Lecture Notes

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GENERATION OF RADIO WAVES IN SPACE. II.

by

T. K. Menon
National Radio Astronomy Observatory
Green Bank, West Virginia

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We shall discuss next the origin of spectral lines in radio astronomy. The frequency ν of an atomic or molecular spectral line can be predicted from the familiar Bohr condition

$$\nu = \frac{E_1 - E_2}{h} \tag{1}$$

where E_1 and E_2 are the initial and final energies of the atom and h is Planck's constant. In order to compute E_1 and E_2 in a specific case, we have to use the methods of quantum mechanics. One of the basic postulates of quantum theory is that the energy states of an atom can have only certain discrete values of a set of values. This is usually done by putting restrictions on the possible values of the various angular moments of the atom. For our purposes, these angular moments are: (1) the total electronic orbital angular momentum $\bar{L}$, (2) the total electronic spin angular momentum $\bar{S}$, (3) the total electronic angular momentum $\bar{J} = \bar{L} + \bar{S}$, (4) the nuclear spin angular momentum $\bar{I}$, and (5) the total angular momentum $\bar{F} = \bar{I} + \bar{J}$. Quantum mechanics also requires that the magnitudes of these vectors are restricted to the values $\sqrt{L(L+1)}\, h/2\pi$ where L is an integer, and similar expressions for the other angular moments, except that S and I may be multiples of a 1/2. For given values of L and S, the permitted values of J are $L + S$, $L + S - 1$, $L + S - 2 \ldots L - S$, and similarly for F.

It is a fundamental property of the electron that associated with its spin angular momentum is a magnetic moment $\bar{\mu}_e$ given by

$$\bar{\mu}_e = -2 \frac{eh}{4\pi m} \bar{S} \qquad (2)$$

The minus sign implies that the magnetic moment is oriented opposite to the electron spin. Similarly, the magnetic moment $\bar{\mu}_I$ is associated with the atomic nucleus and may be related to its spin $\bar{I}$ in an analogous manner.

$$\bar{\mu}_I = g_I \frac{eh}{4\pi m} \bar{I} \qquad (3)$$

where g_I is known as the "nuclear g factor" and is characteristic of a particular nucleus. The value of g_I may be positive or negative. M is the mass of the proton.

The existence of the two moments implies that a system will have different energies for different orientations of the total electron moment relative to the nuclear moment. This energy difference is the so-called "hyperfine energy" and is the most important of the interactions within the atom for radio astronomy, as it is the origin of the 1420 Mc/s line of atomic hydrogen.

We shall only very roughly sketch the procedure for computing the interaction energy for such interactions. The energy of the magnetic interaction between the electron and the nucleus follows from the usual expression for a magnetic dipole in a field.

$$E_m : -\bar{\mu}_I \cdot \bar{B} = -\bar{\mu}_I \cdot (\mu_o \bar{H}) \qquad (4)$$

where μ_o is the Bohr magneton $eh/4\pi mc$ and $\bar{H}$ is the field at the nucleus due to all the electrons. The field at the nucleus consists of two parts: (1) the field due to the electron's magnetic moment, and (2) the field resulting from the orbital motion of the electron. The details of these calculations can be found in a number of books on atomic spectra, particularly in Condon and Shortley "Theory of Atomic Spectra" and Nuclear Moments" by Kopferman.

The interaction energy is found to be

$$E_m = g_I \left(\frac{\mu_o c^2}{4\pi}\right)\left(\frac{m}{M}\right) \frac{\alpha^2 hcRZ^3}{n^3} \left[\frac{F(F+1) - I(I+1) - J(J+1)}{J(J+1)\ (2L+1)}\right] \qquad (5)$$

For the ground state of atomic hydrogen n = 1, S = 1/2, L = 0, J = 1/2, I = 1/2, and hence F = 1 or 0, and eq. (5) gives two values of E_m for two values of F. This illustrates how a single energy level is split into two closely spaced levels by magnetic hyperfine interaction. Substituting the appropriate values of F, I, J, and L for the two cases in (5) and using eq. (1), we get

$$\nu = \frac{8}{3} g_I \left(\frac{\mu_o c^2}{4\pi}\right)\left(\frac{m}{M}\right) \alpha^2 cR \qquad (6)$$

where R is the Rydberg constant, α the fine structure constant. The value of ν turns out to be very close to 1420 Mc/s. In the above formula, higher order quantum mechanical effects have been neglected. The accurately measured laboratory value is 1420 · 405726 $\pm$ 0.000030 Mc/s.

In addition to the hyperfine structure lines we have also the so-called fine structure lines. The physical basis for these lines lies in the fact that the electron is in motion in the electric field of the nucleus; thus it experiences not only the electric field, but also a magnetic field, which in turn interacts with the electron magnetic moment to produce different energies, depending on the orientation of the moment with respect to the field. Tables of number of fine structure and hyperfine structure lines have been given by Barrett (Proc. I.R.E., Jan. 1958). Of these, only the hydrogen line at 1420 Mc/s has been observed so far.

Radio frequency lines are also produced by the many possible interactions in the case of molecules. Since hydrogen is the most abundant element in the universe, we might expect that diatomic molecules containing hydrogen, i.e., the hydrides, will be the most important. We shall consider only two of these, the H_2^+ and OH ions. A more detailed discussion will be found in Shklovsky's book.

The radio frequency lines in the case of H_2^+ ion arises in a manner similar to the hyperfine structure line of hydrogen. However, because of the many possible interactions in a molecule the line is not a single one but has many components. Unfortunately we do not have a reliable theoretical or experimental determination of this line.

In the case of the OH molecule, the mechanism for the existence of radio frequency lines is known as Λ doubling. The quantum number $\Lambda = \Sigma\lambda_i$ where λ is the projection of the total orbital angular momentum of an electron on the axis of the ion. Hence, Λ determines the absolute value of the projection of the total orbital angular momentum on the axis of the molecule. In the case of a molecule in rotation and $\Lambda > 0$ the energy levels are split. For OH the two strongest lines which arise due to this doubling have been measured in the laboratory and from the measured transition probabilities the two other components were assigned their frequencies. The frequencies are near 1612, 1665, 1667, and 1720 Mc/s. The four lines have been observed, in absorption against radio sources, by radio astronomers in the USA and in Australia. From these measurements many surprising results have been found.

The problem of excitation of the 21 cm hydrogen line has been treated in the following references:

1. Field, Proc. I.R.E., 46, 240, 1958.

2. Field, Ap. J., 129, 536, 1959.

3. Purcell and Field, Ap. J., 124, 542, 1956.

4. Shklovsky, Cosmic Radio Waves.