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JET PROPULSION LABORATORY **INTEROFFICE MEMORANDUM**

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To: J. Smith

From: R. Linfield

Subject: Several potential error sources for round trip Doppler measurements in space VLBI

SUMMARY

This memo presents calculations of the Doppler on the phase transfer link between a ground telemetry station and an orbiter. Contributions from two physical effects (Retarded Doppler and Relativistic Doppler) and two types of parameter errors (the clock and the location of the ground telemetry station) are considered. The Retarded Doppler and Relativistic Doppler effects are sufficiently large that they must be included in the model used to generate the uplink and downlink phases. The Doppler contributions from clock errors at the telemetry stations are too small to cause problems. The Doppler contributions from station location errors are acceptable if the station locations are known to within 11 m (VSOP) and 65 m (RADIOASTRON).

ORBIT PARAMETERS AND NAMING CONVENTIONS

The VSOP orbit assumed in this memo has a perigee altitude of 1000 km, an apogee altitude of 20000 km, a maximum velocity (at perigee) of 9.2 km/s, and a maximum acceleration (at perigee) of 7.3 m/s^2. The RADIOASTRON orbit has a perigee altitude of 5000 km, an apogee altitude of 66700 km, a maximum velocity (at perigee) of 7.8 km/s, and a maximum acceleration (at perigee) of 3.1 m/s^2.

For the round trip phase transfer, define the epoch of broadcast from the ground as t_0, the epoch of reception (and transponding) at the spacecraft as t_1, and the epoch of reception at the ground as t_2. The time-dependent locations of the orbiter and the ground telemetry station are $\mathbf{X}_{orb}(t)$ and $\mathbf{X}_{tel}(t)$. The distance between the orbiter and the ground telemetry station is ρ. In this memo, vectors will be designated with bold type face.

RETARDED DOPPLER

Because the signal is in contact with the spacecraft at only one epoch (t_1), the only spacecraft parameters which affect the round trip phase are the position and velocity at that epoch. However, the position and velocity of the ground telemetry station at both t_0 and t_2 affect the round trip phase. Specifically, $\dot{\mathbf{X}}_{orb}(t_1) - \dot{\mathbf{X}}_{tel}(t_0)$ and $\dot{\mathbf{X}}_{tel}(t_2) - \dot{\mathbf{X}}_{orb}(t_1)$ determine the two way link Doppler. Previous work [1] assumed that $\dot{\mathbf{X}}_{tel}(t_0) = \dot{\mathbf{X}}_{tel}(t_1) = \dot{\mathbf{X}}_{tel}(t_2)$, ignoring the acceleration of the ground station during the two-way link delay. This assumption introduces a one way "Retarded Doppler" error ΔD_{ret} on board the orbiter of

$$\Delta D_{ret} = -\left(\dot{\mathbf{X}}_{tel}(t_1) - \dot{\mathbf{X}}_{tel}(t_0)\right) \cdot \frac{(\mathbf{X}_{orb}(t_1) - \mathbf{X}_{tel}(t_1))}{\|\mathbf{X}_{orb}(t_1) - \mathbf{X}_{tel}(t_1)\|} \tag{1}$$

This error cannot be calibrated with round trip link phase measurements, because $\dot{\mathbf{X}}_{tel}(t_1) - \dot{\mathbf{X}}_{tel}(t_0) \approx \dot{\mathbf{X}}_{tel}(t_2) - \dot{\mathbf{X}}_{tel}(t_1)$. The errors in the uplink and downlink models will therefore approximately cancel. The position of a ground telemetry station as a function of time is

$$\mathbf{X}_{tel}(t) = R_{spin}[\cos(\Omega t - \gamma)\hat{\mathbf{x}} + \sin(\Omega t - \gamma)\hat{\mathbf{y}}] + Z\hat{\mathbf{z}},$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are unit vectors in a geocentric, non-rotating frame, R_{spin} is the distance between the telemetry station and the spin axis of the earth, γ is the west longitude of the telemetry station, Z is the distance of the station northward from the plane of the equator, and $\Omega = (2\pi/86400)\mathrm{s}^{-1}$ is the rotation rate of the earth. The acceleration of the ground station due to earth rotation is

$$\ddot{\mathbf{X}}_{tel}(t) = -\Omega^2 R_{spin}[\cos(\Omega t - \gamma)\hat{\mathbf{x}} + \sin(\Omega t - \gamma)\hat{\mathbf{y}}] \tag{2}$$

The magnitude of the acceleration is

$$\|\ddot{\mathbf{X}}_{tel}(t)\| \approx \Omega^2 R_{\oplus} \cos\phi,$$

where $R_{\oplus}$ is the radius of the earth and ϕ is the latitude of the station (the relation would be an equality for a uniformly rotating spherical earth). Therefore, the maximum value for $\|\dot{\mathbf{X}}_{tel}(t_1) - \dot{\mathbf{X}}_{tel}(t_0)\|$ is

$$\|\Delta\dot{\mathbf{X}}_{max}\| \approx \Omega^2 R_{\oplus} \cos\phi \frac{\rho_{max}}{c}, \tag{3}$$

where ρ_{max} is the maximum distance between the ground telemetry station and the orbiter. Setting ρ_{max} equal to the height of apogee and assuming that $\Delta\ddot{\mathbf{X}}_{tel}$ is along the telemetry station-orbiter direction gives:

$$\Delta D_{ret_{max}} \approx 0.22 \text{ cm/s } \cos\phi \qquad \text{(VSOP)} \tag{4}$$

(5) $$\Delta D_{ret_{max}} \approx 0.75 \text{ cm/s } \cos\phi \qquad \text{(RADIOASTRON)}$$

Because the magnitude of the Retarded Doppler effect is comparable to the maximum allowed velocity error for the reconstructed orbit in the most stringent case [2], it must be avoided by the used of the correct model at the station. Since the predicted orbit state vectors are supplied in a geocentric frame, this will be feasible.

RELATIVISTIC DOPPLER

Previous work [1] has assumed that the ratio D of received to transmitted frequency is $D = 1 - \dot\rho/c$, where c is the velocity of light. This is true only in the low velocity limit. The relativistic formula for the ratio D_{rel} of received to transmitted frequency is

(6) $$D_{rel} = \frac{1}{\Gamma(1+\beta\cos\theta)},$$

where $\beta = \dot\rho/c$, Γ is the Lorentz gamma factor $\left(\Gamma = 1/\sqrt{1-\beta^2}\right)$, and θ is the angle between the vector from the ground telemetry station to the orbiter and the corresponding relative velocity vector. For $\theta = 90°$ the correction in the two way Doppler due to special relativity is

$(1-\beta^2) - (1-\beta)$

$(\beta + \beta^2)$

$\beta(1-\beta)$

(7) $$\Delta D_{rel} = (D_{rel}^2 - D^2)\, c = \left[\frac{1}{\Gamma^2} - (1-\beta)\right] c = \beta(1-\beta)\, c = \dot\rho\,(1-\beta).$$

At perigee, when the spacecraft velocity is at its maximum,

(8) $$\Delta D_{rel_{max}} = 28 \text{ cm/s} \qquad \text{(VSOP)}$$

(9) $$\Delta D_{rel_{max}} = 20 \text{ cm/s} \qquad \text{(RADIOASTRON)}$$

Depending on the exact non-relativistic approximation used for the link phase models, part or all of this would be removed by round trip link phase measurements. However, the magnitude of ΔD_{rel} is comparable to the maximum allowed two-way residual Doppler [3], and should be avoided by the used of the correct relativistic model.

CLOCK ERRORS AT THE GROUND TELEMETRY STATION

An error Δt in the clock at a ground telemetry station will cause an error ΔD_{clock} in the round trip Doppler of

$$\Delta D_{clock} = 2\Delta t \|\ddot{\mathbf{X}}_{orb} - \ddot{\mathbf{X}}_{tel}\| \cos\theta, \text{ where} \tag{10}$$

$$\cos\theta = \frac{\left(\ddot{\mathbf{X}}_{orb} - \ddot{\mathbf{X}}_{tel}\right) \cdot (\mathbf{X}_{orb} - \mathbf{X}_{tel})}{\|\ddot{\mathbf{X}}_{orb} - \ddot{\mathbf{X}}_{tel}\| \ \ \|\mathbf{X}_{orb} - \mathbf{X}_{tel}\|}$$

Setting $\theta = 0$ and $\|\ddot{\mathbf{X}}_{orb} - \ddot{\mathbf{X}}_{tel}\|$ equal to $\|\ddot{\mathbf{X}}_{orb_{max}}\|$ gives

$$\Delta D_{clock_{max}} \approx 1.5 \times 10^{-3}\text{cm/s/}\mu\text{sec} \qquad \text{(VSOP)} \tag{11}$$

$$\Delta D_{clock_{max}} \approx 6.2 \times 10^{-4}\text{cm/s/}\mu\text{sec} \qquad \text{(RADIOASTRON)} \tag{12}$$

This is too small to cause a problem for clock errors of 30 μsec or less.

STATION LOCATION ERRORS

To calculate the effect of an error $\Delta\mathbf{X}_{tel}$ in the knowledge of the telemetry station location on the predicted velocity of the orbiter, begin with the expression for the range ρ:

$$\rho = \sqrt{\mathbf{X}_{orb} \cdot \mathbf{X}_{orb} + \mathbf{X}_{tel} \cdot \mathbf{X}_{tel} - 2\,\mathbf{X}_{orb} \cdot \mathbf{X}_{tel}}$$

$$\dot{\rho} = \frac{\mathbf{X}_{orb} \cdot \dot{\mathbf{X}}_{orb} + \mathbf{X}_{tel} \cdot \dot{\mathbf{X}}_{tel} - \mathbf{X}_{orb} \cdot \dot{\mathbf{X}}_{tel} - \dot{\mathbf{X}}_{orb} \cdot \mathbf{X}_{tel}}{\rho}$$

$$\frac{\partial \dot{\rho}}{\partial \mathbf{X}_{tel}} = \frac{\left(\dot{\mathbf{X}}_{tel} - \dot{\mathbf{X}}_{orb}\right) + (\mathbf{X}_{tel} - \mathbf{X}_{orb}) \cdot \frac{\partial \dot{\mathbf{X}}_{tel}}{\partial \mathbf{X}_{tel}} - \dot{\rho}\frac{\partial \rho}{\partial \mathbf{X}_{tel}}}{\rho} \tag{13}$$

In order to set an upper limit to the right side of (13), note that

$$\left\| \frac{\partial \dot{\mathbf{X}}_{tel}}{\partial \mathbf{X}_{tel}} \right\| \leq \Omega$$

At perigee, $\dot{\mathbf{X}}_{orb} \approx 20\ \Omega\ \mathbf{X}_{orb}$ for VSOP, and $\dot{\mathbf{X}}_{orb} \approx 10\ \Omega\ \mathbf{X}_{orb}$ for RADIOASTRON. Therefore, the second term on the right side of (13) is much smaller than the first, and will be neglected.

$$\left\| \frac{\partial \rho}{\partial \mathbf{X}_{tel}} \right\| = 1$$

$$\dot{\rho} \leq \|\dot{\mathbf{X}}_{tel} - \dot{\mathbf{X}}_{orb}\|$$

Therefore, (13) can be reduced to the following approximate upper limit:

(14)
$$\left\| \frac{\partial \dot{\rho}}{\partial \mathbf{X}_{tel}} \right\| < \frac{2\|\dot{\mathbf{X}}_{tel} - \dot{\mathbf{X}}_{orb}\|}{\rho} \approx \frac{2\ \|\dot{\mathbf{X}}_{orb}\|}{\rho}$$

The change $\Delta\dot{\rho}_{stat}$ in $\dot{\rho}$ due to an uncertainty in the telemetry station location is:

(15)
$$\Delta\dot{\rho}_{stat} < \frac{2\ \|\dot{\mathbf{X}}_{orb}\|\ \|\Delta\mathbf{X}_{tel}\|}{\rho}$$

The maximum value $\Delta\dot{\rho}_{stat_{max}}$ occurs at perigee, so that

$$\Delta\dot{\rho}_{stat_{max}} < \frac{2\|\Delta\mathbf{X}_{tel}\|}{h_{per}} v_{per},$$

where h_{per} is the altitude of perigee and v_{per} is the velocity of the orbiter at perigee. Using $\Delta\mathbf{X}_{tel}(\mathrm{m}) \equiv \Delta\mathbf{X}_{tel}/1\ \mathrm{m}$,

(16)
$$\Delta\dot{\rho}_{stat_{max}} < \|\Delta\mathbf{X}_{tel}(\mathrm{m})\|\ 1.8\ \mathrm{cm/s} \qquad \text{(VSOP)}$$

(17)
$$\Delta\dot{\rho}_{stat_{max}} < \|\Delta\mathbf{X}_{tel}(\mathrm{m})\|\ 0.31\ \mathrm{cm/s} \qquad \text{(RADIOASTRON)}$$

Setting this component to be 1/2 of the total velocity error budget of 40 cm/s [3] gives maximum allowed station location errors of

(18)
$$\|\Delta\mathbf{X}_{tel_{max}}\| \approx 11\ \mathrm{m} \qquad \text{(VSOP)}$$

(19)
$$\|\Delta\mathbf{X}_{tel_{max}}\| \approx 65\ \mathrm{m} \qquad \text{(RADIOASTRON)}$$

The locations of all the telemetry stations are expected to be known more accurately than this. However, it should be noted that the correction UT1−UTC must be included in the link phase model. Otherwise, an apparent station location error $\Delta\mathbf{X}_{tel_{ap}}$ of

$$\Delta\mathbf{X}_{tel_{ap}} = R_{\oplus}\Omega\cos\phi[\mathrm{UT1}-\mathrm{UTC}] = 463\ \mathrm{m}[\mathrm{UT1}-\mathrm{UTC}](\mathrm{s})\cos\phi$$

results. As UT1−UTC can be as large as 0.5 s, failure to include it in the model would cause the round trip residual Doppler to be outside the allowed value much of the time.

REFERENCES

[1] "A Discussion of Two-Way Doppler Compensation Principles for OVLBI," J. Springett, Aug. 23, 1990, NeoComm Systems Technical Memorandum.

[2] "Revised orbit determination requirements for VSOP and RADIOASTRON," R. Linfield, Jan. 2, 1990, JPL IOM 335.6–90–1.

[3] "Orbit prediction requirements for VSOP," R. Linfield, April 11, 1990, JPL IOM 335.6–90–17.

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