Lecture Notes

NRAO Lecture Program -- Summer 1964

RADIO INTERFEROMETRY

by

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Introduction

The structural limitations on the size of large radio telescopes limit the angular resolution of these instruments. Hence, we must either look for other methods of obtaining high resolution observations (lunar occultations) or we must artificially build an antenna of the large aperture required ("aperture synthesis"). The fundamental telescope in aperture synthesis is the interferometer. The method of obtaining the equivalent very large aperture beam is to obtain the phase and amplitude of each interferometer record, and add all interferometer records together as a Fourier series[(1)]. For convenience we will consider a one-dimensional antenna.

If we consider a 300-foot one-dimensional radio antenna operating at λ = 10 cm, the "fan-beam" width (to half-power) is $\approx \frac{\lambda}{d}$ radians $\simeq$ 4 arc minutes. If we wish to obtain narrower beams at a given wavelength, we have to increase the diameter of the antenna. For a 10 arc second fan-beam at λ = 10 cm, a 7200-foot long antenna is required!

The Fourier Transformation

It is necessary to consider the properties of Fourier transformations in order to obtain a clearer picture of high resolution techniques in radio astronomy. A simple account of Fourier transformations is given by Jennison[(2)]. More rigorous treatments are given by Arsac[(3)] and Sneddon[(4)]. From the law of superposition of electromagnetic waves, the far-field pattern of an antenna on the sky is the Fourier transformation of the aperture (illumination) function of the antenna. Similarly, the Fourier transformation of brightness distribution of a source (at infinity) is the field distribution (due to the source) on the observing plane. In the same way that the conjugate Fourier quantities in electrical theory are <u>time</u> and <u>frequency</u>, so are the conjugate Fourier quantities in interferometry <u>angle</u> and <u>spatial frequency</u>.

Let us consider some general properties of the Fourier transformation. For <u>any</u> antenna of one-dimensional angular beam pattern $G(\theta)$, observing a source with one-dimensional brightness distribution $T(\theta)$, the recorded output power is given by

$$R(\theta_o) = \int_{\text{all } \theta} G(\theta-\theta_o)\ T(\theta)\ d\theta \tag{1}$$

where θ_o represents the direction in which the power is $G(\theta-\theta_o)$. This is known as a "convolution", and is sometimes abbreviated to

$$R(\theta_o) = G(\theta_o) * T(\theta).$$

The Fourier transformation of $R(\theta_o)$ is

$$r(u) = g(u) \cdot t(u) \tag{2}$$

where u is the spatial frequency and $g(u)$ and $t(u)$ are the Fourier transformations of G and T, respectively. For the Fourier pair $r(u)$ and $R(\theta)$, we have the important general relationship.

$$r(u) = g(u) \cdot t(u)$$
$$R(\theta) = G(\theta) * T(\theta)$$

This may be compared with the resultant two linear electrical filters. In this case we have

$$f(\omega) = f_1(\omega) \cdot f_2(\omega)$$
$$F(t_o) = F_1(t_o) * F_2(t)$$

$F(t_o)$ is known as the (unnormalized) cross-correlation coefficient, and is the Fourier transformation of the resultant filter function $f(\omega)$. Because of the close resemblance between $f(\omega)$ and $r(u)$, and due to the limited spatial frequency band observed at one time by an interferometer, we may consider the interferometer to be a spatial frequency filter.

Interferometer Geometry

Let us consider a two-antenna interferometer whose individual antennas are on an East-West baseline. Interferometer fringe maxima will be observed when path differences (via each antenna) from source to central point are an integral number of wavelengths (fig. I). The instrumental meridian of the two antenna interferometer is the direction in which the paths (via each antenna) are equal. The meridian is sometimes referred to as the position of the "central fringe". For East-West interferometer baselines, sources of all declinations pass through the local meridian and interferometer meridian simultaneously (see fig. II); for non-East-West baselines, the coincidence of meridians only occurs for sources which pass through the local zenith (i.e., whose declination is equal to geographic latitude).

Away from the instrumental meridian two important effects occur.

(a) The apparent baseline, as seen by the source, is reduced. This is known as "baseline forshortening", and is proportional to cos θ (see fig. I).

(b) For sources at declinations other than zero, non-meridian observations result in an apparent rotation of the projected interferometer fringes and baseline (on the source). We call this angle ρ: ~~it is~~ the angle between celestial North-South and the baseline projection on the source is $90° - \rho$.

~~First~~ fringes drawn on the celestial sphere comprise a series of semi-circular arcs, whose planes are parallel (see fig. II). Since sources at all declinations move at the same hour-angle rate, and fringes at higher declinations have larger hour-angle separations, the rate at which a source crosses the fringes is lower at higher declinations. If δ is the source declination, the rate of crossing the fringes is proportional to cos δ sin ρ.

For an East-West interferometer, the rotation of the projected baseline is given by

$$\tan \rho = \cos \delta \cot H$$

where H is the hour angle of the source.

Example. An interferometer comprises two antennas on an East-West baseline. Determine the observed fringe frequency when viewing a source of $\delta = 30°$

two hours before meridian transit ($H = 0$). The maximum fringe frequency (at $\delta = 0$, $H = 0$) is 0.2 cycle per second.

$H = 2$ hours, i.e., $\theta = 30°$

$$\text{Hence fringe rate} = 0.2 \cos\delta \sin\rho \cos\theta$$

$$= 0.2 \cos^2(30°) \sin\rho$$

$$\tan\rho = \operatorname{cosec} 30° \cot 30°$$

$$\rho = 73° \; 54'$$

$$\text{fringe rate} = \underline{0.144} \text{ cycles per second}$$

The Treatment of Interferometer Data

The interferometer output is fringes, whose phase and amplitude must be retained in order that all spatial frequency contributions may be combined to give the synthetic antenna beam. Again, we consider a one-dimensional antenna.

Since the interferometer is a spatial frequency filter, its output represents one component of the Fourier transform of the brightness distribution. In fig. III several one-dimensional source distributions are shown, together with their Fourier transformations: the transformation amplitudes are known as "fringe visibility curves". For zero spatial frequency the fringe visibility is unity ($V = 1$).

The Interferometer System

The N.R.A.O. interferometer system is shown in block schematic form in fig. IV. Analog multiplication followed by integration is equivalent to correlation, and Blum[5] has shown that two advantages accrue from correlation (rather than addition and detection). They are

(a) a slight improvement in system signal-to-noise ratio, and

(b) large reduction in system gain fluctuations.

In order to fix interferometer fringes relative to the baseline, signal and image bands are both accepted prior to superheterodyning the R.F. It has been shown theoretically (by Read[6]) and experimentally that, for equal response to signal and image bands, sources may be tracked by inserting delays in the I.F. circuit, whilst fringes remain fixed on the sky.

Interferometer Signal-to-Noise Ratio

The output of the interferometer is a sine wave of amplitude proportional to the unresolved source flux: i.e., proportional to fringe visibility and total source flux density. Two typical records are shown in fig. V. The system noise (assumed gaussian) has an r.m.s. value proportional to $\tau^{-1/2}$ for a given I.F. bandwidth, whose τ is the integration time. For a sine wave plus gaussian noise, the maximum signal-to-noise ratio for a given source and system is when

$$\tau = \frac{\text{sine wave period}}{2\pi}$$

assuming integration is in a low pass RC filter. For the present N.R.A.O. interferometer (July 1964), the signal-to-noise ratio is

$$\frac{T_A}{\Delta T_{RMS}} = \frac{3.2\ VS}{\sqrt{\tau}}$$

where T_A is the amplitude of the interferometer fringes, ΔT_{RMS} is the r.m.s. noise, V is the fringe visibility, S is the total source flux density (in flux units) and τ is the integration time constant in seconds.

Final Note

No attempt has been made here to consider interferometer baselines oblique to East-West, as at N.R.A.O., due to the considerable complexity of the geometry. Further information on the N.R.A.O. interferometer may be found in reference (7).

References

(1) Ryle, M., Nature, 180, 110 (1957)

(2) Fourier Transforms and Convolutions for the Expermentalist, R. C. Jennison; Pergamon (1961)

(3) Transformation de Fourier et Théorie des Distributions, J. Arsac; Dunod (1961)

(4) Fourier Transforms, I. N. Sneddon, McGraw-Hill (1951)

(5) Blum, E. J., Annales d'Astrophysique, 22, 139-162 (1959); English translation at N.R.A.O.

(6) Read, R. B., Astrophysical Journal, 138, 1 (1963)

(7) Keen, N. J., N.R.A.O. Electronics Division Internal Report, No. 14 (1963)

Examples

1. An interferometer uses two 85-foot antennas (as at NRAO) separated by 900 meters on an East-West baseline. The operating wavelength is 1420 MHz. Determine the fringe rate for a radio-source at $\delta = 0°$

    (a) at instrumental meridian transit angle (zero hour angle);

    (b) 3 hours away from instrumental meridian transit.

2. Determine the fringe rate for the above interferometer observing a source of $\delta = 40°$ at 4 hours from the instrumental meridian transit. Why may we not use this point on an East-West visibility curve?

3. For the interferometer in Example 1, calculate

    (a) the time constant to give maximum signal-to-noise ratio in 1(a) and 2;

    (b) the signal-to-noise ratio for a point source of 8 flux units in 1(a) and 2, using these time constants.

4. Determine the time constant and signal-to-noise ratio in case 1(b) where the source has 80 flux units, and is in fact two equal point sources separated by one minute of arc in an East-West direction. Repeat the calculation with a source of equal flux, which is in fact a double gaussian in an East-West direction with half-power widths and separation equal to one minute of arc.

[Hint: Use fig. IV in both cases, remembering that the scale on the brightness distribution varies inversely as the scale on the visibility curve.]

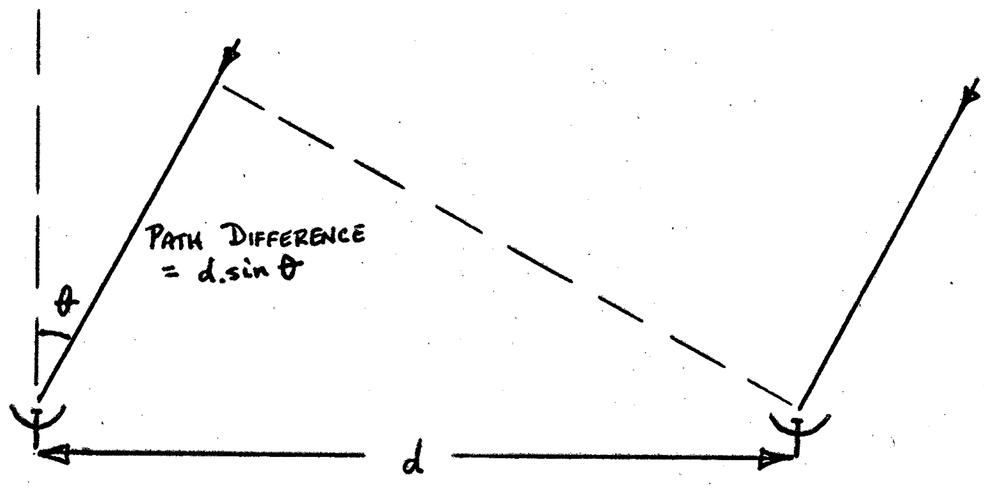

FIGURE I

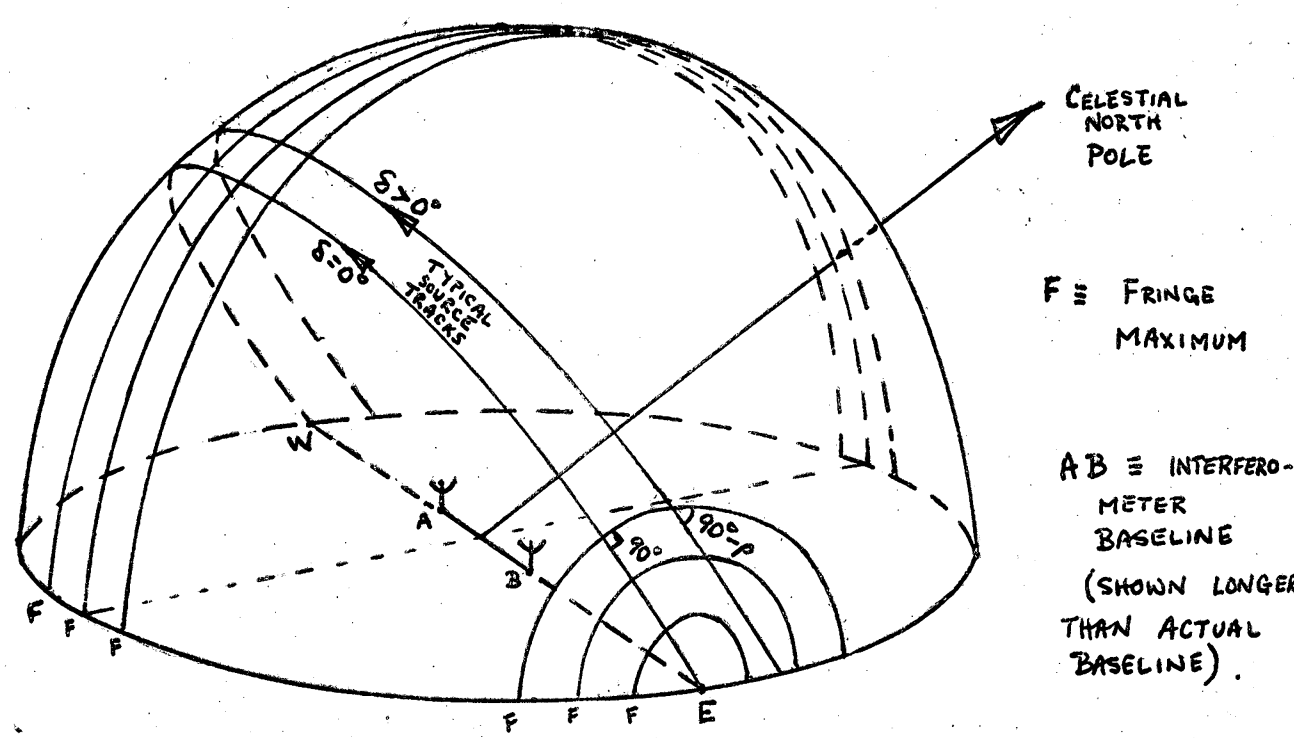

FIGURE II

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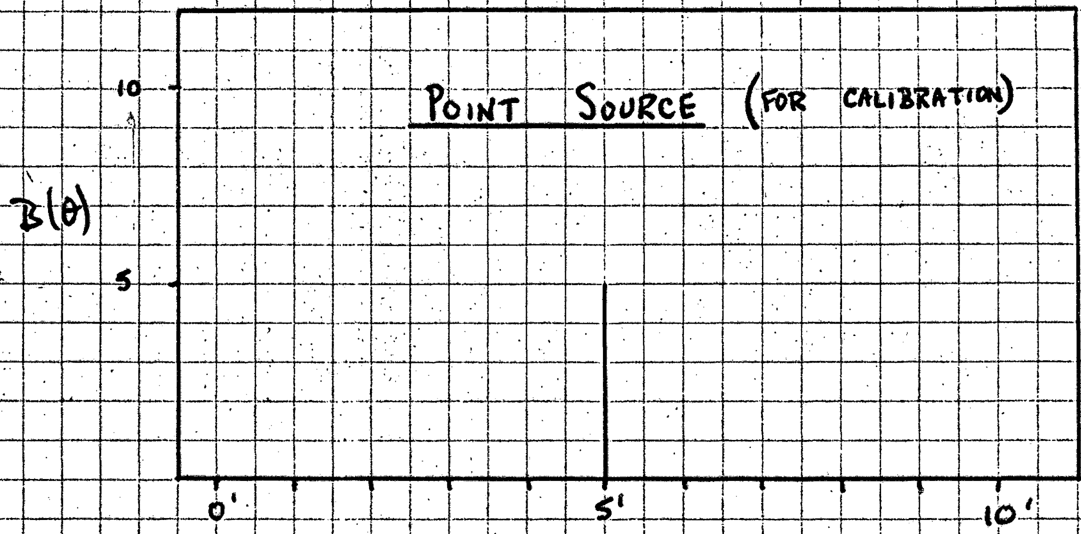
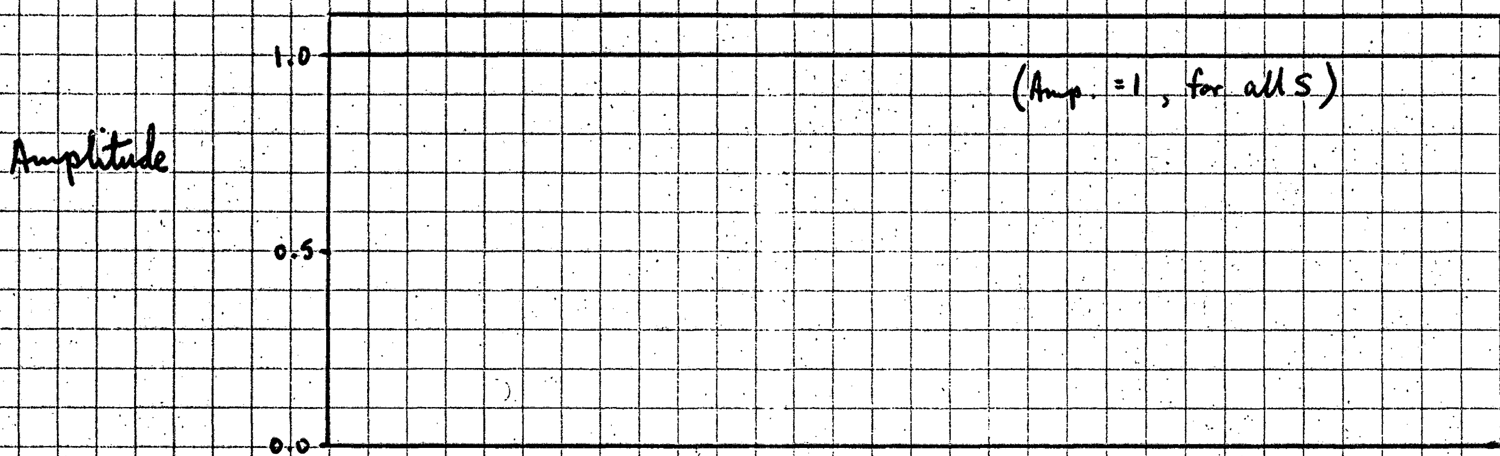
FIGURE III (a)

Source Model



Fourier Transform



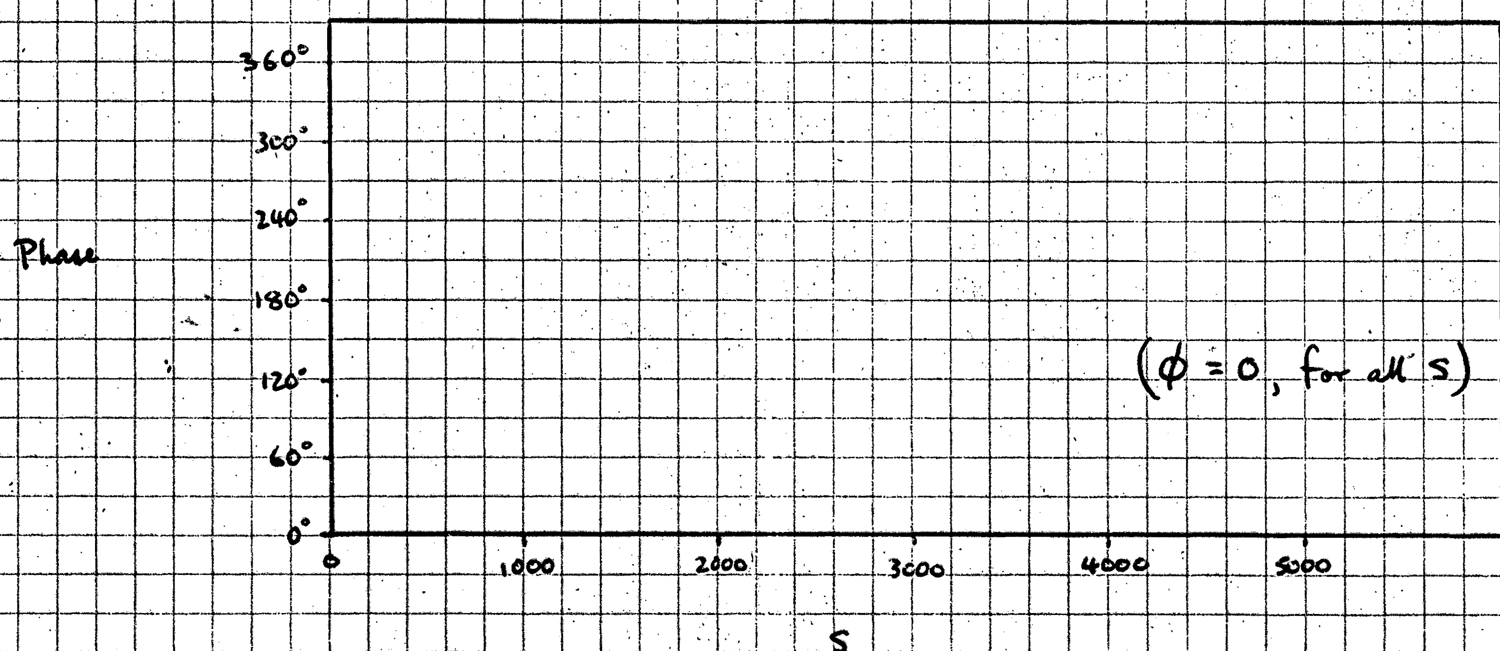
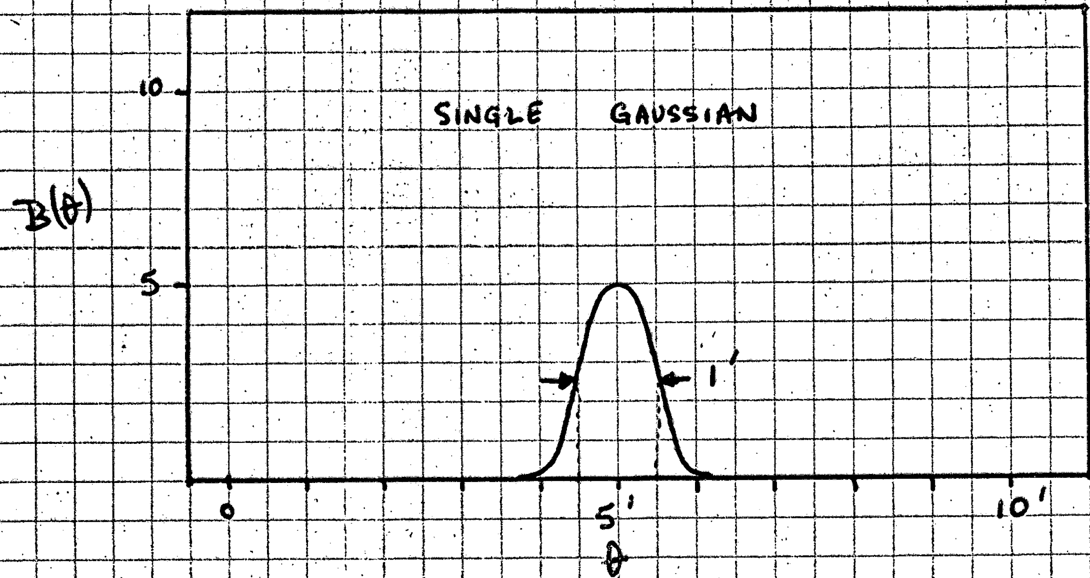
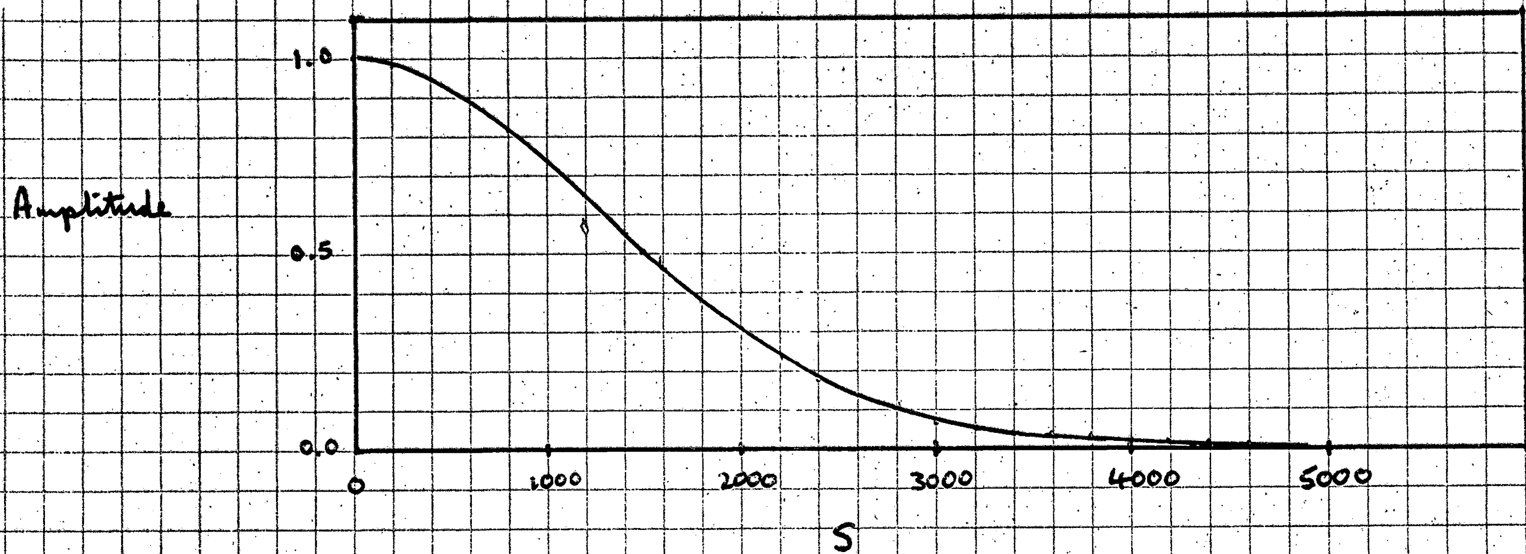


FIGURE III (b)

Source Model



Fourier Transform



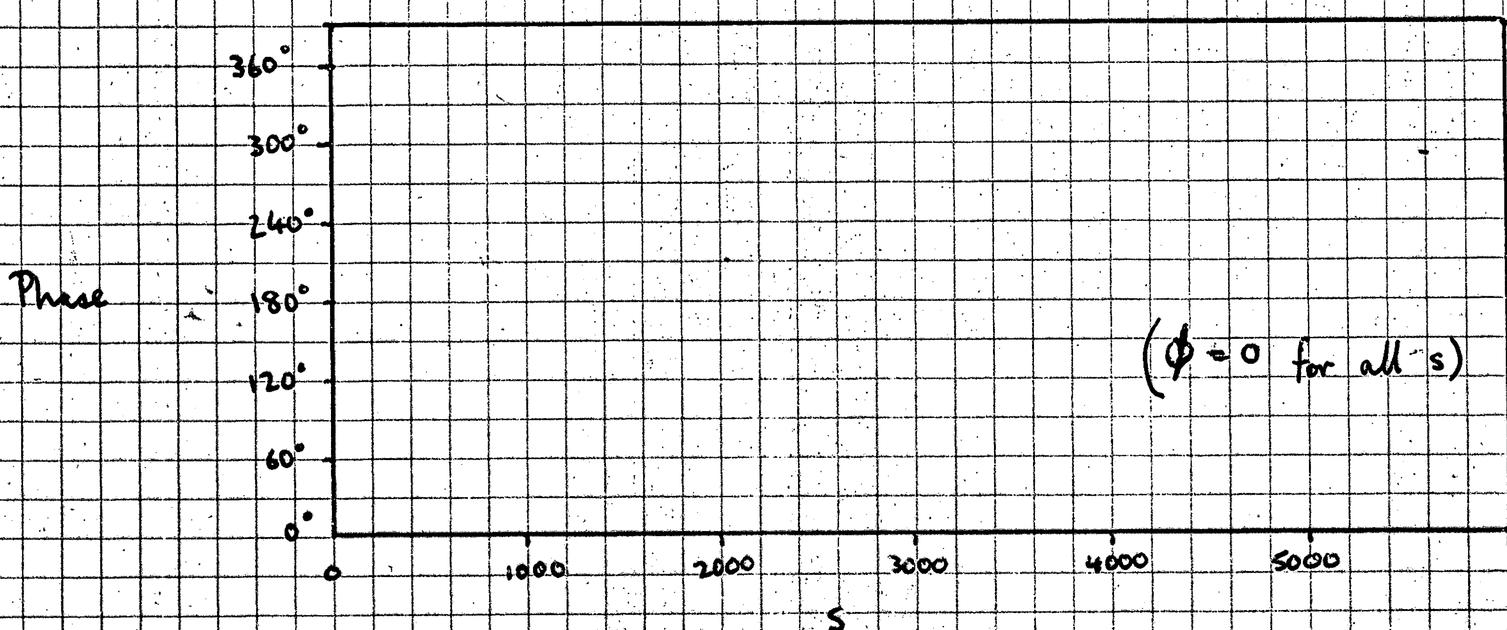
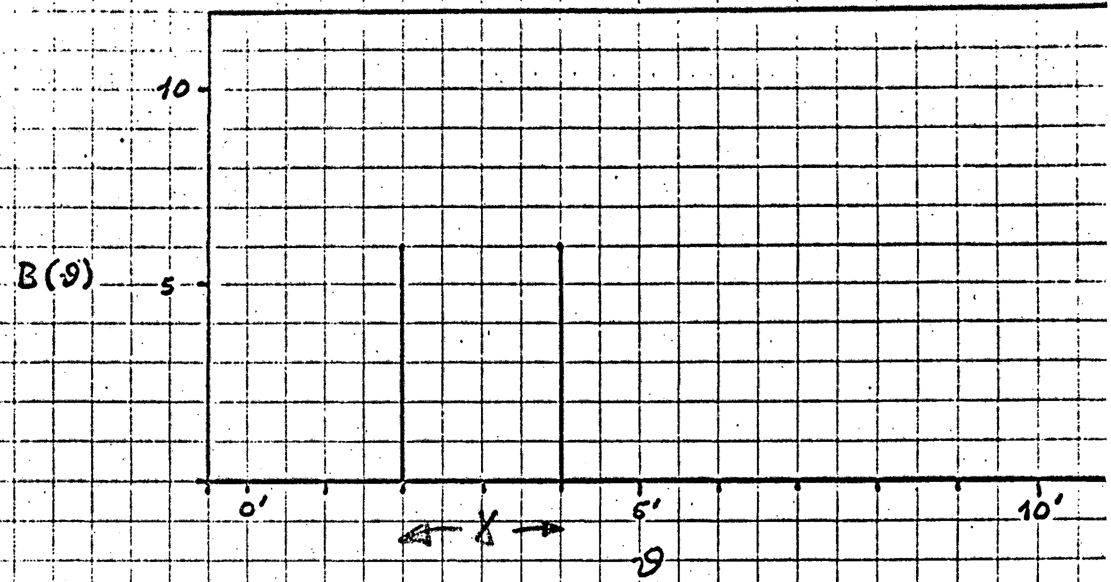
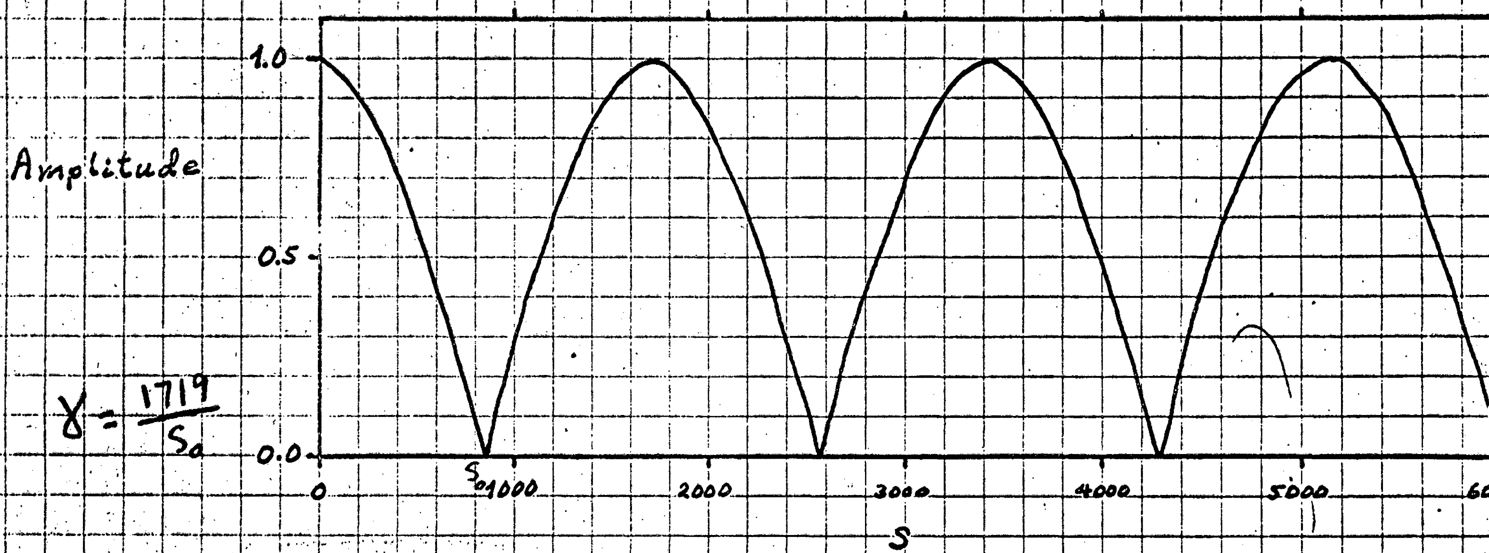


FIGURE III (c)

Source Model



Fourier Transform



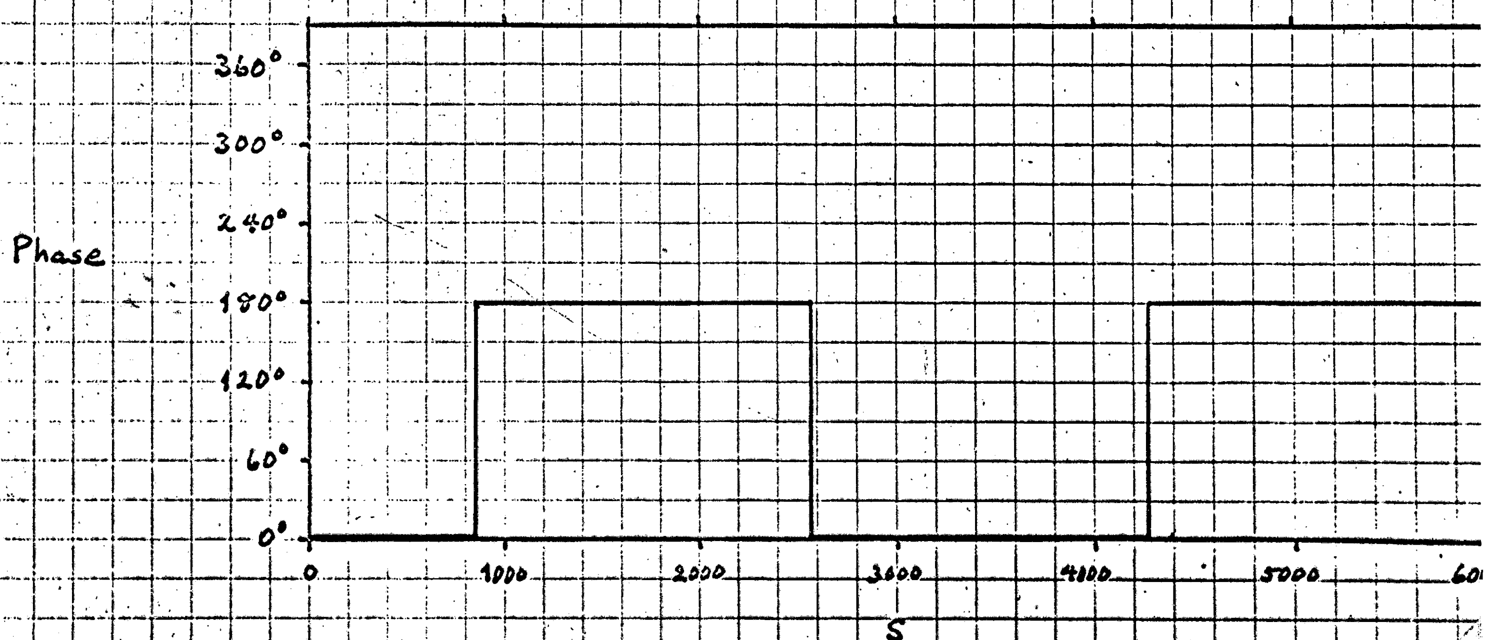
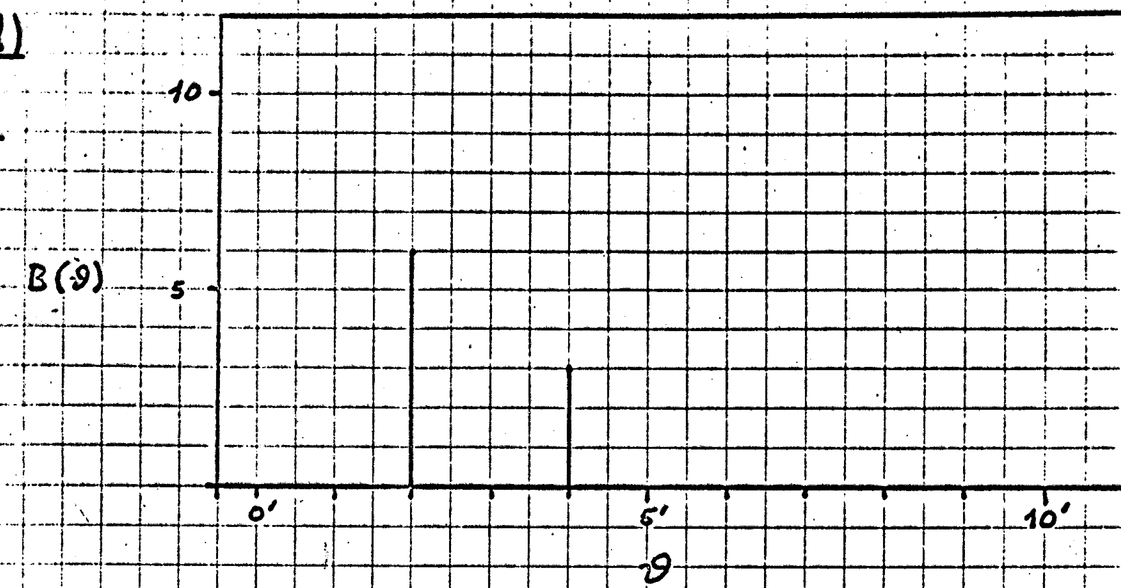
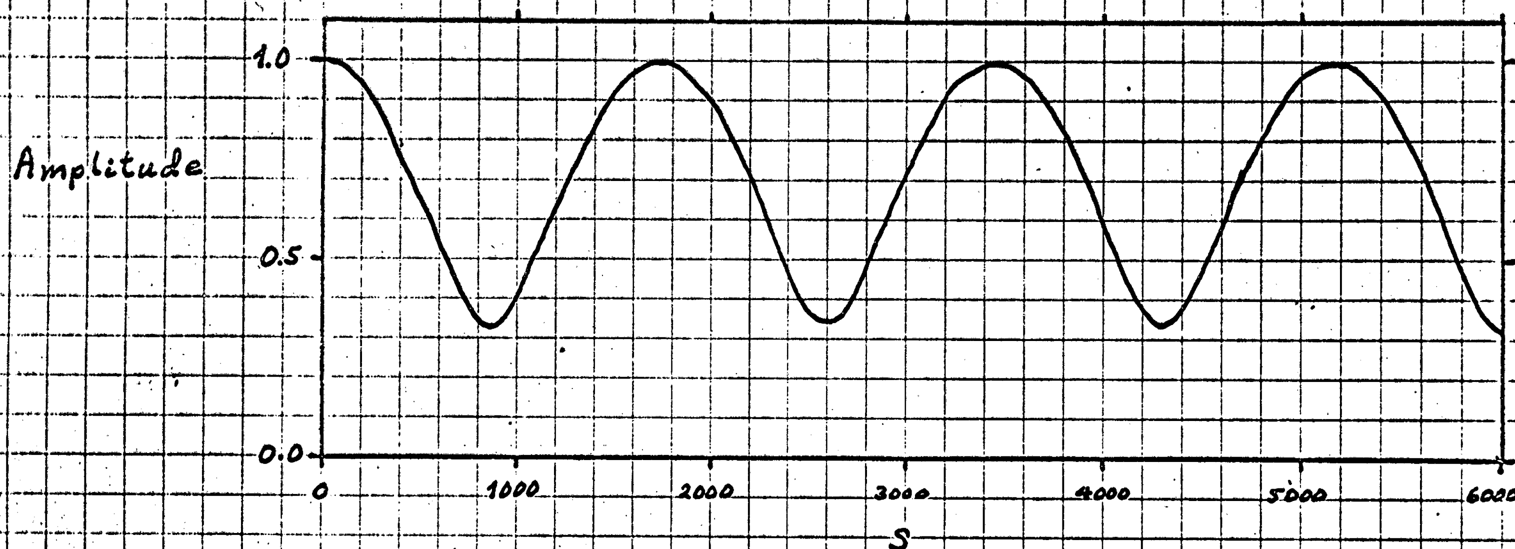


FIGURE III (d)

Source Model



Fourier Transform



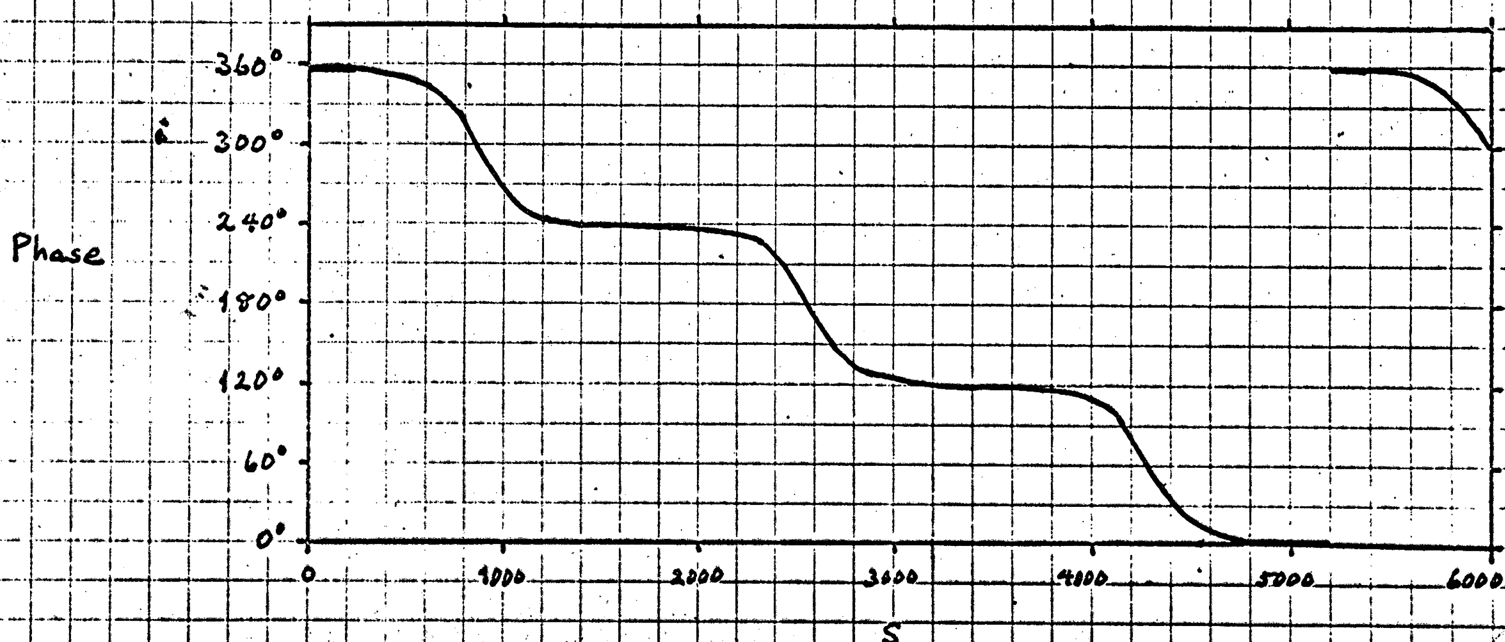


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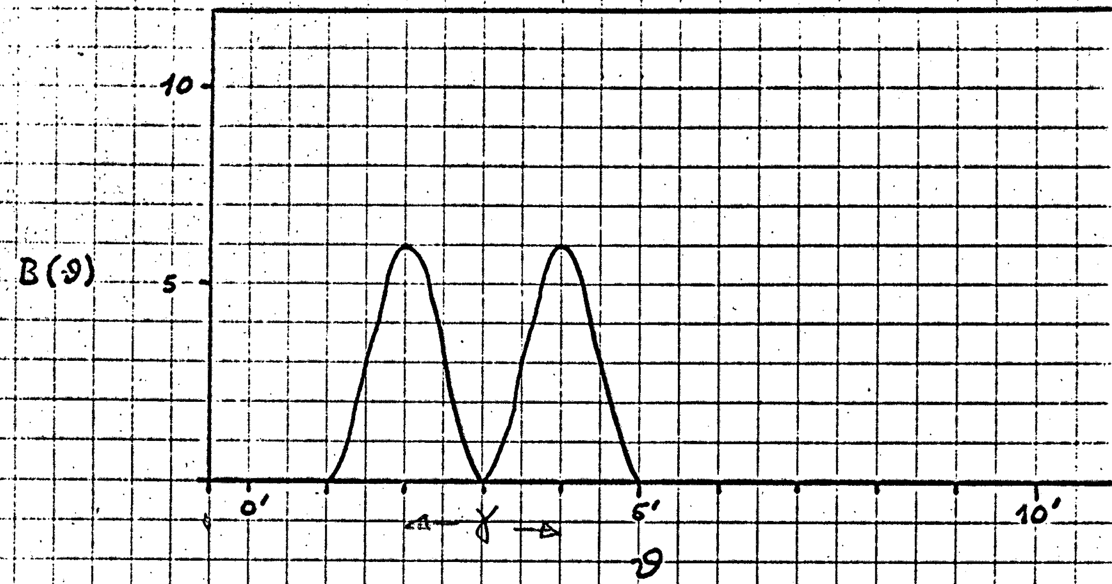
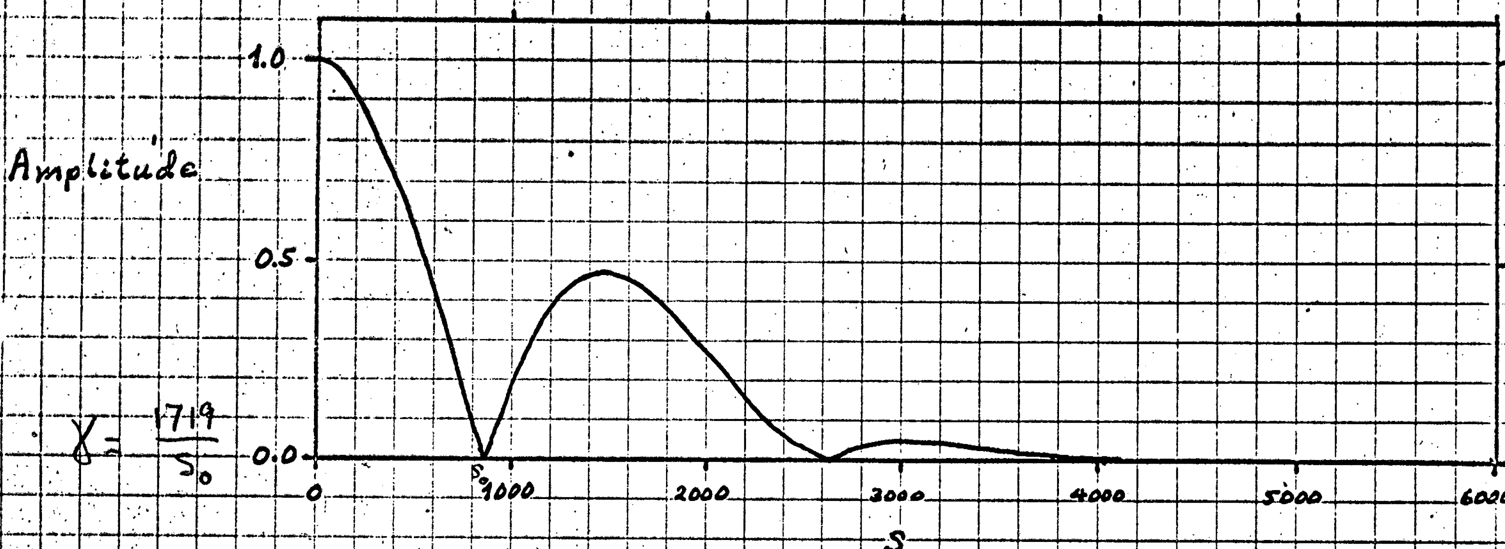
FIGURE III (e)

Source model



Fourier Transform



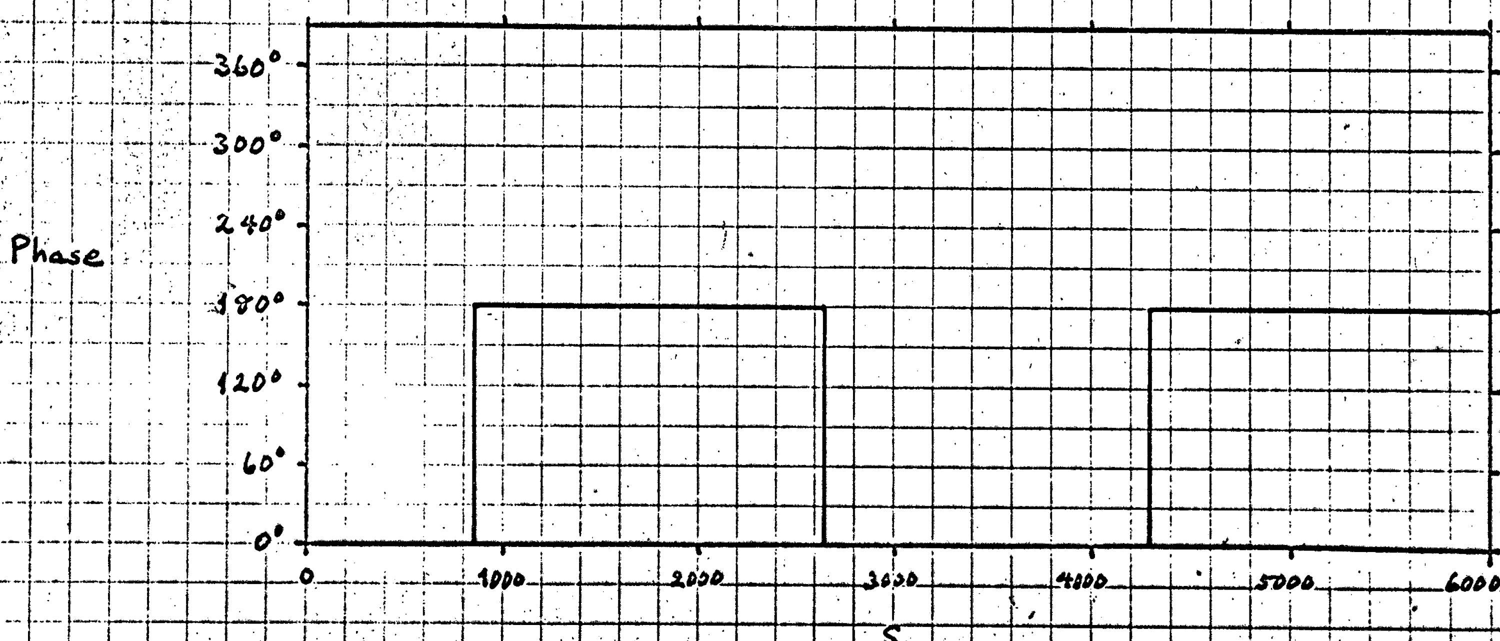


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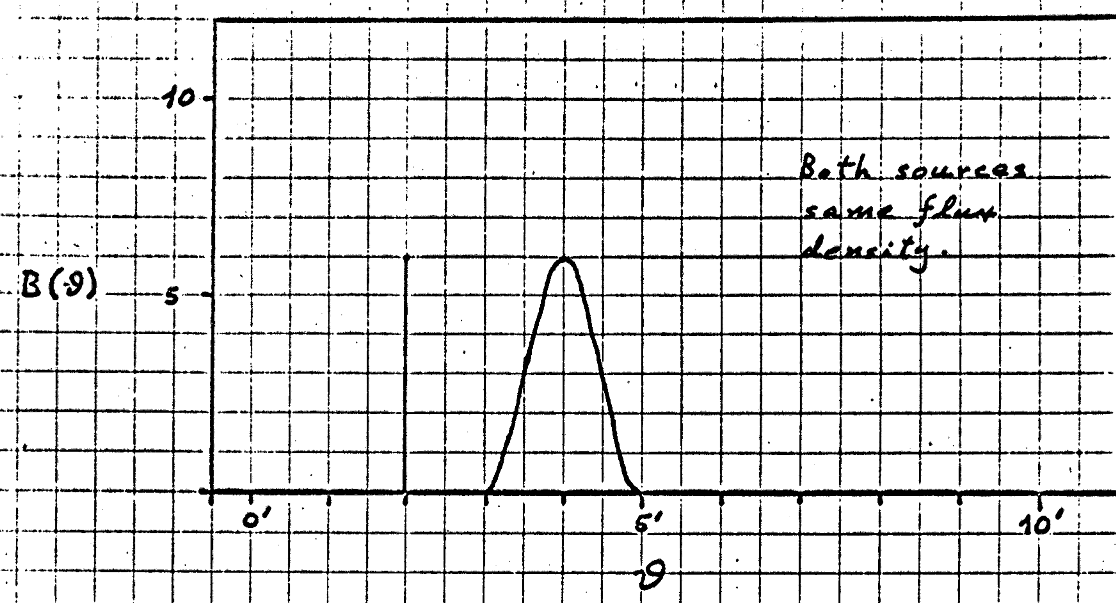
FIGURE III (f)

Source model



Fourier Transform

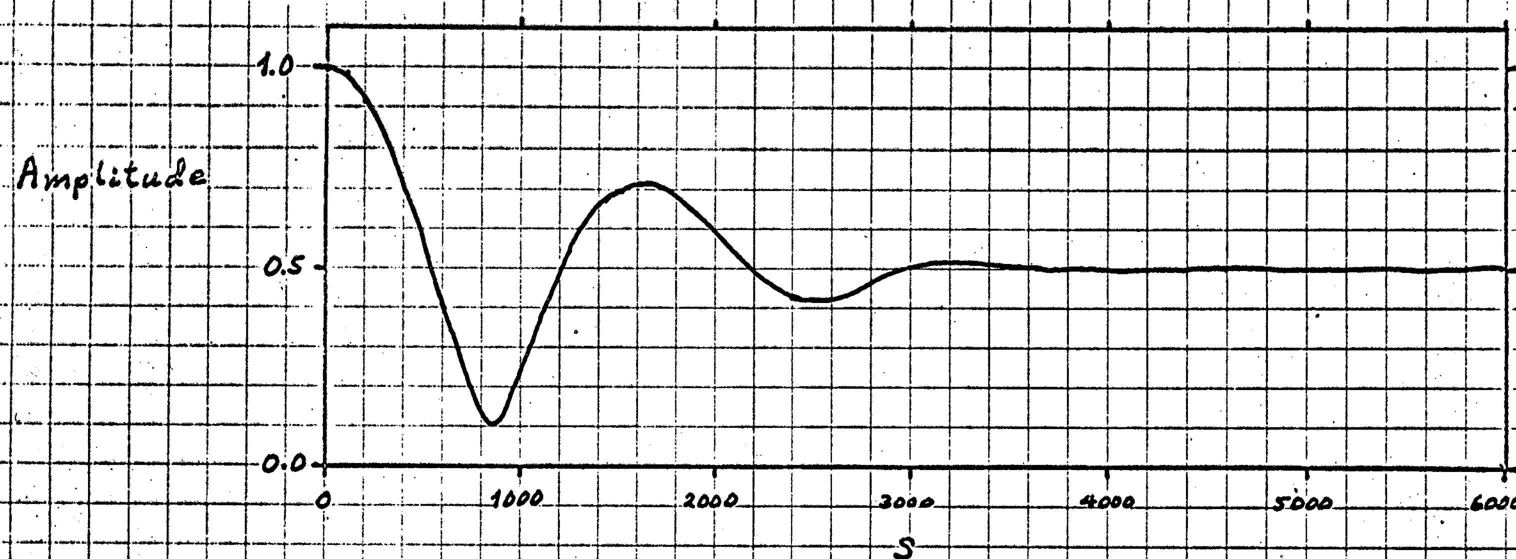
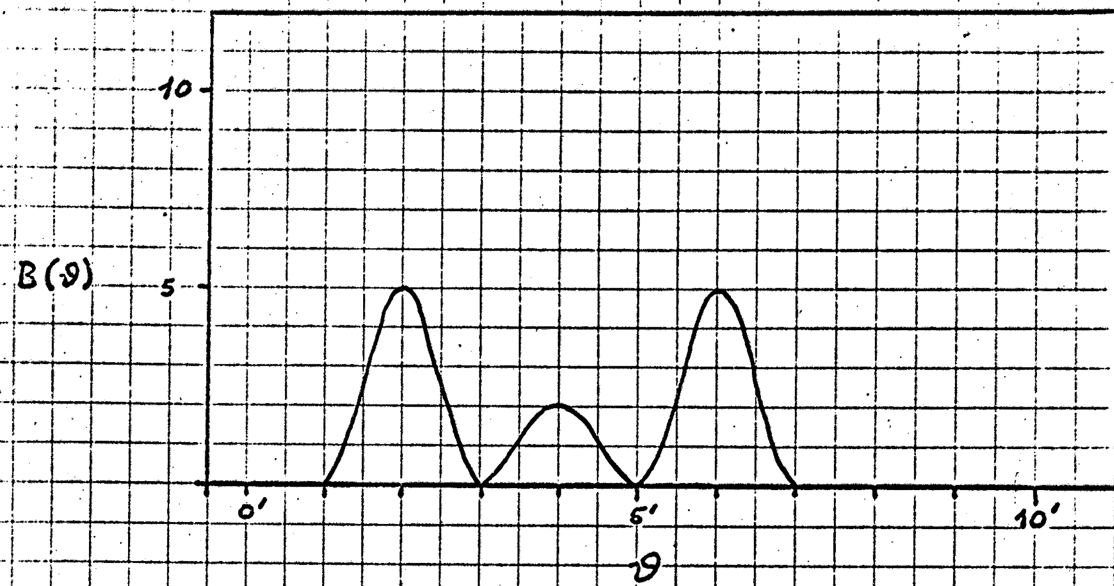
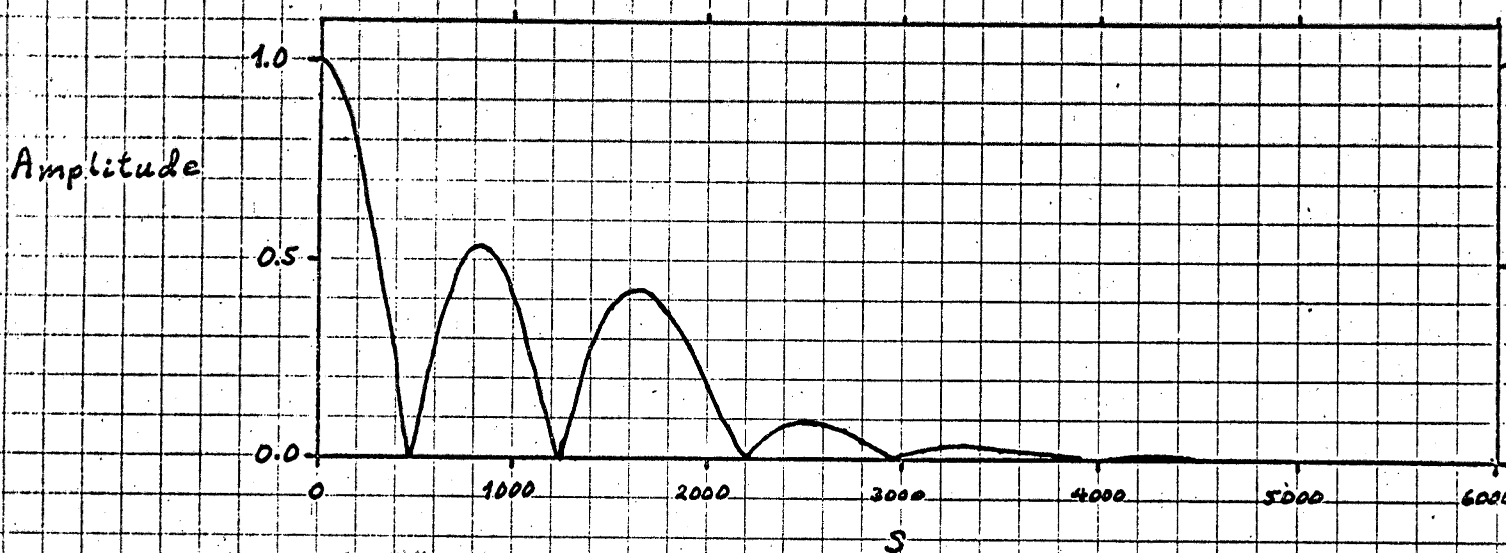


FIGURE III (q)

Source Model



Fourier Transform



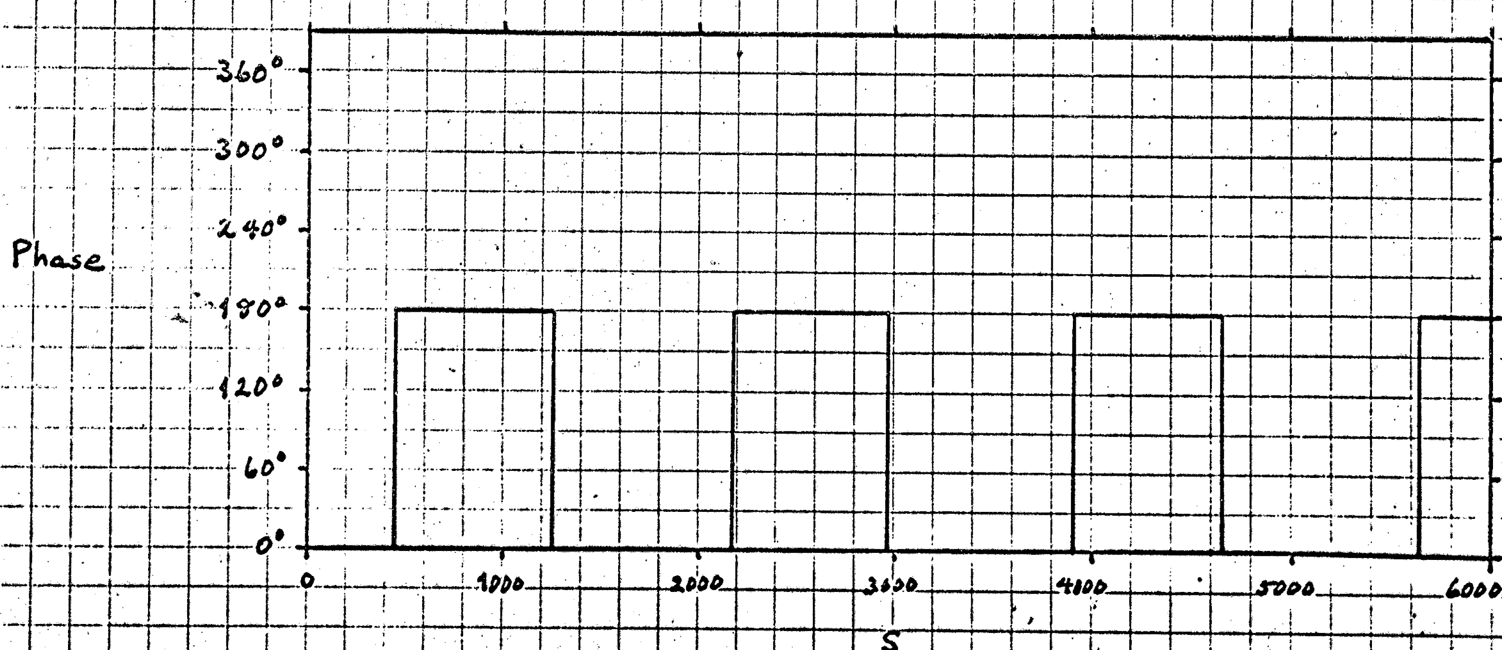


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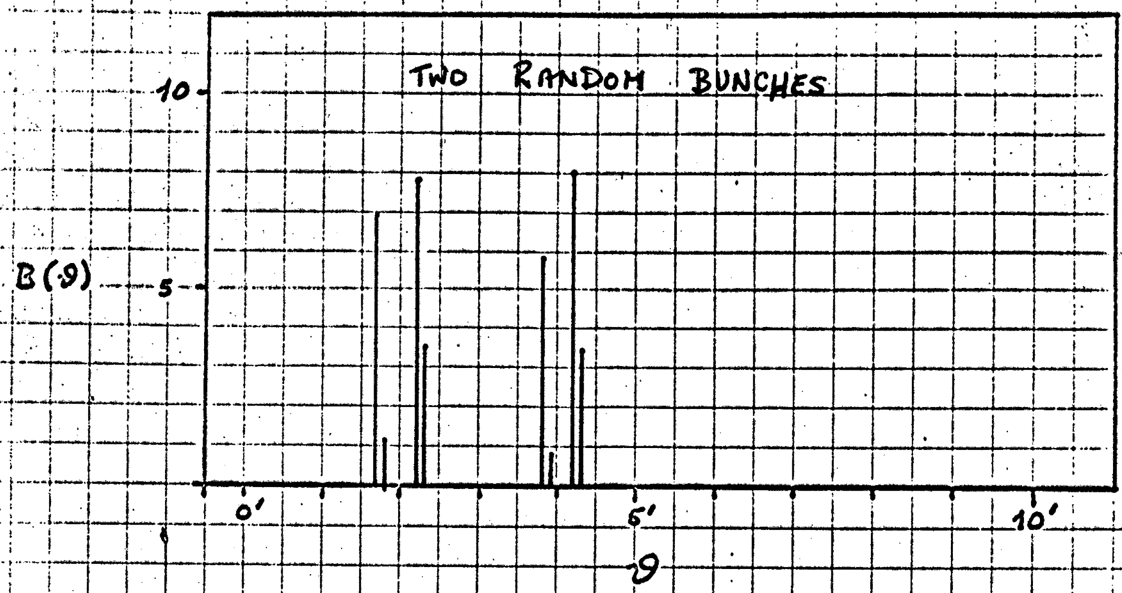
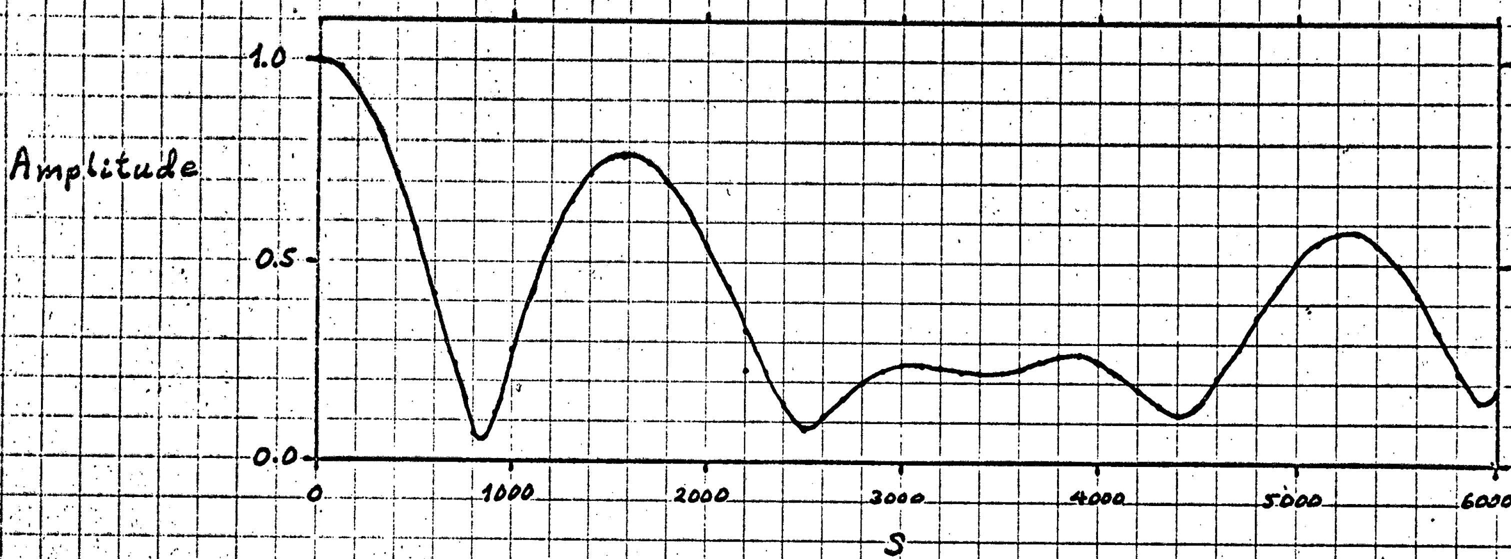
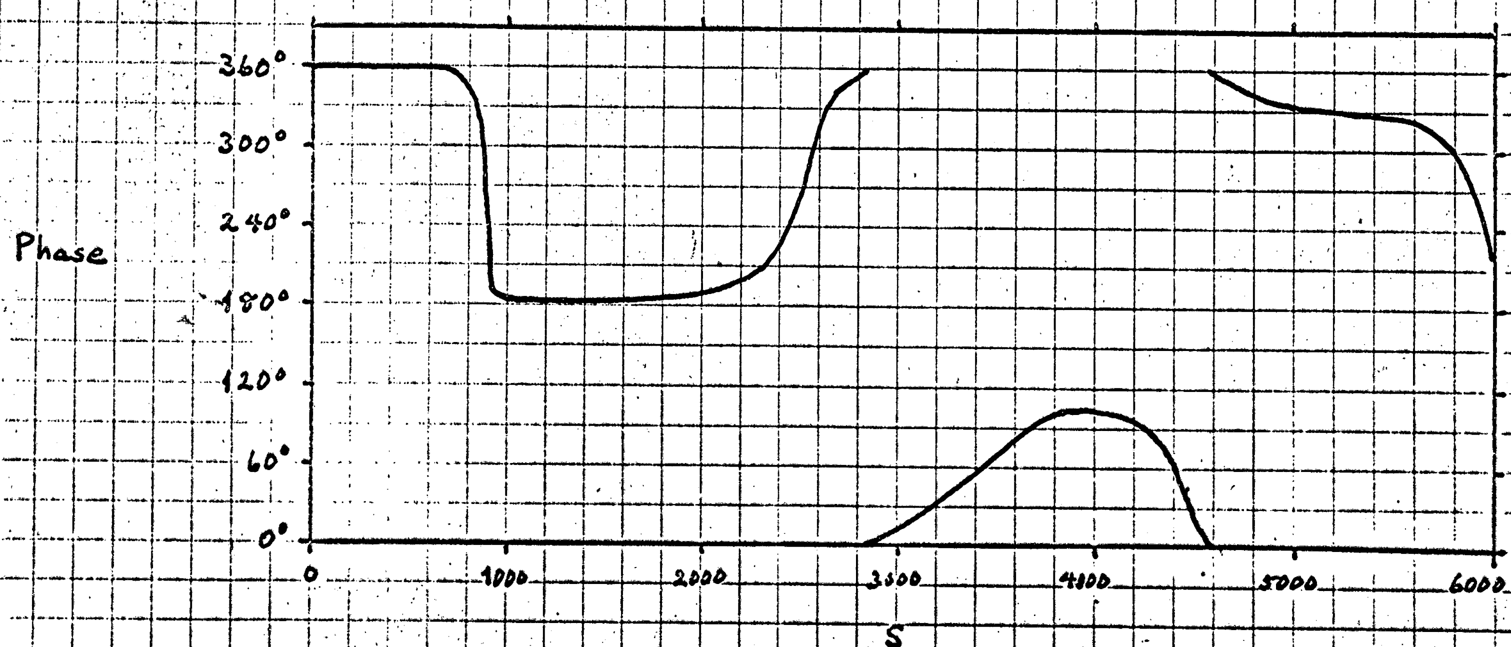
FIGURE III (h)

Source Model



Fourier Transform





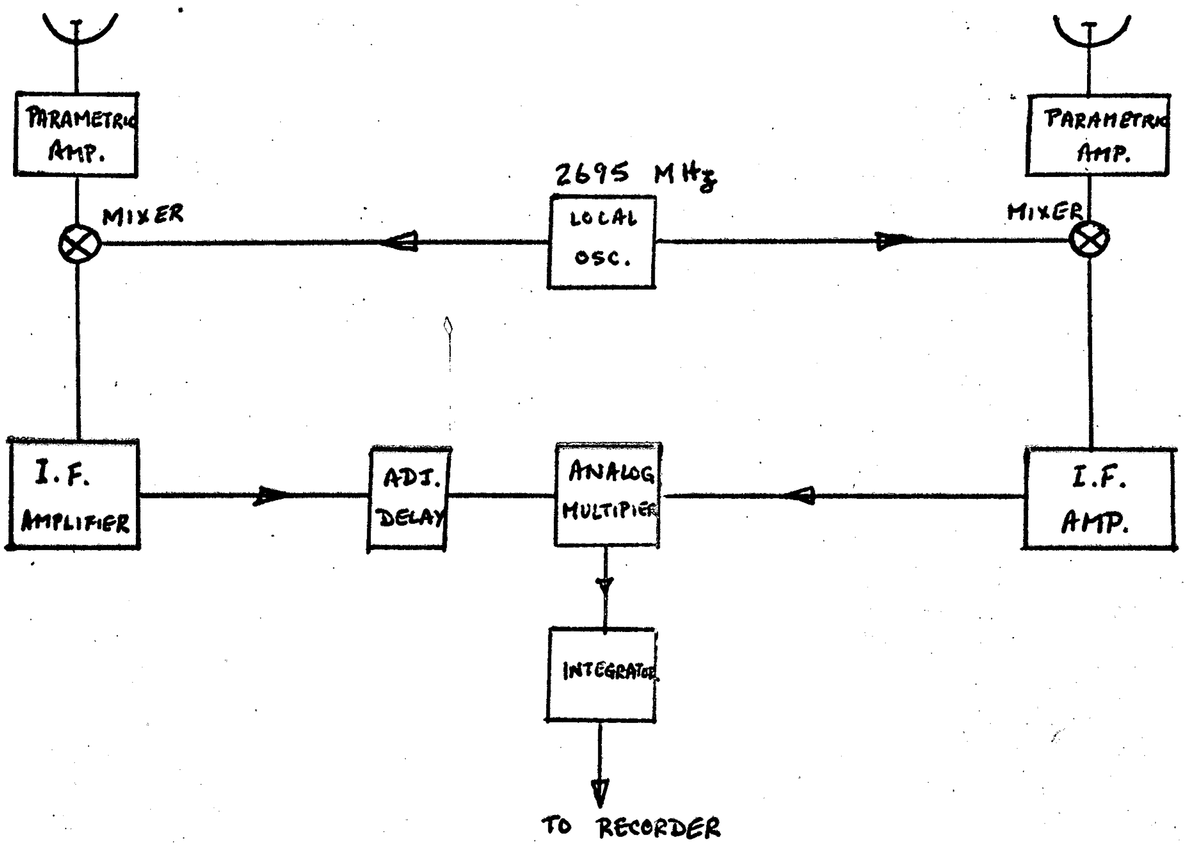

PARAMETRIC AMP.
PARAMETRIC AMP.
2695 MHz
LOCAL OSC.
MIXER
MIXER
I.F. AMPLIFIER
ADJ. DELAY
ANALOG MULTIPIER
I.F. AMP.
INTEGRATOR
TO RECORDER

FIGURE IV

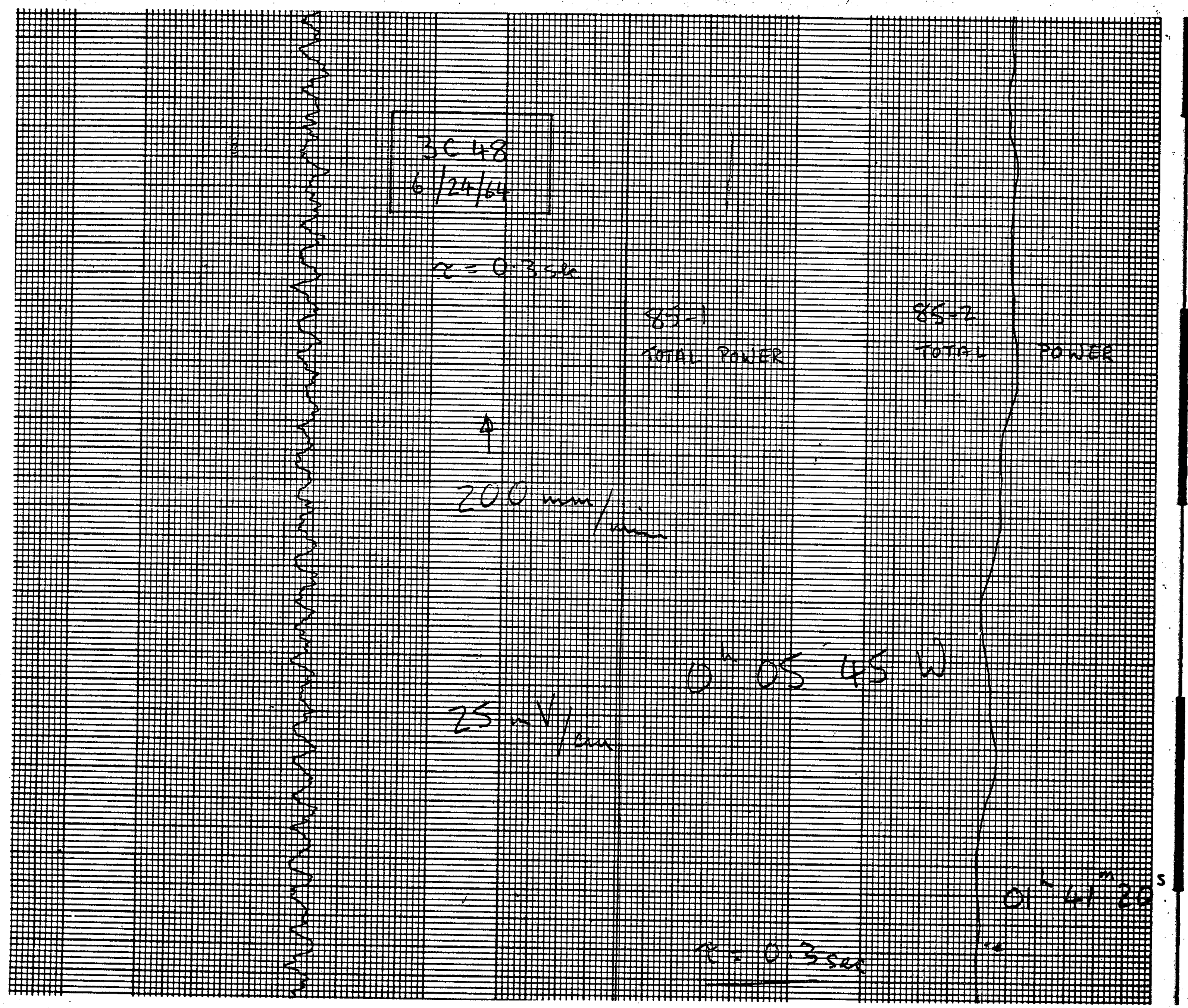


FIGURE V(a) — A POINT SOURCE (V = 1)

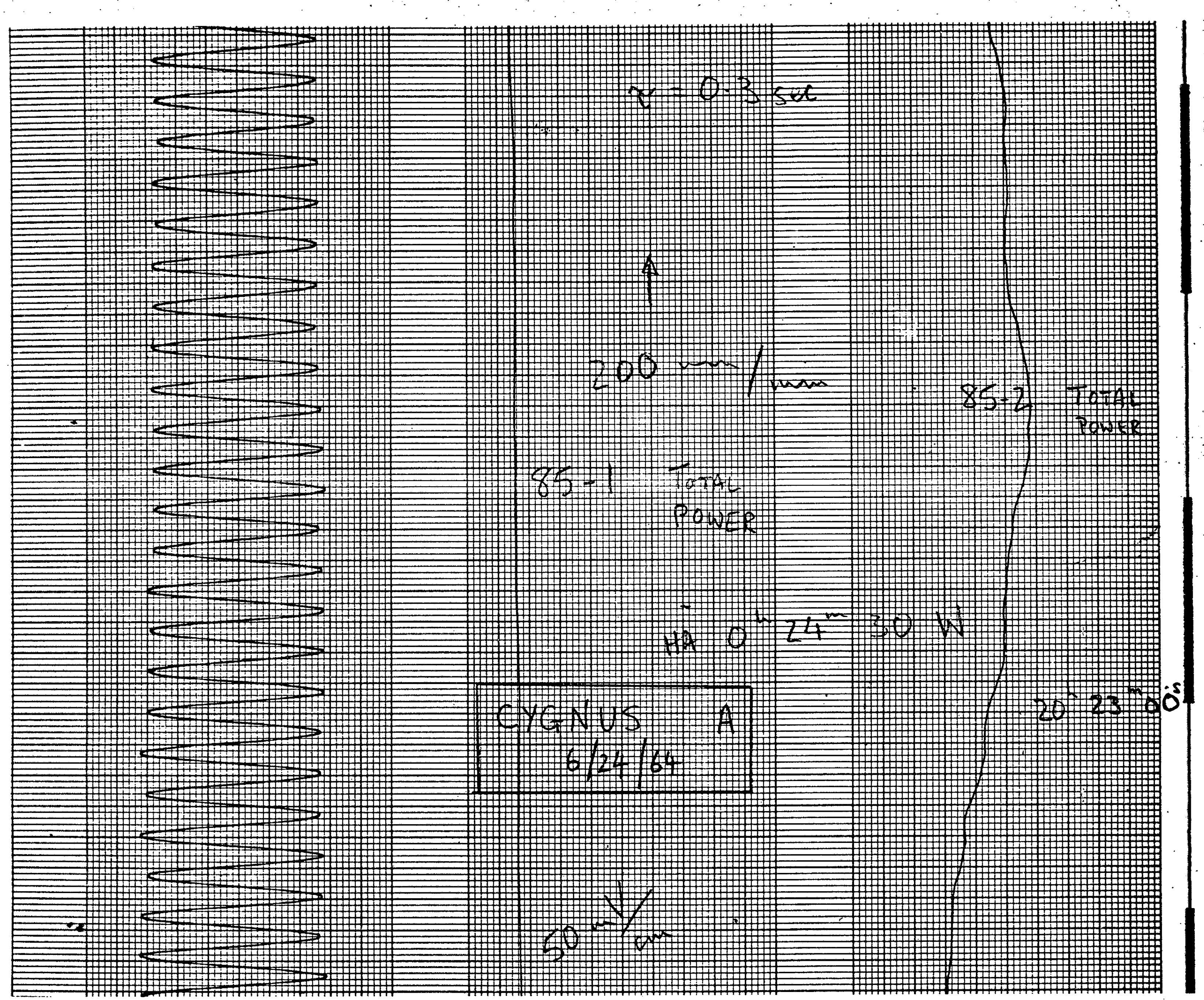


FIGURE V(b) — A LARGE STRONG SOURCE WITH $V \simeq 0.2$