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ORBIT DETERMINATION REQUIREMENTS FOR VSOP AND RADIOASTRON

by

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ABSTRACT

Orbit determination requirements for VSOP and RADIOASTRON fall into two categories: prediction and reconstruction. Successful tracking of the spacecraft requires knowledge of the position and velocity 3–7 days in advance. Successful correlation of data from the spacecraft requires accurate reconstruction of the orbit with an ephemeris available 1–2 weeks after the epoch of observations. The derivation of these requirements is presented and the numerical values are given. The orbit prediction requirement for position is very stringent for VSOP and will probably prevent tracking for altitudes below about 2500 km.

INTRODUCTION

Five previous documents on space VLBI orbit determination [references 1–5] presented a variety of specifications, many of them incorrect for current mission parameters. This document updates earlier work and presents the complete orbit determination requirements of both missions, based on our best current knowledge of the spacecraft and ground systems. The altitude of perigee of the RADIOASTRON orbit has not yet been decided. A value of 5000 km has been used for the calculations presented here.

Orbit determination requirements fall into several categories. The first consists of orbital parameters which need to be known in advance of the observations, in order for the ground telemetry stations to successfully track the spacecraft. These will be referred to as *orbit prediction* requirements. All other requirements apply to knowledge of the orbit after the observations, used to analyze the data (correlation, fringe-fitting, and astrometry). These will be referred to as *orbit reconstruction* requirements. Orbit reconstruction requirements fall into two general classes. Astrometric and phase-reference observations have very stringent orbit determination requirements (typically a few cm to 1 m in position), and will not be treated in this memo. It appears very unlikely that either VSOP or RADIOASTRON could achieve useful scientific results in these areas (except for differential astrometry over small angular fields). Astronomical imaging and modeling observations, where correlated flux density and closure phase are the observables, have more modest orbit determination requirements, and will comprise the vast majority of observations with VSOP or RADIOASTRON. These requirements will be presented here.

The requirements for both orbit prediction and orbit reconstruction consist of allowed errors in one dimension. If the error ellipsoid is spherical, the corresponding three dimensional errors will be larger by a factor of $\sqrt{3}$.

ORBIT PREDICTION

In order to successfully track VSOP or RADIOASTRON (two-way phase transfer link and IF downlink), the position and velocity of the spacecraft must be known at the time of the observations. The position of the spacecraft must be known in order to point the ground telemetry antenna. Knowledge of both the postion and the velocity of the spacecraft is needed for the phase transfer process. The length of time required to send Doppler data from the ground telemetry stations to a central site, process the data into an ephemeris, and send predictions back to the telemetry stations for use in future tracking is estimated to be 3–7 days when done on a regular basis. Therefore, the following orbit prediction requirements should be met for an ephemeris extrapolated 3–7 days into the future.

One requirement on the position accuracy is set by the allowable signal loss due to pointing errors of the ground telemetry stations. This is important because the Ku-band IF downlink has a narrow power margin for both missions. The following specification has been adopted:

The pointing loss for a 3σ ephemeris error should be no more than 3 dB for a 10 m tracking antenna.

If we assume a Gaussian beam with full width at half maximum (FWHM) of 8 arcminutes for a 10 m antenna at 15 GHz, the allowed pointing error $\Delta\theta$ is 4.0 arcminutes (3σ). The resulting requirement on orbit prediction position accuracy Δx in the plane perpendicular to the orbiter-telemetry station direction is $\Delta x = d\Delta\theta$, where d is the range to the orbiter. Because the orbiter altitude is always less than or equal to the range, this requirement is satisfied if Δx is less than $400\text{m} \cdot (\frac{\text{Altitude}}{1000\text{km}})$ (1σ) for both missions. However, the phase-transfer process imposes a requirement on position knowledge which is approximately 10 times as stringent as this for the case of VSOP, and 4 times as stringent for RADIOASTRON.

The phase transfer process imposes accuracy requirements for orbit prediction due to 1) the allowed tolerance in the frequency of the on-board phase-lock loop and 2) the maximum round trip link phase residual which can be adequately sampled on the ground. J. Springett has given a requirement for the allowable error on the phase-lock loop of 20 Hz

(1σ). The sampling rate on the ground will be 400 Hz, so that the round-trip link phase residual can be as large as 200 Hz (3σ) and still allow sampling at least as often as the Nyquist rate. The phase-lock loop requirement is therefore more stringent.

The error $\Delta\nu_{received}$ in the on-board phase-lock loop frequency is

(1)
$$\Delta\nu_{received} = \frac{\Delta v_{predicted} \cdot \nu_{uplink}}{c}$$

ν_{uplink} is the uplink frequency for phase transfer (16.5 GHz for VSOP and 7.2 GHz for RADIOASTRON), $\Delta v_{predicted}$ is the error in the predicted radial velocity between the orbiter and the ground telemetry station, and c is the velocity of light. $\Delta v_{predicted}$ can be represented as the sum of two components. The first is due to an error in the orbiter position, which changes the orbiter-telemetry station direction and therefore the radial velocity. This component will be labeled $\Delta v_{position}$. The second component is due to an error in the orbiter velocity, such that the radial velocity would be in error even if the predicted position of the orbiter were perfect. This component will be labeled $\Delta v_{velocity}$. $\Delta v_{velocity} = v_t\Delta\theta$, where v_t is the component of the orbiter-telemetry station velocity which is transverse to the line of sight. The corresponding components of the frequency error in the phase-lock loop will be labeled $\Delta\nu_{position}$ and $\Delta\nu_{velocity}$.

(2)
$$\Delta\nu_{position} = \frac{v_t\Delta\theta \cdot \nu_{uplink}}{c} = \frac{v_t\Delta x \cdot \nu_{uplink}}{cd}$$

(3)
$$\Delta\nu_{velocity} = \frac{\Delta v_{velocity}\nu_{uplink}}{c}$$

It appears that the constraint on predicted position given in (2) will be much more difficult to meet than the constraint on predicted velocity given in (3). Therefore, 80% of the error budget will be allocated to $\Delta\nu_{position}$ and 20% will be allocated to $\Delta\nu_{velocity}$. The resulting 1σ errors $\Delta X_{1\sigma}$ and $\Delta V_{1\sigma}$ in the predicted orbit are

(4)
$$\Delta X_{1\sigma} = \frac{20\text{Hz} \cdot cd\sqrt{0.8}}{v_t\ \nu_{uplink}}$$

(5)
$$\Delta V_{1\sigma} = \frac{20\text{Hz} \cdot\ c\sqrt{0.2}}{\nu_{uplink}}$$

Both $\Delta X_{1\sigma}$ and $\Delta V_{1\sigma}$ are errors in one dimension. $\Delta X_{1\sigma}$ is the predicted position error along the velocity vector of the orbiter when it is projected onto the plane of the sky (as seen from the telemetry station). $\Delta V_{1\sigma}$ is the predicted velocity error along the range vector. The most stringent values of $\Delta X_{1\sigma}$ occur when the orbiter passes directly over the telemetry station at perigee. These values are 35 m for VSOP and 480 m for RADIOASTRON. The worst case value for VSOP is so severe that the dependence of $\Delta X_{1\sigma}$ upon location within the orbit must be studied. Table 1 gives the values for $\Delta X_{1\sigma}$ as a function of spacecraft altitude for both VSOP and RADIOASTRON. Two simplifications have been used to calculate these values, both of which make the entries in Table 1 worst-case values for those altitudes. First, v_t has been set equal to v_{orb}, the velocity of the orbiter with respect to the center of the earth. v_t will probably only ever exceed v_{orb} slightly, if at all, but will sometimes be substantially smaller. Second, the range has been set equal to the altitude. In practice the range can be significantly larger than the altitude. For an elevation limit of 10° at a telemetry station, the maximum range is 2800 km for an altitude of 1000 km, 4400 km for an altitude of 2000 km, and 5900 km for an altitude of 3000 km. The *typical* value of $\Delta X_{1\sigma}$ for a given altitude may therefore be larger than that given in Table 1 by a factor of 1.5–2. However, numerical studies are needed to calculate the true distribution of $\Delta X_{1\sigma}$ for realistic tracking scenarios. This would allow the determination of the loss in tracking coverage as a function of orbit prediction accuarcy.

It seems likely that tracking VSOP below an altitude of about 2500 km will be difficult or impossible. Because much of this data would be lost in any case because of a limited ground telemetry network, and because the *u-v* coverage generated by VSOP at these low altitudes will be duplicated by earth baselines, the science penalty of losing this data may be fairly mild. If the perigee of the RADIOASTRON orbit should be reduced below about 2500 km, tracking at perigee would not be possible.

The values of $\Delta V_{1\sigma}$ are 16 cm/s for VSOP and 37 cm/s for RADIOASTRON. These are listed in Table 2. Note that for altitudes where the position prediction error is significantly less than $\Delta X_{1\sigma}$, the velocity prediction error can be larger than the value in Table 2 by a factor as large as $\sqrt{5}$.

The requirement for the predicted value of acceleration arise from the details of the phase transfer process. More study is needed to determine these requirements. Preliminary calculations indicate that the requirement may be difficult to meet, especially for RADIOASTRON.

ORBIT RECONSTRUCTION

Correlation of VLBI data requires a knowledge of the vector baseline between two antennas, along with the time evolution of that baseline. The time evolution of the geometric delay, which is proportional to the baseline component parallel to the direction to the source, must be known accurately. *The specifications of the VLBA correlator have been used in the calculations for position and velocity requirements presented below. For other correlators, the values will be different.*

The 'standard continuum mode' of the VLBA correlator will have 64 delay lags per frequency channel. Due to the Nyquist sampling used in VLBI, the spacing $\Delta\tau$ of the delay lags is $\Delta\tau = 1/(2\mathrm{BW})$, where BW is the bandwidth of each frequency channel. VSOP has 16 MHz channels, resulting in a lag spacing of 31 nsec and a delay window of 2 μsec (full width). RADIOASTRON has channel bandwidths of 2–8 MHz, so that the delay window will be at least 4 μsec (full width). Assume a minimum buffer of 4 lags between the peak of the delay spectrum and the edge of the correlator window, and allow for the possibility that the initial fringe-search in an experiment could occur when the (time-dependent) position error is at a maximum (*i.e.* the peak-to-peak delay range must fit between the center of the correlator window and 4 lags from its edge). The allowed peak-to-peak range (3σ) in delay is then 28 lags, which equals $\pm 0.44\mu$sec for VSOP and $\pm 0.88\mu$sec for RADIOASTRON. The corresponding one dimensional position errors (along the source direction) are 45 m (1σ) for VSOP and 90 m (1σ) for RADIOASTRON. These position requirements for orbit reconstruction are given in Table 3.

The orbit reconstruction requirements on velocity arise from the need to limit the output rate from the correlator. The specification for the VLBA correlator is a maximum output rate of 0.5 Mbyte/s. This in turn imposes a minimum correlator integration time (this is the duration of time over which the correlator averages its output, and should not be confused with the much longer coherent integration time used during fringe-fitting). The correlator integration time imposes a maximum residual fringe rate which can occur without loss of signal-to-noise ratio (S/N). For a correlator integration time τ_{corr} and a residual fringe rate ν_{res}, the fractional loss $\Delta S/N$ of S/N is

$$\Delta S/N = \mathrm{sinc}\,(\pi \tau_{corr} \nu_{res}) \qquad (6)$$

$\mathrm{sinc}(x) \equiv (\sin x)/x$. The residual fringe rate is determined by the velocity error in the direction of the source.

In the calculations below, information provided by J. Romney regarding the VLBA correlator output rate for various configurations has been used. For a 20 station spectral line experiment with 1024 frequency bins (2048 delay lags) in 1 frequency channel, the minimum correlator integration time is 3.5 s. For a 20 station continuum experiment with 64 lags per channel and 8 channels, the minimum integration time is 0.92 s. Note that the number of 'baselines' saved by the correlator is equal to $n(n+1)/2$, where n is the number of stations. This exceeds the true number of baselines $n(n-1)/2$ because of n 'self-spectra' taken by the correlator. It has been assumed that the integration time on baselines between ground telescopes will be 5 times longer than on baselines between the orbiter and a ground telescope. The allowed maximum S/N loss (3σ) due to high fringe rate for these calculations is 5% (there is no associated calibration error). The calculations have allowed for the possibility that the initial fringe-search will center the fringe-rate window at one extreme of the fringe-rate variation (*i.e.* the peak-to-peak variation in fringe-rate must lie in the region between the center of the fringe-rate window and its edge). For water maser spectral line experiments, 0.2 km/s velocity resolution has been assumed to be adequate (this is the best velocity resolution available with the 16 MHz channels of VSOP, but RADIOASTRON could achieve a factor of 2 better with its 8 MHz channels, at the expense of a tighter velocity requirement).

The calculations required a crucial assumption about the capability of the VLBA correlator which is not yet determined. This assumption concerns the possible existence of an output filter which can increase the correlator integration time by a factor of 4 with no loss in S/N. This output filter is currently being designed by NRAO. Its cost is unknown. Depending upon the results of the design process, this filter may be implemented by NRAO in the initial construction of the correlator, it may be left as an option which could be constructed with some funding from NASA, or it may be dropped from the correlator design entirely. Results are therefore presented for two different cases: 1) output filter *not* available 2) output filter available. These two cases could be considered as 'guaranteed' and 'optimistic' respectively.

Table 4 presents the one dimensional velocity requirements (along the source direction) for a number of different types of experiments. I believe that this list includes the most demanding experiments which are likely to be made with either mission. The most stringent velocity requirements arise from 22 GHz spectral line observations of water maser sources. Table 4 also lists a number of experiment types for which the velocity requirements are quite loose. The numerical values for these cases may be of use in estimating the orbit knowledge required for a real-time fringe search.

For observations at a wavelength λ over an integration time τ, the coherence C is

$$C = 1 - \frac{\pi^2 a^2 \tau^4}{360\lambda^2} + \frac{\pi^4 a^4 \tau^8}{362880\lambda^4} + O\left(\frac{a^6\tau^{12}}{\lambda^6}\right) \tag{7}$$

a is the error in the orbit acceleration (along the source direction) used in the correlator model. The error introduced by using only the constant and quadratic (in a) terms of this formula is 0.1% for $C = 95\%$, 0.5% for $C = 90\%$, and 1.5% for $C = 80\%$.

Table 5 gives the requirements for the one dimensional acceleration knowledge in the reconstructed orbits for both VSOP and RADIOASTRON at all their observing frequencies. An integration time of 300 s has been used for all calculations. The assumed requirements on coherence for a 3σ orbit determination error used for these calculations are included in

Table 5. Note that these values include only coherence losses due to orbit reconstruction errors, and do not include losses in the phase transfer process. For VSOP at 22 GHz, a coherence value of 99% is specified in Table 5. This is lower than the value of 99.5% which has been set as the requirement for the coherence of the phase transfer link. The class of observations for which a 99.5% coherence is required consist of mapping strong sources. For these observations, a somewhat shorter integration time (250 s) can be used to give a coherence of 99.5% without requiring a more accurate acceleration knowledge. However, an acceleration knowledge of $2.2 \times 10^{-8} \mathrm{m/s}^2$ is *desired* for these observations, to allow the use of a 300 s integration time. Because observations with RADIOASTRON are expected to be primarily survey observations, the required calibration accuracy is less stringent, and the corresponding acceleration requirements are looser. If it should prove possible to obtain very good *u-v* coverage with RADIOASTRON using observations near perigee, then the requirements listed for VSOP would apply. Note that the requirements on acceleration in the orbit reconstruction do not depend on any details of the correlator, unlike the situation for position and velocity.

Because of the difficulty of tracking VSOP at low altitudes, the requirements for the reconstructed orbit of VSOP given in Tables 3–5 will probably not need to be met for altitudes below about 2500 km.

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REFERENCES

[1] "Orbit determination requirements for VSOP and RADIOASTRON," R. Linfield, Nov. 21, 1989, JPL IOM 335.6–89–115.

[2] "VSOP orbit determination requirements," R. Linfield, in *Proceedings of VSOP Symposium* (in press)

[3] "Revised orbit determination requirements for VSOP and RADIOASTRON," R. Linfield, Jan. 2, 1990, JPL IOM 335.6–90–1.

[4] "Orbit prediction requirements for VSOP," R. Linfield, April 11, 1990, JPL IOM 335.6–90–17.

[5] "Corrected acceleration requirements for space VLBI orbit determination," R. Linfield, Aug. 27, 1990, JPL IOM 335.6–90–31.

Table 1

Orbit Prediction Requirements In Position

(3–7 days before observations)

Altitude	Position Requirement in 1-D (1σ) (VSOP)	Position Requirement in 1-D (1σ) (RADIOASTRON)
1000 km	35 m	
1500 km	55 m	
2000 km	76 m	
2500 km	100 m	
3000 km	125 m	
3500 km	150 m	
4000 km	180 m	
5000 km	240 m	480 m
7000 km	380 m	740 m
10000 km	650 m	1200 m
15000 km	1300 m	2000 m
20000 km	2500 m	3000 m
40000 km		10000 m
60000 km		30000 m

Table 2

Orbit Prediction Requirements in Velocity

Mission	Velocity Requirement in 1-D (1σ)
VSOP	16 cm/s
RADIOASTRON	37 cm/s

Table 3

Orbit Reconstruction Requirements In Position Along the Source Direction

Mission	Position Requirement in 1-D (1σ)
VSOP	45 m
RADIOASTRON	90 m

Table 4

Orbit Reconstruction Requirements In Velocity Along the Source Direction

Mission	Experiment Type	1-D Requirement without output filter (1σ)	1-D Requirement with output filter (1σ)
	CONTINUUM		
VSOP	5 station 1.67 GHz	70 cm/s	270 cm/s
VSOP	20 station 1.67 GHz	8 cm/s	33 cm/s
VSOP	20 station 5 GHz	2.7 cm/s	11 cm/s
VSOP	20 station 22 GHz	0.6 cm/s	2.5 cm/s
VSOP	10 station 22 GHz	2 cm/s	8 cm/s
RADIOASTRON	5 station 0.33 GHz	170 cm/s	700 cm/s
RADIOASTRON	5 station 1.67 GHz	34 cm/s	140 cm/s
RADIOASTRON	5 station 5 GHz	11 cm/s	46 cm/s
RADIOASTRON	10 station 5 GHz	4.2 cm/s	17 cm/s
RADIOASTRON	5 station 22 GHz	2.6 cm/s	10 cm/s
RADIOASTRON	10 station 22 GHz	1.0 cm/s	3.8 cm/s
	SPECTRAL LINE		
VSOP	10 station 22 GHz	0.13 cm/s	0.51 cm/s
VSOP	5 station 22 GHz	0.34 cm/s	1.4 cm/s
RADIOASTRON	4 station 22 GHz	0.91 cm/s	3.6 cm/s

Table 5

Orbit Reconstruction Requirements In Acceleration Along the Source Direction

Mission	Observing Frequency	Required Coherence for 3σ orbit error	Acceleration Requirement in 1-D (1σ)
VSOP	1.67 GHz	99.5%	$2.8 \times 10^{-7} m/s^2$
VSOP	5 GHz	99.5%	$9.5 \times 10^{-8} m/s^2$
VSOP	22 GHz	99%	$3.0 \times 10^{-8} m/s^2$
RADIOASTRON	0.33 GHz	98%	$2.9 \times 10^{-6} m/s^2$
RADIOASTRON	1.67 GHz	98%	$5.7 \times 10^{-7} m/s^2$
RADIOASTRON	5 GHz	98%	$1.9 \times 10^{-7} m/s^2$
RADIOASTRON	22 GHz	95%	$6.8 \times 10^{-8} m/s^2$