OVLBI-ES MEMO NO. 23

Gravitational Effects on OVLBI Timing Link

Randy Hudson* D.A. Czenkusch

December 30, 1991

Very Long Baseline Interferometry (VLBI) requires an extremely precise timing reference at each receive site so that coherence can be assured during later correllation of the recorded signal. In OVLBI, when the actual receiver is an orbiting satellite, practical considerations require the timing reference to remain on the ground and to transmit its timing signal to the satellite. The satellite then transmits to the ground a second reference which is phase related to the uplink. The ground station uses this second timing signal to resolve uncertainties in the parameters of the two-way timing link.[1]

Because of the stringent phase stability requirement, many phenomena which affect this two-way timing link must be taken into account.[2,3] Most phenomena which affect the link are reciprocal; they produce the same effect on the uplink as they do on the downlinked signal. This note identifies one phenomenon with a complementary effect on the uplink and downlink paths.

It is a well known consequence of general relativity that the apparent frequency of an electromagnetic wave is affected by the gravitational potential of the observer. If a wave is transmitted from location T with frequency f_T and is received at location R, then an observer at R will see a wave with apparent frequency f_R given by:

$$\frac{f_R}{f_T} = \frac{1 - U_R}{1 - U_T} \tag{1}$$

Where we can use $U_X = -\frac{GM}{c^2 R_X}$, the Newtonian gravitational potential.[4] G is the gravitational constant, 6.673×10^{-11} nt-m^2/kg^2; R_X is the radius from the center of mass; M is the mass of the object producing the gravitational field; and c is the velocity of light, 3×10^8m/s.

If $-U_X \ll 1$ then the change in frequency is given by:

$$f_R - f_T = f_T(U_T - U_R) \tag{2}$$

In this example, we are concerned with the potential at R and T caused by a single mass $M_e = 5.976 \times 10^{24}$kg – the mass of the earth. The earthstation is located approximately $R_R = 6.37 \times 10^6$m from the center of the earth, while the maximum distance of the Russian satellite is about $R_T = 7.5 \times 10^7$m. The frequency difference is thus:

$$f_R - f_T = f_T \frac{GM}{c^2} \frac{R_T - R_R}{R_T R_R} \tag{3}$$

The timing downlink frequency is $f_T = 8.47$ GHz, so $f_R - f_T = 5.4$ Hz. This is of the same order as other effects which were previously identified as being significant. This is a complementary effect; the uplink experiences a *redshift* while the downlink experiences a *blueshift*, so there should be no observable effect on the two-way time.

It should be noted that constant effects on the timing path are irrellevant, as they are accounted for in later signal processing. Effects which vary during a tracking pass are important; however, they can be removed to the extent they can be predicted. The question thus becomes: What are the errors involved in computing the gravitational frequency shifts? They are the uncertainty in the orbit of the spacecraft and the variability in the gravitational field of the Earth. An estimate of these errors will be the subject of a later note.

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*National Radio Astronomy Observatory, Green Bank WV

†Hewlett-Packard Corp., Voltmeter division, Loveland CO

[1] L.R. D'Addario, "Time Synchronization in Orbiting VLBI," *IEEE Trans. Instr. & Meas.*, vol. IM-40, pp. 584-590, 1991.

[2] L.R. D'Addario, "Two-Way Timing Measurement With Uplink Compensation," OVLBI-ES Memo No.11, 24 April 1991.

[3] R. Linfield, "Several Potential Error Sources for Round-Trip Doppler Measurements in Space VLBI," OVLBI-ES Memo No.7, 31 Aug. 1991.

[4] Theodore Frankel *Gravitational Curvature*, W.H.Freeman, 1979