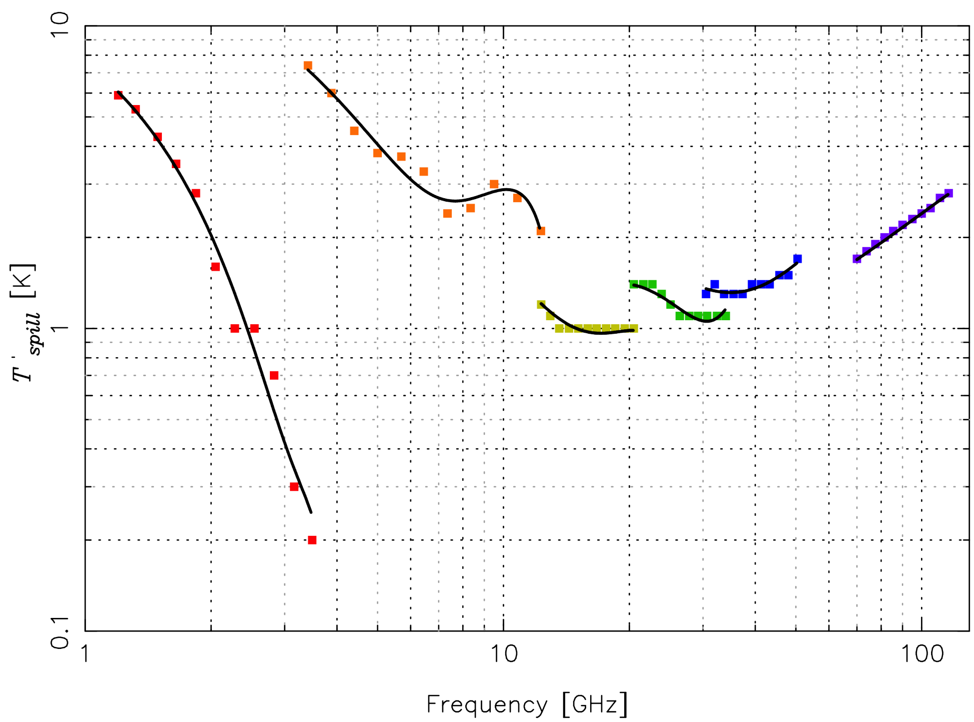

Figure 8. Values of T'_{spill} as a function of frequency. Calculated values are the colored dots (the six receiver bands for ngVLA are each a different color); the polynomial fits are the solid black lines.

3. Receiver Temperature

The receiver temperature contribution as a function of frequency is a combination of many terms, as explained in detail in Grammer (2021). Note that the receiver temperatures from that analysis must also have the C&W correction applied. Similar to the values of T'_{spill}, there are eleven values of T'_{rx} per band (but not as a function of elevation angle), and we fit them with a polynomial in each band. The values and fits are shown in Figure 9.

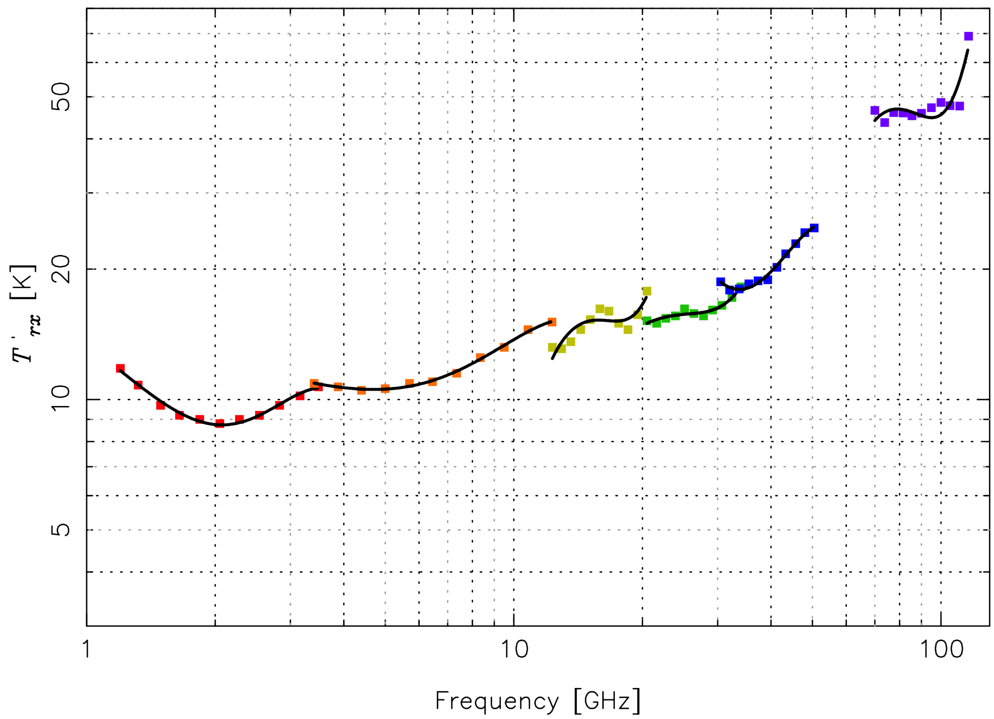

Figure 9. Values of T'_{rx} as a function of frequency. Calculated values are the colored dots (the six receiver bands for ngVLA are each a different color); the polynomial fits are the solid black lines.

4. Results

Combining all of the above results, we can now calculate T_{sys} for any combination of atmospheric conditions and elevation, as a function of frequency. Figure 10 shows an example calculation where for frequencies below 60 GHz, a PWV value of 6 mm is used, while for frequencies above 60 GHz, a PWV value of 1 mm is used. A forward efficiency of $\eta_f = 0.97$ was assumed. Band edges are apparent, and expected, because of the inclusion of T_{spill} and T_{rx}, which are band-dependent and have those edge effects (see Figures 8 and 9).

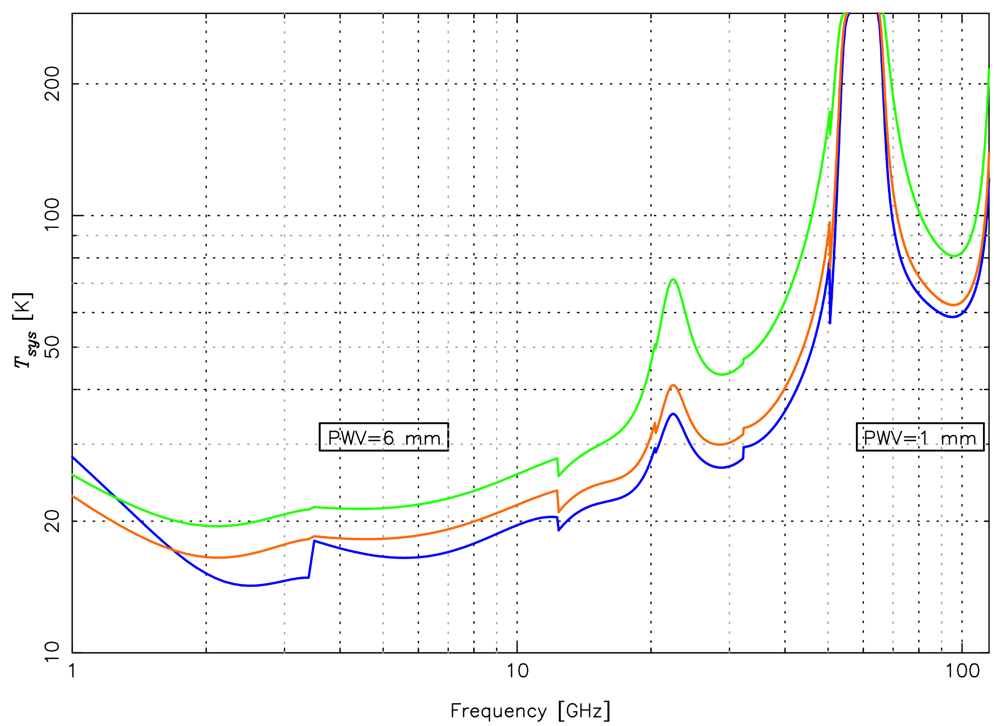

Figure 10. Calculation of Tsys vs. frequency, taking into account atmospheric emission, spillover, and receiver temperature. Color represents zenith angle: blue = 0^o; orange = 45^o; green = 75^o.

5. Conclusion

We have presented a complete description of how to calculate the antenna (system) temperature for ngVLA antennas, and shown some examples of both the sky contribution, and the total system temperature calculated in this way. We note that in order to turn this into a noise flux density that can be used to compare to the expected flux density of any source, one should correct this system temperature value to the top of the atmosphere, i.e., multiply by $e^{\tau_{atm}}$, because the source emission will be attenuated by that value as it passes through the atmosphere. Such a calculation also must include an aperture efficiency term (η_a). These are both described in Butler et al. (2019).

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