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SYSTEM PHASE NOISE REQUIREMENTS

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1. GENERAL THEORY

Our earth station's electronics should degrade the coherence of the space-ground interferometer by a negligible amount compared with other effects. This is the basic requirement. It is affected mainly by the phase stability of the electronics in the two-way timing path.

It is convenient to divide the phase stability specifications into short-term (<1 sec) and long term (>1 sec) parts [Weinreb 1983]. The long term part should be dominated by the drift in our knowledge of the hydrogen maser's phase; this will be true if we correctly compensate for delay changes in the cable, and if phase shifts induced in the electronics by temperature and antenna elevation changes are sufficiently small. (There is some dispute between NRAO and JPL on the maximum time scale for which such long term drifts are significant. JPL has specified to Scientific Atlanta that all drifts on time scales >300 sec can be ignored, no matter how large; we contend that any drift over a full tracking pass must be considered.) In this memo, we consider only the short-term phase stability, often called "phase noise."

If we let $1-\epsilon$ be the coherence of the interferometer, where ϵ is called the coherence loss, we can decide on an acceptable value for ϵ and allocate parts of it to various causes in the system, including the atmosphere, ionosphere, spacecraft, and ground equipment. If the instantaneous system phase error is ϕ, then

$$1-\epsilon = \langle \cos\phi \rangle \approx 1 - \langle \phi^2 \rangle /2.$$

The phase error can be regarded as an error voltage δV added to the ideal signal, with $\delta V/V_0 = \phi$, where V_0 is the magnitude of the complex amplitude of the signal. Then the total relative power in the phase noise is

$$P_n/P_0 = \langle (\delta V/V_0)^2 \rangle = \langle \phi^2 \rangle.$$

Note that the various contributions to the phase noise, if independent of each other, sum in the mean square. Somewhat arbitrarily, we budget $\epsilon_g < .01$ for the ground portion of the coherence loss, and we budget equal amounts of this for the uplink and downlink parts of the electronics, and take them to be independent. In addition, we must account for the fact that phase errors on the two-way timing link are scaled to the observing frequency on the spacecraft; for the worst-case observing frequency of 22 GHz, the scale factors are 22/15.3 for VSOP and 22/7.2 for Radioastron.

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Using all of the above leads to the specification

$$\begin{aligned}
\phi_{rms} &< 0.14\,\text{radian at 22 GHz} \\
&< 0.09\,\text{radian at 15.3 GHz} \\
&< 0.046\,\text{radian at 7.2 GHz} \\
\phi_{rms,1W} &< 0.033\,\text{radian worst case} \\
P_n/P_0 &< -29.9\,\text{dB worst case.}
\end{aligned}$$

The phase noise generally occurs on a range of time scales, and as mentioned we are concerned here with time scales <1 sec, or noise frequencies >1 Hz. On the uplink, both satellites will lock to the signal using PLLs with noise bandwidths the order of 1 kHz, so phase noise at frequencies greater than this is not significant unless it is very large. Similarly, on the downlink the GBES phase detector has a bandwidth limit of 1.8 kHz. To allow some margin, we take the above specification to apply from 1 Hz to 5 kHz. To ensure that no surprises occur from non-linear effects in wideband parts of the electronics (prior to the phase detectors), we impose the additional spec that the phase noise from 1 Hz to 1 MHz should not exceed −10 dBc.

2. Measurable Quantities

It is possible to measure roughly the phase noise of a nominally sinusoidal signal using a spectrum analyzer. We observe how much power is scattered into sidebands around the signal, assuming that all of it is due to phase noise (and not thermal or amplitude-only noise). For this to be meaningful, the phase noise of the spectrum analyzer's L.O. must be negligible. A better measurement can be made by applying the signal to a phase detector having a well controlled, variable input bandwidth and a very low noise reference.

Here we investigate what the power spectrum should look like if our specs are to be met.

In our system, we are concerned mainly with oscillators that are in phase locked loops. Outside the loop bandwidth, the phase noise spectral density is essentially that of the unlocked oscillator; inside, it usually falls off as $|f - f_0|^2$ (for a second-order loop [Gardner 1979]) until the noise of the reference signals begins to dominate. If the unlocked oscillator has a $1/|f - f_0|^2$ spectrum and the references are perfect, we get a total phase noise power (see Fig. 1) of

$$P_n = (8/3)S(f_n)f_n$$

where f_n is the noise bandwidth of the loop, and $S(f)$ is the oscillator's noise PSD at offset f. If $f_1 < f_n$ is our close-in spec limit (5 kHz) and $f_2 > f_n$ is our far-out spec limit (1 MHz), then the noise for $f < f_1$ is

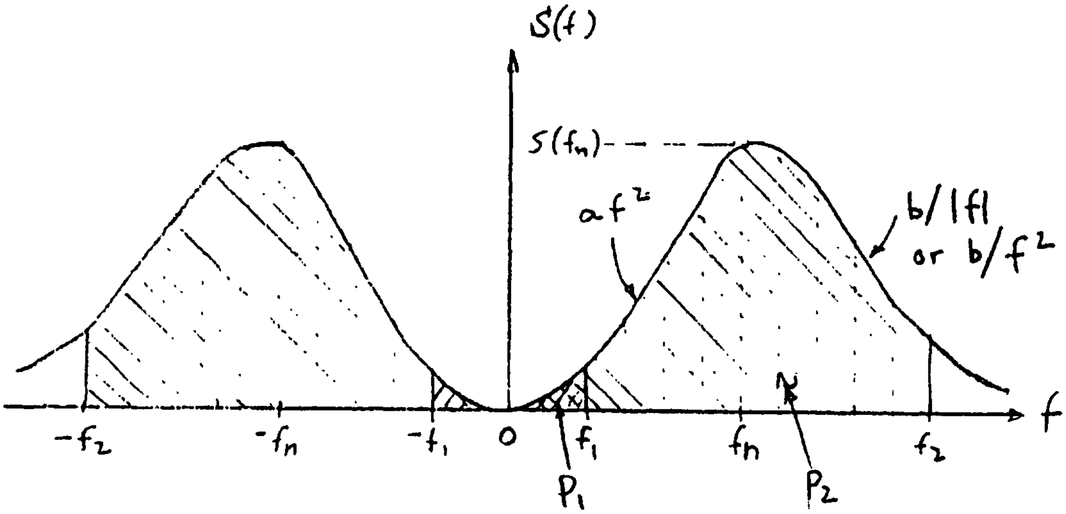
$$P_1 = (2/3)S(f_n)(f_1^3/f_n^2) = (2/3)S(f_n)f_n(f_1/f_n)^3$$

and the noise for $f < f_2$ is

$$P_2 = 2S(f_n)f_n(4/3 - f_n/f_2).$$

However, if the unlocked oscillator's noise does not fall off so rapidly but instead has a $1/|f - f_0|$ form, we find

$$P_{2a} = 2S(f_n)f_n[1/3 + \log(f_2/f_n)].$$



$$P_1 = 2\int_0^{f_1} S(f)\,df = 2\int_0^{f_1} S(f_n)\left(\frac{f}{f_n}\right)^2 df;$$

$$P_2 = 2\int_0^{f_2} S(f)\,df = \frac{2}{3} f_n S(f_n) + 2\int_{f_n}^{f_2} S(f_n)\left|\frac{f_n}{f}\right|^K df,$$

$$(K = 1 \text{ or } 2)$$

Fig. 1: Typical power spectrum of a locked oscillator, where f is the offset from the carrier frequency. P_1 and P_2 are the close-in and wideband noise powers considered in the text; they are derived by integrating the functions shown here.

All of these results come from integrating the double-sided spectra.

Let's evaluate these expressions using numbers that are typical of our system. Let f_n = 50 kHz, f_1 = 5 kHz, f_2 = 1 MHz, and S(50kHz) = −80 dBc/Hz. Then

$P_1 = -64.7$ dBc,

$P_2 = -28.9$ dBc, and

$P_{2a} = -24.7$ dBc.

All of these are well within our specifications.

For the P_2 spec, a worst-case requirement can be set by assuming that the loop bandwidth will always be at least 1 kHz. Then the $P_2 < -10$dBc spec is met for any

$$S(f_n)f_n < -21.6\text{dB}, f_n > 1\text{kHz}.$$

Thus, oscillators with noise < -51.6 dBc/Hz at 1 kHz offset or < -71.6 dBc/Hz at 100 kHz offset are needed at the corresponding loop bandwidths.

The $P_1 < -30$ dBc spec can be met for almost any oscillator by selecting sufficiently large loop bandwidth. For example, if the oscillator has a $1/f^2$ spectrum with −50 dBc/Hz noise at 10 kHz, then we need f_n >55 kHz.

Compared with available oscillators, these requirements are remarkably loose. If met, they will result in the noise inside 5 kHz being dominated by the references.

3. Reference Signals

The reference signals to which the PLLs are locked originate at the hydrogen maser.

Very close to the carrier (within a few Hz), we should certainly be tracking the maser very accurately; but from there to a few kHz there could be degradation from the transmission system. Flat-spectrum additive noise as large as −70 dBc/Hz within 5 kHz would still allow meeting the spec. However, the reference for our microwave oscillators is a harmonic of a 500 MHz signal transmitted from the maser. Phase noise power degrades as N^2 for harmonic number N. Thus, at 15 GHz we need to have the 500 MHz noise 30^2=29.5 dB lower, or −99.5 dBc/Hz. This should be easily achievable in the optical fiber system, but it needs to be verified.

Additional references in the system are at 10 MHz and 100 MHz. The 10 MHz is transmitted by optical fiber and is re-synchronized to the 500 MHz, so it should have the same close-in phase noise as the 500 MHz. The 10 MHz reference is multiplied to 500–700 MHz in the second LO (640 MHz for RA, 610 for VSOP), causing a degradation of 34 to 37 dB, tightening the flat noise limit on the received 500 MHz to −107 dBc/Hz. The 100 MHz reference is derived from the 10 MHz in a PLXO, but the resulting 100 MHz is not further multiplied; therefore the requirements it imposes on the references are not as stringent.

References

Weinreb, S., 1983, "Short-term phase stability requirements for interferometer coherence." Electronics Division Internal Report No. 233, NRAO, Charlottesville, Virginia 22903.

Gardner, F., 1979, *Phaselock Techniques* (2nd Ed.). John Wiley & Sons, New York.