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JET PROPULSION LABORATORY INTEROFFICE MEMORANDUM

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To: Space VLBI Team

From: R. Linfield

Subject: DATA LOSS AS A FUNCTION OF ORBIT PREDICTION ACCURACY FOR VSOP

SUMMARY

The effect of orbit prediction errors upon loss of data from VSOP is quantified. The fractional data loss is largest at perigee, and drops rapidly as the altitude increases. The total loss of science data is very small, even for 8-day advance predictions. The currently planned sampling rate at ground tracking stations of 400 Hz will be completely adequate for tracking VSOP. (No data loss occurs for Radioastron, due to its higher perigee and lower link frequencies.)

INTRODUCTION

On-board local oscillator phase for VSOP will be transferred from a ground tracking station to the spacecraft via a two-way link. This phase transfer process requires that the true downlink phase be compared to its predicted value, and the difference (residual) recorded. These measured residual phases will be used in VLBI data correlation and in the calculation of total Doppler values for (non real time) orbit determination. The currently planned design for DSN ground stations involves sampling the downlink residual phase 400 times per second. This in turn places an upper limit $\Delta\nu_{down_{max}}$ of 175 Hz on the downlink frequency error $\Delta\nu_{down}$ in order to allow successful tracking (J. Springett, private communication). An earlier document [1] gave requirements on the accuracy of the predicted orbit used to generate the predicted downlink phase. These requirements were based on simplistic calculations of worst-case tracking geometries, rather than on a requirement to allow tracking for a specified fraction of the time. A recent study of the orbit prediction capability using Doppler tracking with the DSN [2] concluded that meeting the position prediction requirement will be difficult, and will not be possible in all cases. Specifically, the worst-case values from [1] can be met only 1 day in advance. A more careful study of the requirements is presented in this memo.

THE EFFECT OF GEOMETRY UPON TRACKING PARAMETERS

The downlink frequency error $\Delta\nu_{down_x}$ due to an error Δx in the predicted position of VSOP is

$$\Delta\nu_{down_x} = \frac{2v_{orb}\Delta x \, \sin^2\omega \cdot \nu_{down}}{cd} \tag{1}$$

v_{orb} is the velocity of the orbiter relative to that of the tracking station, ω is the angle between the spacecraft-tracking station position vector and the spacecraft-tracking station velocity vector, c is the velocity of light, d is the distance between the tracking station and the orbiter (also known as the 'slant-range'), and ν_{down} is the downlink frequency (14.2 GHz for VSOP). Equation (1) assumes that the position error Δx of the predicted orbit lies entirely along the path of the orbiter (an 'along-track' error). In practice, this is approximately true (C. Christensen, private communication). Furthermore, for the geometry of nearly all cases of interest, an along-track error will cause a larger contribution to $\Delta\nu_{down_x}$ than will a position error along either of the two orthogonal directions ('cross-track' and 'radial'). Therefore, assuming purely along-track position errors should give fairly accurate results for $\Delta\nu_{down_x}$. A further approximation will be to neglect the geocentric velocity of the tracking station (maximum value of 400 m/s for tracking stations at least 30° from the equator) compared to the geocentric velocity of the orbiter (> 6.8 km/s for altitudes below 5000 km).

The downlink frequency error $\Delta\nu_{down_v}$ due to an error Δv in the predicted velocity of VSOP is

$$\Delta\nu_{down_v} = \frac{2\Delta v \, \cos\omega \cdot \nu_{down}}{c} \tag{2}$$

The velocity error has been assumed to be purely along-track. In fact, the velocity error ellipsoid is more spherical than the position error ellipsoid. However, $|\Delta\nu_{down_x}| > 10 \; |\Delta\nu_{down_v}|$ in nearly all cases (*i.e.* the position error dominates), so that the specific treatment of velocity errors has only a minor effect on the final results. Because only the magnitude of the downlink frequency error is of concern (see eqn. (8) and (9)), the sign conventions for $\Delta\nu_{down_x}$ and $\Delta\nu_{down_v}$ are not important. Gaussian distributions of Δx and Δv, with zero mean, have been assumed.

Figure 1 shows the tracking geometry. $R_\oplus$ is the radius of the earth, h is the altitude of the orbiter, el is the elevation angle of the orbiter, as seen from the tracking station, θ is the angle between the tracking station and the sub-earth point, as seen from the orbiter, η is the angle between the velocity vector of the orbiter and a purely tangential velocity vector (*i.e.* one which would maintain a constant altitude), and ϕ (not shown on the diagram) is the azimuthal angle, defined as the angle between the plane containing the center of the earth, the tracking station, and the orbiter, and the plane containing the center of the earth and the orbiter velocity vector. ψ is the angle between the orbiter and the tracking station, as seen from the center of the earth. From the law of cosines,

$$(R_\oplus + h)^2 = R_\oplus^2 + d^2 - 2dR_\oplus \cos(el + 90°)$$

$$d_{max} = \sqrt{R_\oplus^2 \sin^2 el_{min} + h^2 + 2R_\oplus h} - R_\oplus \sin el_{min} \tag{3}$$

el_{min} is the minimum elevation angle for the tracking station and d_{max} is the slant range to the orbiter for el_{min}. The maximum value ψ_{max} of ψ is

$$\cos\psi_{max} = \frac{2R_\oplus^2 + 2R_\oplus h + h^2 - d_{max}^2}{2R_\oplus(R_\oplus + h)} \tag{4}$$

For $\psi \leq \psi_{max}$,

$$d = \sqrt{(R_\oplus + h)^2 + R_\oplus^2 - 2R_\oplus(R_\oplus + h)\cos\psi} \tag{5}$$

$$\cos\theta = \frac{2R_\oplus h + h^2 + d^2}{2(R_\oplus + h)d} \tag{6}$$

$$\cos\omega = \sin\theta\ \cos\phi\ \cos\eta + \cos\theta\ \sin\eta \tag{7}$$

Except for its sign, η is a function only of h.

In order to calculate the fraction of data lost (due to $\Delta\nu_{down} > \Delta\nu_{down_{max}}$) as a function of orbiter altitude h, a two dimensional numerical integration (with uniform weighting) was performed over the area on the earth where the orbiter is visible at altitude h. (*i.e.* where $el \geq el_{min}$). This is equivalent to performing an ensemble average over all possible tracking geometries (at altitude h) at a given tracking station. For each integration element, the values of $\Delta\nu_{down_x}$ and $\Delta\nu_{down_v}$ from 1σ errors in both position (σ_x) and velocity (σ_v) were calculated from eqns. (1)–(7). Two different procedures were used to calculate the total downlink frequency error $\Delta\nu_{down_{tot}}$. In the first, the position and velocity errors were assumed to be uncorrelated, so that the two components of $\Delta\nu_{down}$ were added in quadrature:

$$\Delta\nu_{down_{tot}}(uncorr) = \sqrt{\Delta\nu_{down_x}^2 + \Delta\nu_{down_v}^2} \tag{8}$$

In the second procedure, the position and velocity errors were assumed to be perfectly correlated, and the components of $\Delta\nu_{down}$ were added, using the same sign for both:

$$\Delta\nu_{down_{tot}}(corr) = |\Delta\nu_{down_x}| + |\Delta\nu_{down_v}| \tag{9}$$

These two procedures could be roughly characterized as 'best-case' and 'worst-case', respectively. Because the position error dominates $\Delta\nu_{down_{tot}}$, the 'correlated' and 'uncorrelated' results are fairly similar. The fraction of data lost for an integration element is erfc $\left[\Delta\nu_{down_{max}} / \left(\Delta\nu_{down_{tot}}\sqrt{2}\right)\right]$, where erfc is the Gaussian error function and $\Delta\nu_{down_{tot}}$ can be either the correlated or uncorrelated value.

$$\mathrm{erfc}x \equiv \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2}\, dt$$

The factor of $\sqrt{2}$ arises because of the difference between the exponent in a Gaussian distribution with unit standard deviation and the exponent in the erfc integrand. Note that both the positive and negative tails of the Gaussian distribution are included in the calculation of data loss.

The values of σ_x and σ_v were derived from reference [2] and from more recent work which used a finer time grid. (J. Estefan, private communication). This latter work provided detailed information on the dependence of σ_x and σ_v with altitude. Empirical fits to these results, valid for 1000 km $< h <$ 10000 km, were used in the calculations of data loss.

$$\sigma_x(h) \approx \sigma_x(per)\left[1 - 8.69 \times 10^{-4}\left(\frac{h - 1000\text{ km}}{1\text{ km}}\right)^{0.686}\right]$$

$$\sigma_v(h) \approx \sigma_v(per)\left[1 - 3.33 \times 10^{-2}\left(\frac{h - 1000\text{ km}}{1\text{ km}}\right)^{0.343}\right]$$

$\sigma_x(per)$ and $\sigma_v(per)$ are the values of σ_x and σ_v at perigee.

RESULTS

The fractional data loss as a function of altitude h is given for four cases in Table 1 for uncorrelated position and velocity errors, and for the same four cases in Table 2, but with correlated position and velocity errors. A brief description of each case is given at the bottom of the two tables. The results for correlated and uncorrelated position and velocity errors are fairly similar, differing by only $\sim 20\%$. The fractional data loss is largest at perigee, and drops off rapidly at higher altitudes. A summary of the actual amount of tracking time which would be lost due to imperfect orbit predictions is presented in Tables 3 and 4. The first row in both Tables 3 and 4 gives the total amount of time lost (on average) per orbit. Rows 2 and 3 give the total amount of time lost per orbit at altitudes above 2000 and 3000 km, respectively. For comparison, the average total tracking time per orbit is 4–5 hours. Therefore, the integrated loss of tracking data at altitudes > 2000 km is < 1% of the total tracking time, even for the worst case studied here (8-day advance predicts with correlated position and velocity errors). The integrated loss of tracking data at altitudes > 3000 km is < 0.5% of the total tracking time. Data lost due to spacecraft eclipses by the earth are not included in the 4–5 hour total. However, these eclipses will diminish the tracking coverage at all altitudes approximately equally (on average) and will therefore cause only minor changes in the fractional losses given by Tables 1–4.

The values in Tables 3 and 4 reflect the amount of time that the orbiter spends at different altitudes, as well as the fraction of time during which the orbiter is visible from the network of ground tracking stations, again as a function of altitude. The fraction of the earth's surface over which the orbiter is visible (*i.e.* $el \geq el_{min}$) as a function of altitude, for both $el_{min} = 10°$ and $el_{min} = 6°$, is given in Table 5. This fraction is given by the formula $(1 - \cos\psi_{max})/2$. For VSOP, the percentage visibility as a function of altitude (ignoring spacecraft constraints) can be estimated by multiplying the values in Table 5 by a factor of 4 (3 DSN stations and Kagoshima — Green Bank adds very little coverage), and this estimate was used for the calculations presented in Tables 3 and 4. For altitudes of 1000–2000 km, this estimate of the tracking visibility is too low by as much as $\approx 10\%$ because the polar regions of the earth (where there are no tracking stations) are never visible. The values in row 1 of Tables 3 and 4 are therefore slight underestimates.

The loss of tracking data at altitudes below approximately 3000 km will cause very little science penalty because: 1) Tracking in this region may be impossible for other reasons, such as dumping of momentum wheels on the orbiter 2) u–v smearing may result in this data being unusable 3) Space-ground baselines at these low altitudes will not provide any data which cannot be obtained in other ways. The u-v coverage can be obtained from ground baselines, while crossing points (overlap between space-ground and ground-ground coverage, needed for calibration) can be obtained from data taken at altitudes of 4000–6000 km.

The numerical value of the critical altitude, below which data loss is of little consequence, is not known. It probably lies in the range 2000–4000 km. A better determination of this value (perhaps from imaging simulations) would help in selecting the maximum orbit prediction period to be employed for VSOP (*e.g.* if data down to 2000 km are desired, the maximum prediction period might be 7 days, while if data below 4000 km can be discarded, prediction periods as long as 9–10 days could be used).

In Tables 1 and 2, the different cases are described as '4-day advance,' '6-day advance,' and '8-day advance.' These prediction periods refer to the duration of time between the end of the tracking used for orbit determination and the epoch of tracking for which the value of data loss is desired. This differs from the definition used in [2]. The '8-day advance' values are derived from an extrapolation of the curves provided by J. Estefan, which extended only up through 7 days.

J. Ulvestad made many helpful comments on a draft of this memo. C. Christensen suggested including Tables 3 and 4, and also calculated the values in these two tables.

REFERENCES

[1] "Orbit determination requirements for VSOP and RADIOASTRON," R. Linfield, June 6, 1991.

[2] "Japanese and DSN Orbit Prediction Capability for the VSOP Mission," J. Estefan, C. Christensen, and J. Ellis, JPL IOM 314.5–1551, July 1, 1991.

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Table 1

Fractional Data Loss Due to Orbit Prediction Errors for Uncorrelated Position and Velocity Errors

Altitude (km)	Case 1	Case 2	Case 3	Case 4
1000	19%	41%	36%	55%
1200	14%	34%	30%	49%
1400	9.9%	29%	26%	44%
1600	7.0%	25%	22%	40%
1800	4.8%	20%	18%	35%
2000	3.2%	17%	15%	30%
2200	2.0%	14%	12%	27%
2400	1.2%	11%	9.4%	24%
2600	0.7%	8.5%	7.3%	20%
2800	0.4%	6.5%	5.6%	17%
3000	0.2%	4.8%	4.2%	14%
3500	<0.1%	2.1%	1.8%	8.7%
4000		0.7%	0.6%	4.8%
4500		0.2%	0.2%	2.3%
5000		<0.1%	<0.1%	0.9%
5500				0.3%
6000				<0.1%

Case 1: 4-day advance prediction (400 m, 32 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 2: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 3: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 6°$
Case 4: 8-day advance prediction (1050 m, 90 cm/s 1σ errors at perigee), $el_{min} = 10°$

Table 2

Fractional Data Loss Due to Orbit Prediction Errors for Correlated Position and Velocity Errors

Altitude (km)	Case 1	Case 2	Case 3	Case 4
1000	22%	46%	42%	61%
1200	16%	39%	35%	54%
1400	12%	33%	30%	49%
1600	8.3%	28%	25%	45%
1800	5.8%	24%	21%	40%
2000	3.9%	20%	18%	36%
2200	2.5%	16%	14%	32%
2400	1.6%	13%	12%	28%
2600	1.0%	11%	9.3%	25%
2800	0.5%	8.3%	7.3%	21%
3000	0.3%	6.4%	5.6%	18%
3500	<0.1%	3.0%	2.6%	12%
4000		1.2%	1.0%	7.0%
4500		0.4%	0.4%	3.9%
5000		<0.1%	<0.1%	1.9%
5500				0.9%
6000				0.4%
6500				0.1%
7000				<0.1%

Case 1: 4-day advance prediction (400 m, 32 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 2: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 3: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 6°$
Case 4: 8-day advance prediction (1050 m, 90 cm/s 1σ errors at perigee), $el_{min} = 10°$

Table 3

Total Data Loss Per Orbit Due to Orbit Prediction Errors for Uncorrelated Position and Velocity Errors

Altitude Range	**Case 1**	**Case 2**	**Case 3**	**Case 4**
All	0.6 minutes	2.0 minutes	2.2 minutes	3.7 minutes
> 2000 km	0.1 minutes	0.6 minutes	0.6 minutes	1.6 minutes
> 3000 km	< 0.1 minutes	0.1 minutes	0.1 minutes	0.6 minutes

Case 1: 4-day advance prediction (400 m, 32 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 2: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 3: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 6°$
Case 4: 8-day advance prediction (1050 m, 90 cm/s 1σ errors at perigee), $el_{min} = 10°$

Table 4

Total Data Loss Per Orbit Due to Orbit Prediction Errors for Correlated Position and Velocity Errors

Altitude Range	**Case 1**	**Case 2**	**Case 3**	**Case 4**
All	0.7 minutes	2.3 minutes	2.6 minutes	4.4 minutes
> 2000 km	0.1 minutes	0.7 minutes	0.8 minutes	2.1 minutes
> 3000 km	< 0.1 minutes	0.2 minutes	0.2 minutes	0.9 minutes

Case 1: 4-day advance prediction (400 m, 32 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 2: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 10°$
Case 3: 6-day advance prediction (720 m, 58 cm/s 1σ errors at perigee), $el_{min} = 6°$
Case 4: 8-day advance prediction (1050 m, 90 cm/s 1σ errors at perigee), $el_{min} = 10°$

Table 5

Fraction Of Earth's Surface Visible From Different Altitudes

Altitude (km)	Fraction Of Surface Which Is Visible ($el_{min} = 10°$)	Fraction Of Surface Which Is Visible ($el_{min} = 6°$)
1000	3.5%	4.6%
1200	4.3%	5.5%
1400	5.1%	6.4%
1600	5.9%	7.3%
1800	6.6%	8.1%
2000	7.3%	8.9%
2200	8.0%	9.7%
2400	8.7%	10.5%
2600	9.4%	11.2%
2800	10.0%	11.9%
3000	10.6%	12.5%
3500	12.0%	14.1%
4000	13.3%	15.5%
4500	14.5%	16.8%
5000	15.6%	18.0%
5500	16.6%	19.0%
6000	17.5%	20.0%
6500	18.4%	21.0%
7000	19.2%	21.8%
7500	20.0%	22.6%

Figure 1
Tracking Geometry