Lecture Notes

NRAO Lecture Program -- Summer 1965

ANTENNAS

by

J. W. M. Baars
National Radio Astronomy Observatory
Green Bank, West Virginia

ANTENNAS

J. W. M. Baars
National Radio Astronomy Observatory
Green Bank, West Virginia

INTRODUCTION

One might define an antenna (aerial) as the structure associated with the region of transition between guided and free-space waves or vice versa.

Thus an antenna transforms a received electro-magnetic field into an electrical voltage or current, which is proportional to the field strength. The antenna is a linear device.

The antenna is indispensable for observational radio astronomy. There exist several types of antennas depending on a specific need. I shall describe these quickly. (See Findlay's article.)

In order to subtract from the observations reliable and trustworthy results it is important that the characteristics of the antenna are thoroughly known by the astronomer. So we shall spend most time discussing antenna theory and describing methods to determine specific parameters by experiment.

Also we shall deal with the problem of how to extract knowledge about the source of radiation from the output signal of the antenna.

Astronomers have two major requirements for a radio telescope:

1. Large collecting area, which gives high sensitivity.
2. Narrow beam yielding a high resolution.

Both requirements ask for a large physical size of the antenna.

TYPES

1. The most common and versatile antenna is the paraboloidal reflector antenna. We shall deal with this type in detail.

2. Horn antenna. Because the collecting area is theoretically calculable to a high accuracy, the horn is used as a calibration instrument. Example: Little Big Horn at NRAO.

3. Fixed reflector antennas:

   a. Spherical reflector with moving feed, (300 m diameter), Arecibo.

   b. Parabolic cylinder (180 x 120 m), University of Illinois.

   c. Standing paraboloidal or spherical reflector with flat tiltable mirror, Ohio State (80 m); Nancay (France) (200 m long).

We do not talk about interferometers and synthesis type antennas in this part of the lecture series.

THEORY OF THE PARABOLOIDAL REFLECTOR ANTENNA

We shall concentrate our theoretical treatment on the paraboloidal reflector, because it is well suited for a general approach and moreover the most widely used antenna in radio astronomy. One knows the property of a paraboloid to change a plane wave, falling in along the axis of the paraboloid, upon reflection into a spherical wave converging towards the focal point of the reflector. So we place a feed (commonly a horn or a dipole) in that point and we receive the power intercepted by the reflector. We call this the primary focus arrangement. (Fig. 1)

Another type is the Cassegrainian antenna. It has a second reflector near the focus of the paraboloid and the focal point of the combination lies near the vertex of the paraboloid. One can show that the secondary reflector, if it lies between the paraboloid and its focal point, has a hyperboloidal shape with one of its focal points lying in the paraboloid's focus and the other focus being the focal point of the complete antenna. The eccentricity e of the hyperbola determines the position of the final focus. The effective focal length of the Cassegrain is $(e + 1)/(e - 1)$ times the focal length of the paraboloid. We call the multiplicative factor the magnification of the Cassegrain. The geometry of the parabola is given by

$$\tan\frac{\psi}{2} = D/4F \tag{1}$$

where ψ is the aperture angle, D the diameter and F the focal length. It is clear that the geometry is entirely determined by the F/D ratio. In the Cassegrain, however, we have more freedom in geometry by varying the magnification or the diameter of the secondary reflector.

In the theoretical treatment of the antenna we will find it often convenient to consider a transmitting antenna at one moment and a receiving antenna at another. All the results, however, are valid for both modes of operation by virtue of a reciprocity theorem due to Lord Rayleigh, which states that, if a power generator connected to antenna A gives a voltage V in antenna B, the same generator connected to B will induce the same voltage V in antenna A.

Considering the antenna from the standpoint of pure geometrical optics it is found that a spherical wave leaving the focal point will after reflection form a tube of rays with a diameter of the reflector. At infinite distance from the antenna the beam would consequently be a point.

However, in reality the beam spreads out at large distances because there is a diffraction effect. Now you know the Huygens-Fresnel principle stating that each point on a wave surface is a source of elementary fields; you may also know Green's theorem which says that the field in a point can be calculated as an integral of the known field over a closed surface. Without going into the details of some assumptions this means that we can find the field at a distant point (the radiation pattern thus) from an integration of the field over the aperture plane of the reflector.

The aperture field is found from the radiation characteristics of the feedhorn in the focal point by geometrical reflection of its rays at the reflector.

Thus we are able to calculate the secondary radiation pattern if we know the radiation pattern of the feed, by performing an integration over the aperture field.

The method requires that the aperture is very large as compared to the wavelength (which is in radioastronomy always the case) and that the aperture field is linearly polarized (the cross-polarization term is actually very small in practical cases).

Inspection of the pertaining integral shows that several approximations can be made depending on the distance of the point of observation from the aperture plane. Thus we distinguish three regions:

1. Near field region, extending to a few aperture diameters. Here no approximations can be made and evaluation of the integral is generally very difficult.

2. Fresnel region, extending to a distance of about $2D^2/\lambda$ (λ is the wavelength); it appears that the field energy flows here in a tube with diameter D up to the so-called Rayleigh distance $D^2/2\lambda$ and widens into a beam of some angular width further out.

3. Fraunhofer or far-field region, going to infinite distances. In this region we have what is called the radiation field; the integral is solvable fairly easily and we shall spend the bulk of our discussion on the characteristics of the far field, because moreover the sources all lie in the Fraunhofer region. The integral to be solved for the far field is

$$g(u, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int F(\xi, \eta) e^{ik(u\xi + v\eta)} \, d\xi \, d\eta \qquad (2)$$

where $g(u, v)$ is the far field in point u, v (amplitude field) (Fig. 2). $F(\xi, \eta)$ is the aperture field. You see that the far field is the two-dimensional Fourier transformation of the aperture distribution. This is a very important result. It clearly indicates that the radiation pattern is completely governed by the aperture field.

Let us apply equation (2) to a circular aperture with normalized radius one. Introducing polar coordinates we obtain

$$g(\Theta, \varphi) = \frac{1}{2\pi} \int_0^1 \int_0^{2\pi} F(\rho, \varphi') e^{ika \sin\Theta \, \rho \cos(\varphi - \varphi')} \rho d\rho \, d\varphi' \qquad (3)$$

where ρ and φ' are polar coordinates in the aperture, Θ and φ the angular polar coordinates of the observation point far away; $k = \frac{2\pi}{\lambda}$ and a the actual radius of the aperture.

It is normal to have a symmetrical beam in the far field and so we shall make the aperture illumination function $F(\rho, \varphi')$ independent of φ'. In that case the φ' integration can be performed and equation (3) becomes (see, e.g., Jahnke-Emde, p. 149)

$$g(u) = \int_0^1 F(\rho)\, J_0(u\rho)\, \rho\, d\rho \tag{4}$$

where $J_0(u\rho)$ is the Bessel function of the first kind and zero order, and $u = ka \sin \Theta$ the normalized form of the polar angle.

There exists a class of functions $F(\rho)$ which approximates practical aperture illumination functions satisfactory and makes a closed form solution of equation (4) possible. It is the function

$$F(\rho) = b + (1 - \rho^2)^n, \tag{5}$$

which we could give the name "hyperparabolic illumination on a pedestal". The pedestal height b determines the level of illumination at the edge of the reflector and is called the taper of the aperture illumination function.

Integration of equation (4) with equation (5) inserted gives (see Watson, Theory of Bessel Functions, p. 373)

$$g(u) = b\, \Lambda_1(u) + \Lambda_{n+1}(u) \cdot \frac{1}{n+1} \tag{6}$$

where the function $\Lambda_n(u) \equiv \dfrac{n!\, J_n(u)}{(u/2)^n}$; it is tabulated in Jahnke-Emde's Tables of Functions, p. 180 ff.

The next table gives some parameters of the resulting radiation patterns for different aperture functions; it should be noted that the values apply to the power pattern $f(u) = \left\{g(u)\right\}^2$.

	Sidelobe ratio (db)			Beamwidth (rad)		
n	$b = 0$	$b = \frac{1}{4}$	$b = \frac{1}{2}$	$b = 0$	$b = \frac{1}{4}$	$b = \frac{1}{2}$
0	17.6	—	—	$1.02\lambda/2a$	—	—
1	24.7	23.7	22.0	$1.27\lambda/2a$	$1.17\lambda/2a$	$1.13\lambda/2a$
2	30.7	32.3	26.5	$1.47\lambda/2a$	$1.23\lambda/2a$	$1.16\lambda/2a$
3	36.1	32.3	30.8	$1.65\lambda/2a$	$1.25\lambda/2a$	$1.16\lambda/2a$

	$\frac{G}{G_0}$			Position first null		
n	$b = 0$	$b = \frac{1}{4}$	$b = \frac{1}{2}$	$b = 0$	$b = \frac{1}{4}$	$b = \frac{1}{2}$
0	1.00	—	—	$1.22\lambda/2a$	—	—
1	0.75	0.87	0.92	$1.116\lambda/2a$	$1.49\lambda/2a$	$1.33\lambda/2a$
2	0.55	0.81	0.88	$2.03\lambda/2a$	$1.68\lambda/2a$	$1.51\lambda/2a$
3	0.45	0.79	0.87	$2.42\lambda/2a$	$1.97\lambda/2a$	$1.56\lambda/2a$

The resulting pattern consists of the main beam and sidelobes. The Half Power Beam Width (HPBW) is the angular width of the pattern at the half power level compared to the maximum. The sidelobe level is important because it can cause confusion about the direction of the received radiation; it should be as low as possible. The table clearly indicates that a stronger taper in the aperture function gives lower sidelobe level, but broader beam and also a decrease in the gain G of the antenna. We will return to this quantity later.

The optimum requirements for a radiotelescope appear now to be incompatible with antenna theory. So we have to establish some kind of compromise between sidelobe level and gain, for example. We can do this by a careful choice of the edge taper of the aperture illumination.

ANTENNA PARAMETERS

As the table indicates already, we can deduce from the knowledge of the radiation pattern several characteristical quantities of the antenna. One of the most important is the DIRECTIVITY D. It is defined as the power per unit solid angle in a specific direction (commonly the direction of maximum radiation) divided by the average power radiated per unit solid angle. In formula

$$D_M = \frac{4\pi}{\iint_{4\pi} f(\Theta, \varphi)\, d\Omega} \quad , \text{ where the maximum of the power pattern } f(\Theta, \varphi) \text{ is normalized to one.} \tag{7}$$

It is very difficult to measure this quantity, because $f(\Theta, \varphi)$ is difficult to measure in regions far from the maximum. But knowing the complete theoretical pattern we could calculate the Effective Solid Angle of the antenna which is defined as

$$\Omega_A = \iint_{4\pi} f(\Theta, \varphi)\, d\Omega \tag{8}$$

and we see that Ω_A is connected to the directivity by the relation

$$\Omega_A = 4\pi/D \tag{9}$$

In practice there will be some loss in the reflector, for which we account by the radiation efficiency η_R. So we arrive at the quantity which indicates to what extent the antenna concentrates the available energy into a specific direction; it is called the GAIN and we have thus

$$G(\Theta, \varphi) = \eta_R\, D(\Theta, \varphi) \quad \text{and} \quad G_M = \eta_R\, D_M \tag{10}$$

Another way to characterize the antenna is its effective absorption area A; it is defined as the power available at the terminals divided by the power crossing a unit area of wavefront. The ratio of A to A_g (the physical area of the aperture) is called the aperture efficiency: $\eta_A = A/A_g$.

From the reciprocity relation it is clear that A is proportional to G; in fact we have the relation

$$G = 4\pi A/\lambda^2 \tag{11}$$

which can be proven for the general case by thermodynamical considerations. From now on we always talk about the gain and effective area in the direction of the maximum of the main beam.

The absorption area (and thus the gain) is maximum for uniform aperture illumination ($b = 0$, $n = 0$ in eq. (5)). The decrease in gain for tapered illumination is due to the smaller value of the illumination function at the outer part of the aperture, which results in a smaller effective aperture diameter and hence a smaller absorption area. Also, the beamwidth increases, because it is proportional to λ/D_{eff}. But the sidelobe level is much lower for a tapered illumination.

Radiation entering directly into the feed (mostly from the ground) is called spillover radiation. It increases the noise temperature of the antenna and can change when the telescope is moved. By tapering the illumination the spillover effect will be decreased.

At our observatory the taper at the edge is about 18 dB below the field at the center. This results in a sidelobe level which is slightly more than 20 dB below the peak of the main beam. On the other hand the maximum obtainable aperture efficiency is only about 65 percent with a perfect reflector; also the HPBW is about 1.2 times the minimum obtainable.

We have talked about the antenna beam which consists of the main beam and side lobes; also we have mentioned the spillover radiation. Now we want to know how large a percentage of the energy entering the antenna is contained in the main beam. If we had an accurate knowledge of the antenna power pattern over the entire sphere, we could simply perform the integration of eq. (8). But generally the pattern is only known in a limited region around the maximum, were it only because the approximations used to obtain eq. (2) are not valid at large angular distances from the beam axis.

But we can write the antenna solid angle in two parts as (Fig. 3)

$$\Omega_A = \int\limits_{\substack{\text{main}\\\text{beam}}}\int f(\Theta,\varphi)\,d\Omega + \int\limits_{\substack{\text{stray}\\\text{region}}}\int f(\Theta,\varphi)\,d\Omega \equiv \Omega_m + \Omega_s \qquad (12)$$

the first part giving the main beam solid angle Ω_m and the second part the side lobe solid angle or stray region solid angle Ω_s. The main beam can generally be measured to a fair degree of accuracy; it turns out in practice that a gaussian curve forms an excellent approximation to the main beam; so we write the main beam

$$f_m(\xi, \eta) = \exp\left\{-\frac{\xi^2}{(0.6\,\Theta_E)^2} - \frac{\eta^2}{(0.6\,\Theta_H)^2}\right\} \tag{13}$$

where Θ_E and Θ_H are the HPBW in the two perpendicular planes (E- and H-plane) and ξ and η are rectangular coordinates replacing the spherical ones in the direct vicinity of the main beam maximum. Integration is easily performed and we obtain the main beam solid angle

$$\Omega_m = 1.133\,\Theta_E\,\Theta_H = 1.133\,\Theta_A^{\ 2} \tag{14}$$

the last part being valid for a symmetrical beam with HPBW $\Theta_A = \Theta_E = \Theta_H$. If we know the gain we can find the antenna solid angle from eq. (9). Then we can introduce the BEAM EFFICIENCY η_B as

$$\eta_B = \Omega_m/\Omega_a \leq 1 \tag{15}$$

It gives the percentage of all the transmitted power that enters the main beam; it is a very important antenna parameter.

From the relations now obtained it is simple to derive the next two:

$$A = \eta_B\,\eta_R\,\frac{\lambda^2}{\Omega_m} \tag{16}$$

Further

$$\eta_A = \frac{\eta_R\lambda^2}{A_g\,\Omega_m}\,\eta_B \approx 0.76\,\eta_B \tag{17}$$

We have made use of eq. (14) and in the last step we have inserted the value for the beamwidth, pertaining to the taper used at NRAO,

$$\Theta_A = 1.21\,\lambda/D \quad \text{(in radians)}$$

Equation (17) connects the aperture efficiency with the beam efficiency and is sometimes useful during measurements of the antenna parameters. Often the radiation efficiency ($\eta_R \approx 0.99$) is taken into η_B and the product $\eta_R\eta_B$ is called beam efficiency, denoted η_B.

ANTENNA TEMPERATURE

Before we spend some time on the methods used to determine experimentally the parameters of the antenna and of the characteristics of the radiosource, we have to introduce the concept of ANTENNA TEMPERATURE.

Consider an antenna surrounded by a black body at temperature T and connected by a lossless line to a resistor R_r, which has the value of the characteristic radiation resistance of the antenna. Let R_r also have a temperature T. All energy received by the antenna is absorbed in R_r but on the other hand each resistor gives a power according to its temperature and this power is radiated by the antenna. There is an exchange of energy and because all is at temperature T, equilibrium exists.

The same situation applies if the antenna is replaced by a resistor R_r at the physical temperature T of the surrounding box. We define now as the Antenna Temperature T_A the temperature to which a matched resistance has to be brought in order to obtain the same noise power at the terminals as the signal received by the antenna does. Thus, we can express any power captured by the antenna in an equivalent effective antenna temperature T_A.

In observing a source we measure some recorder deflection at the output of the radiometer. In order to subtract quantitative information about the strength of the received power the radiometer must be calibrated. The definition of antenna temperature suggests already a method of doing this.

Instead of the antenna we connect a matched load (resistor having the value of the characteristic resistance) to the input of the radiometer. Changing the temperature of the load a well known amount will cause the recorder to have a certain deflection. Because the strength of the received power, expressed in antenna temperature, influences the recorder deflection in the same way as the temperature change of the load we can now connect to a particular recorder deflection a certain value of antenna temperature.

Let us now investigate the relation between the measured antenna temperature T_A and the flux density S_ν (in W m^{-2} Hz^{-1}) or brightness temperature T_b (in °K) of the observed radiosource.

For a particular beamwidth we divide the sources in three classes depending on the ratio of source diameter to HPBW. The flux density of a source is connected to its brightness temperature by the relation

$$S_\nu = \frac{2k}{\lambda^2} \int\limits_{\text{source}} \int T_b \, d\Omega \tag{18}$$

where the integration is performed over the solid angle subtended by the source: $k = 1.38 \cdot 10^{-23}$ Ws/°K is the Boltzmann constant. Bearing in mind the definition of the beam efficiency we find the general formula connecting observed antenna temperature and flux density

$$S_\nu = \frac{2k}{\lambda^2 \eta_B} \int\limits_{\text{source}} \int T_A(\xi, \eta) \, d\xi \, d\eta \tag{19}$$

where the integration is extended over the source region.

If we have additional information about the angular size and the shape of the source, we can reduce the integral of eq. (19) to simpler formulae:

a. If we have a point source, the only significant quantity is the flux density S_ν. The intercepted power is $1/2 \; S_\nu A$ the factor 1/2 coming from the fact that the antenna accepts only one polarization direction. The antenna temperature is thus

$$T_A = \frac{S_\nu \cdot A}{2k} \tag{20}$$

b. On the other hand, a source with an angular diameter two or more times the HPBW will fill the antenna beam completely and we have the relation between brightness temperature T_b and antenna temperature T_A

$$T_A = \eta_B \cdot T_b \tag{21}$$

c. In the intermediate case (source diameter smaller than the beam but too large to be a point source) we must be careful. The antenna temperature will now depend strongly on the distribution of the brightness temperature over the source. If the source solid angle Ω_s is considerably smaller than the beam solid angle Ω_m the radiation pattern has an approximately constant value over the source and the relation holds

$$T_A = \eta_B T_b \frac{\Omega_s}{\Omega_m} \tag{22}$$

where we have for

$$\Omega_s = \iint \psi(\xi, \eta) \, d\xi \, d\eta$$

where $\psi(\xi, \eta)$ is the normalized brightness distribution over the source and the integration is extended over the source size. For a larger source we have to account for the beam pattern $f_m(\xi, \eta)$ and Ω_s transforms into the effective source solid angle

$$\Omega_s' = \int\limits_{\text{source}} \int f_m(\xi, \eta) \, \psi(\xi, \eta) \, d\xi \, d\eta \leq \Omega_s$$

Thus we can write

$$T_A = \eta_B T_b \frac{\Omega_s'}{\Omega_m} \tag{23}$$

In order to be able to calculate the source solid angle Ω_s and the effective source solid angle Ω_s' the functions $f_m(\xi, \eta)$ and $\psi(\xi, \eta)$ must be known.

We see from the discussion that we can deduce source parameters from the measured antenna temperatures once the antenna parameters are known.

On the other hand, if we had available a source with well known angular diameter and brightness temperature, we could find the antenna parameters. The only reliable way to find the antenna parameters is by measurement. This requires a transmitter of well known and stable power output in the far field of the antenna. I have mentioned already that the far field begins at a distance of about $2D^2/\lambda$; you can easily calculate that for the large apertures of most radiotelescopes this distance is too far to make an earthbound calibration source feasible.

N.B. In using these formulae for practical calculations one shall keep in mind that the solid angles are all expressed in steradians. One steradian contains $3.2838 \cdot 10^3$ square degrees or $1.1818 \cdot 10^7$ square arc minutes.

Radio astronomers have taken the natural step to overcome this difficulty by using strong radiosources (which are always in the far field) as calibration transmitters. Two requirements must be met in order to use a radio source:

a. The power output (flux density) at the used frequency must be well known. This is achieved by measurements with a calibrated antenna as a horn, the gain of which can be calculated with high accuracy (Little Big Horn).

b. The source must be strong and have an angular diameter which is small compared to the HPBW of the antenna. Only in that case is it possible to obtain some detailed information about the shape of the beam and the first sidelobes.

The parameters we want to find are:

1. HPBW — one finds it from the width of the recorder tracing if the beam is allowed to move over the source at a known speed.
2. Gain or effective aperture area — by measuring the antenna temperature of a small source with well known flux density.
3. Beam shape and side lobe level — as in 1. A strong source can give information on the level of a few sidelobes directly adjacent to the main beam.
4. Beam efficiency and main beam solid angle — by integrating the measured main beam and comparison with the gain, obtained under 2.

NON-IDEAL SITUATIONS

Until now we have confined ourselves to the theory in which the antenna is an ideal reflector and the feed is a point in the exact focal point. In practice, however, there are several situations which influence the radiation characteristics of the antenna. We shall deal with them quickly.

1. Most important is the deviation of the reflector surface from the mathematical paraboloid as a result of inevitable constructional tolerances. If the r.m.s. deviation of the random errors of the reflecting surface end, the gain of the antenna decreases from its theoretical value G_o to

$$G(\lambda) = G_o \exp\left\{-\left[\frac{4\pi d}{\lambda}\right]^2\right\} \tag{24}$$

This is the basic formula in the "antenna tolerance theory" which was formulated by Ruze. The formula shows how the gain of a particular antenna decreases with decreasing wavelength as a result of the imperfections in the reflecting surface. Figure 4 shows theoretical curves (based on measured surface errors by photogrammetric means) and measured aperture efficiencies using radio sources. Generally one considers antennas to have a perfect surface if $d \leq \frac{\lambda}{16}$.

2. It is possible that the feed is not exactly in the focal point.

   a. A displacement along the axis (axial defocusing) causes a gain decrease, a small beam broadening and a higher sidelobe level. For a feed displacement Δ the gain decreases as

$$G(\Delta) = G_o \left[\frac{\sin\left(\frac{\pi\Delta}{\lambda}\right)}{\frac{\pi\Delta}{\lambda}} \right]^2 \tag{25}$$

   An application of this result is the establishment of the correct focal point by measuring the antenna temperature of a point source for different positions of the feed along the axis. Figure 5 shows such a focusing curve.

   b. A lateral (also called radial) defocusing causes a shift of the beam in the opposite direction. But because of the curvature of the reflector the angular displacement of the beam is smaller than the angular feed displacement by a factor, called the beam deviation factor (BDF), which is typically about 0.85 for the NRAO telescopes. It is, of course, dependent on the F/D ratio of the paraboloid and Fig. 6 illustrates its behavior as a function of this ratio for two values of illumination taper.

There is a decrease in the gain, but the most serious effect of the lateral feed displacement is the strong first sidelobe on the side of the antenna axis, the <u>coma lobe.</u>

Figure 7 shows the coma lobe level and gain decrease as a function of the beam deviation with the edge taper as a parameter. The coma lobe is much weaker for larger F/D ratios. So if one wants to use off-axis feeds, a Cassegrainian antenna with its long effective focal length is of advantage.

3. In the primary focus antenna the feed (and commonly a part of the radiometer, the "front end") are placed near the focus of the

3. (Continued) --

reflector supported by 2, 3, 4 or even more so-called feed support legs. The legs are also needed in a Cassegrain to keep the secondary reflector in place. Feed and receiver box (secondary reflector respectively) plus support legs cause a certain obscuration of the reflector surface for the incoming radiation. This is generally called aperture blocking. Its effect is a decrease in antenna gain and an enhancement of the sidelobe level. Figure 8 illustrates this.

MISCELLANEOUS REMARKS

In concluding these lectures on antennas, I would like to mention a few points which are of considerable interest in radio astronomy but only slightly connected to pure antenna theory.

Feed Design

Much work has been put in feed design recently in an attempt to achieve a more uniform aperture illumination while still preserving a high taper at the edge of the reflector.

In this way it is possible to obtain a higher gain without increasing the sidelobe level.

A new type feed, the so-called "Scalar Feed" is now generally used at NRAO. It gives only a slightly higher aperture efficiency but the spillover radiation is considerably decreased.

MM-Wave Observations

In the mm-wavelength range strong sources are extremely rare (at least not found yet) and moreover receiver sensitivities are still rather low. The sun and the moon are the strongest sources in that frequency region; but both are large sources ($\approx 1/2°$ angular diameter). It is still possible however to find the shape of the antenna beam from a driftcurve over the sun by differentiating the record with respect to the angular coordinate. The brightness distribution over the sun (or moon) is virtually rectangular. So the crossing from the outside onto the source is a step function. The derivative of the step function is the impulse function or Dirac function. Hence, the instantaneous output of the antenna, while the beam is moving over the unit impulse function, will be proportional to the strength of the beam. Consequently, we obtain the antenna beam by differentiation of the driftcurve over the sun. Examples of this procedure are to be found in Electronics Division Internal Report No. 36.

In measuring the beam efficiency of the antenna on such an extended source as the moon or the sun one must be aware of the fact that the first sidelobes also will contribute to the measured antenna temperature.

Influence of Atmosphere

It is well known that the refractivity of the atmosphere is dependent on the temperature and the pressure of the constituent gases. Oxygen and water vapor are the main components. Due to changes in the temperature and water vapor content the atmosphere is certainly not homogeneous and there will occur spatial as well as temporal variations in the refractive index. These variations influence the directivity of the antenna and also the effective noise figure of the radio telescope.

The spatial variations occur in "blobs" of turbulence with scales (distances over which there is a correlation between the variations) ranging from a few decimeter up to over one hundred meters. Blobs larger than the aperture of the antenna will cause a tilt of the beam while smaller turbulences will broaden the beam slightly. This would mean that there is an upper limit for the obtainable resolution set by the atmosphere.

Although there is some small evidence from antenna measurements that the above effect occurs, the subject is in an uncertain position mainly because there is little detailed knowledge about the atmospheric turbulences.

At the very high frequencies (say above 15 GHz) atmospheric effects may impose a limit upon the sensitivity of the radio telescope. In that region are the absorption bands of oxygen and water vapor originating in transitions of the energy levels of the dipole like molecules of water vapor and oxygen. Because absorption is connected to radiation by Kirchhoff's law, the atmosphere will have its own fluctuating radiation with a strength mainly dependent on the amount of water vapor present. The resulting noise fluctuations received by the antenna may well be larger than the inherent noise fluctuations of the radiometer and consequently decrease the overall sensitivity of the radio telescope.

A method to decrease this effect considerably is the following. We use two feeds close to the focal point, which will project two slightly displaced beams in the far field. In the Fresnel region, however, and that is the region in space where the turbulences occur, the beams will overlap each other to some degree depending on the feed separation and the distance from the aperture. Let one of the beams be pointed on the source, then the other will be just beside it. If we switch the radiometer rapidly between the two beams and record the difference signal, we find the strength of the source. But the noise contribution of the part of the atmosphere which fills both beams in the Fresnel region will be cancelled by this procedure, because the temporal variation will be slow compared to the switching frequency. The method has been studied theoretically in some detail and looks promising. We shall investigate it experimentally in the near future.

Calibration Sources

As became clear earlier the calibration of antenna parameters is conveniently performed by using radio sources as radiators. Of course, one has to know the flux density of the source at the specific frequency as accurate as possible.

In an attempt to establish accurate flux densities for a few sources over a large frequency region, especially above 3 GHz, Mezger, Wendker and I have collected all "absolute" flux density measurements of Cas A, Tau A, and Cyg A. The resulting spectra make a calculation of the expected flux density possible with an accuracy of a few percent in the frequency region from 300 MHz to 15 GHz (for Tau A up to 35 GHz). The results are collected in Electronics Division Internal Report No. 35.

LITERATURE

General Antenna Theory:

S. Silver: "Microwave Antenna Theory and Design". Vol. 12 of MIT Rad. Lab. Series. McGraw Hill and Boston Techn. Lithographers (1963).

J. D. Kraus: "Antennas". McGraw-Hill, 1950.

R. C. Hansen: "Microwave Scanning Antennas". Vol. I, Academic Press, 1964.
Chapter 1: Aperture Theory
Chapter 2: Reflecting Systems

Specific Radio Telescope Antennas:

R. C. Hansen: See above; Chapter 4 by H. C. Ko: " Radio-Telescope Antennas".

R. N. Bracewell: "Radio Astronomy Techniques" in Handbuch der Physik, Vol. 54. Springer Verlag, 1962.

J. W. Findlay: "Antennas and Receivers for Radio Astronomy" in Advances in Radio Research, J. A. Saxton, editor. Academic Press, 1964.

NRAO Internal Reports:

J. W. M. Baars: "A Comparison Between Prime Focus and Cassegrain Antennas".

Rama C. Menon: "Atmospheric Absorption in the Wavelength Range Between 10 cm and 1 Micron".

J. W. M. Baars and P. G. Mezger: Electronics Division Internal Report No. 30, "The Characteristics of the 300-Foot Telescope at 10 cm Wavelength".

J. W. M. Baars and P. G. Mezger: Electronics Division Internal Report No. 36, "The Characteristics of the NRAO 85-Foot Telescopes at 2.07 cm Wavelength".

P. G. Mezger: Electronics Division Internal Report No. 17, " Application of Antenna Tolerance Theory to NRAO 300-Foot and 85-Foot Antennas".

EXERCISES

1. Show that the combination of a parabolic and hyperbolic reflector (with one of the foci of the hyperbola coinciding with the parabola's focus F') will focus a plane wave falling in along the axis into the second focus F of the hyperbola (fig. 1, Cassegrain).

2. Calculate the limit of the Fresnel region (see page 4) for
   a. The 300-foot (D = 91.5 m) at 21 cm wavelength.
   b. The 140-foot (D = 42.7 m) at 6 cm wavelength.
   c. The 36-foot (D = 11 m) at 3.5 mm wavelength.

3. Write out the steps on the way from eq. (2) to eq. (3). See fig. 2 and simply take $F(x, y) = 0$ outside the aperture.

4. Make a plot of eq. (6) for $b = 0.2$ and $n = 1$ up to the second minimum in $g(u)$. Use the tables on pages 180 ff of Jahnke-Emde. Normalize it to a maximum of one and also draw the normalized power pattern $f(u) = \left\{ g(u) \right\}^2$. Now plot the gaussian function $f_g(u) = \exp \left\{ - \frac{u^2}{(0.6\ \Theta)^2} \right\}$ taking for Θ the value where $f(u) = 1/2 \cdot f(0)$. Compare the two patterns and calculate the attenuation of the sidelobe in dB.

5. Prove the relation $G = 4\pi A/\lambda^2$ of eq. (11). Consider an antenna with a matched resistor at temperature T in series with a voltage generator connected to its terminals. Let the antenna see a black body also at temperature T under a solid angle Ω in a direction where the gain is G. Using the Rayleigh-Jeans law for blackbody radiation and Nyquist formula for the power delivered by a resistor, we can prove the required result, stating that there exists thermal equilibrium.

6. Integrate $f_m(\xi, \eta)$ of eq. (13) over ξ and η from $-\infty$ to $+\infty$ and obtain eq. (14).

7. Derive the relations of eq. (16) and (17) using earlier results.

8. The antenna solid angle of the 85-ft. antenna is $4 \cdot 10^{-2}$ square degrees at 6 cm wavelength. The HPBW at that wavelength is 10' (arc minutes). Calculate the beam efficiency, aperture efficiency and the effective absorption area.

9. Calculate the factor Ω_s'/Ω_m of eq. (23) for a gaussian antenna beam with circular symmetry and HPBW Θ_A, and a gaussian symmetrical source with a halfwidth Θ_s. We observe the source Tau A (assuming a symmetrical gaussian with $\Theta_s = 3.5'$) with the 85-ft. at 6 cm. The antenna temperature is $T_A = 56\,°K$. Calculate the brightness temperature T_b (eq. (23)) and the flux density S_ν (eq. (18)) at this frequency.

10. The 300-ft. has an offset feed for 21 cm wavelength giving a beam, which is displaced 2 HPBW from the antenna axis.
    a. Find the distance between the feed center and the focal point of the antenna (F/D = 0.425).
    b. What edge taper would you choose if the sidelobe level should stay below -16 dB.
    c. What is the decrease in gain. Use the figures of these notes.

11. Our new 11 m diameter mm-wave antenna shall have a r.m.s. surface error of $d \lesssim 0.06$ mm. Calculate how far the gain will be under the maximum obtainable at a wavelength of 1.2 mm (eq. (24)).

N.B. Please do not forget to note in your calculations that the solid angles in the formulae are in steradians. The factors needed for calculations in square degrees or arc minutes are to be found in the note on page 13.

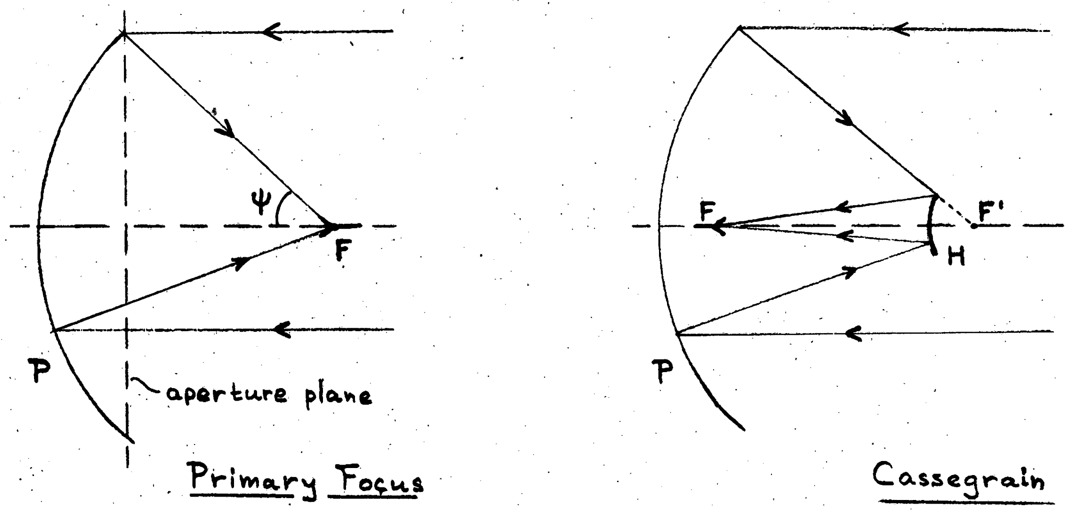

fig. 1

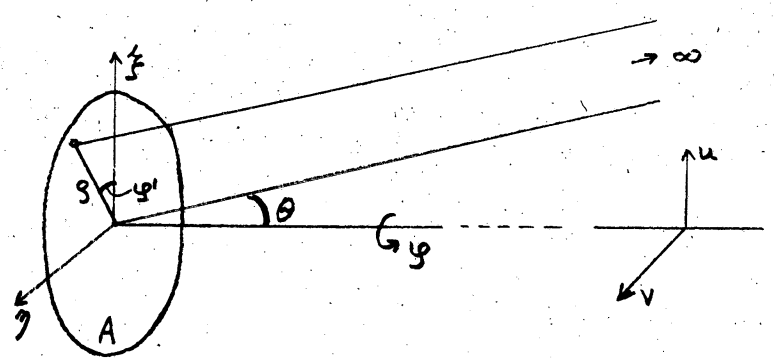

fig. 2 Geometry of the Radiation Problem

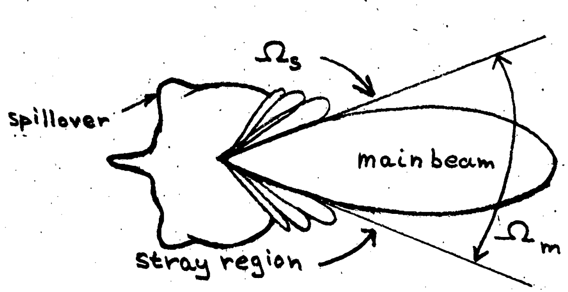

fig. 3 Illustrating eq.(12).

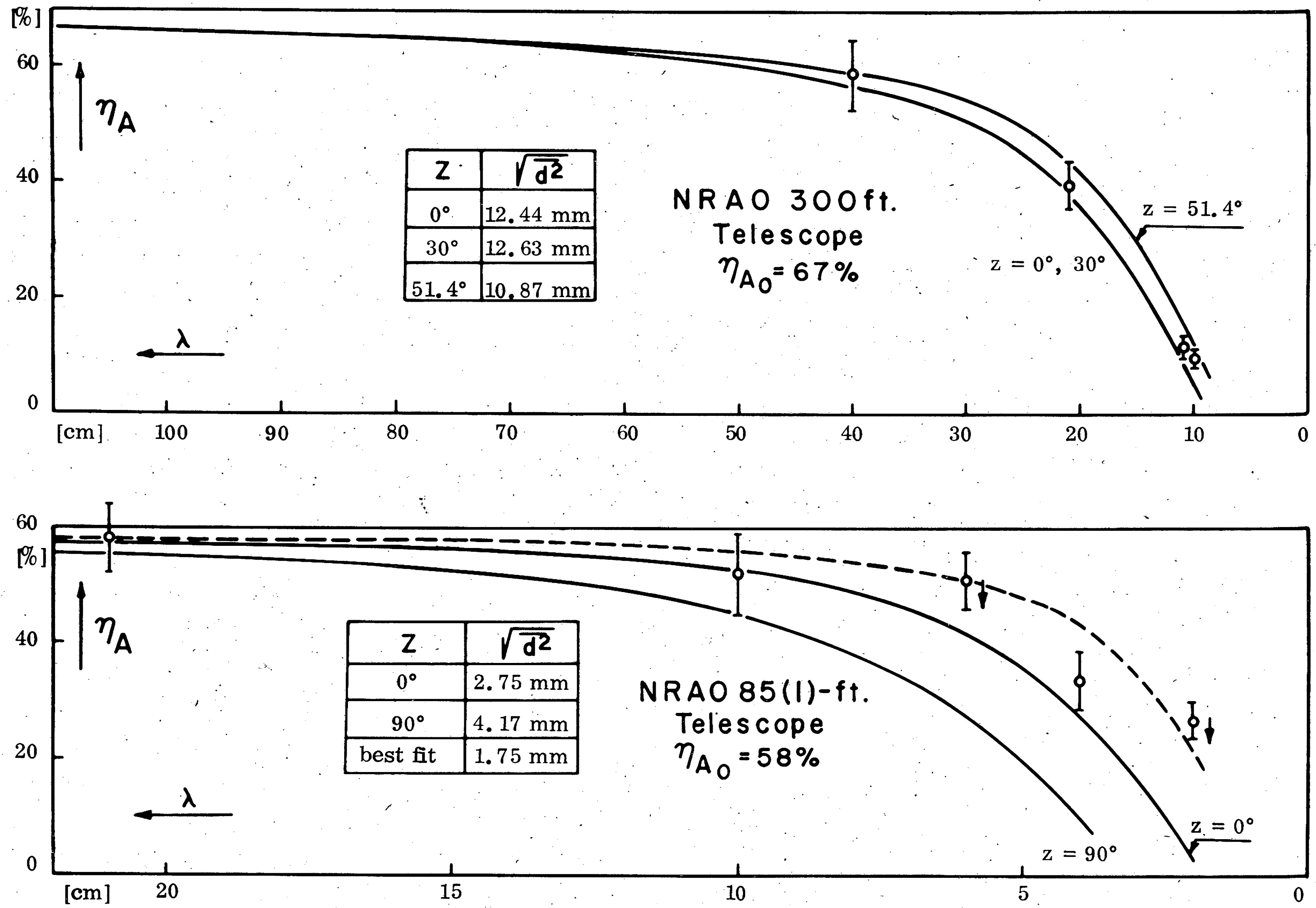

Fig. 4 Aperture Efficiency as Function of Wavelength

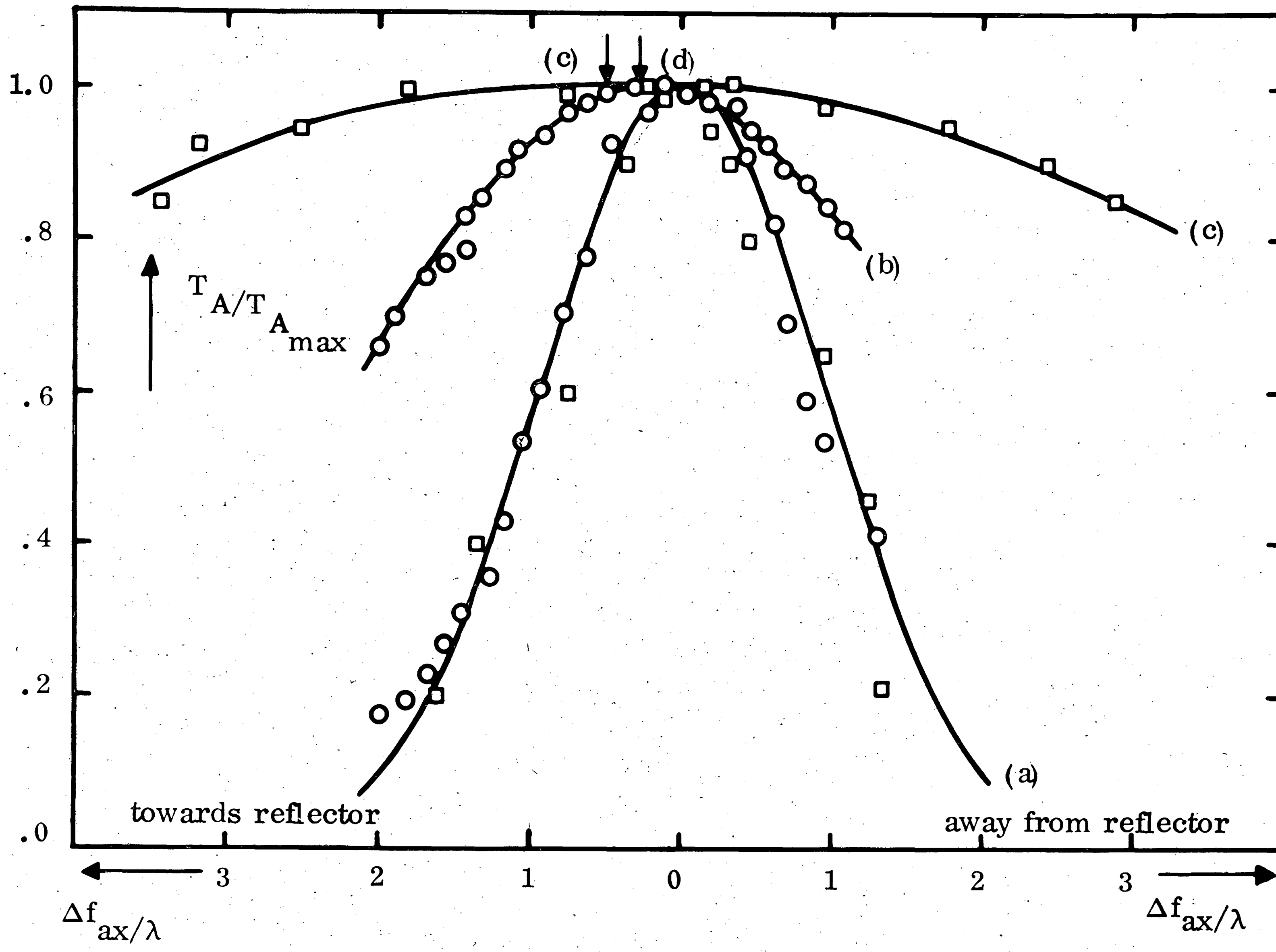

FIG. 5 FOCUSING CURVE OF 85(I). 00 at 6 GHz, ◻◻ at 14. 5 GHz
a: point source, band c: moon.
b y c

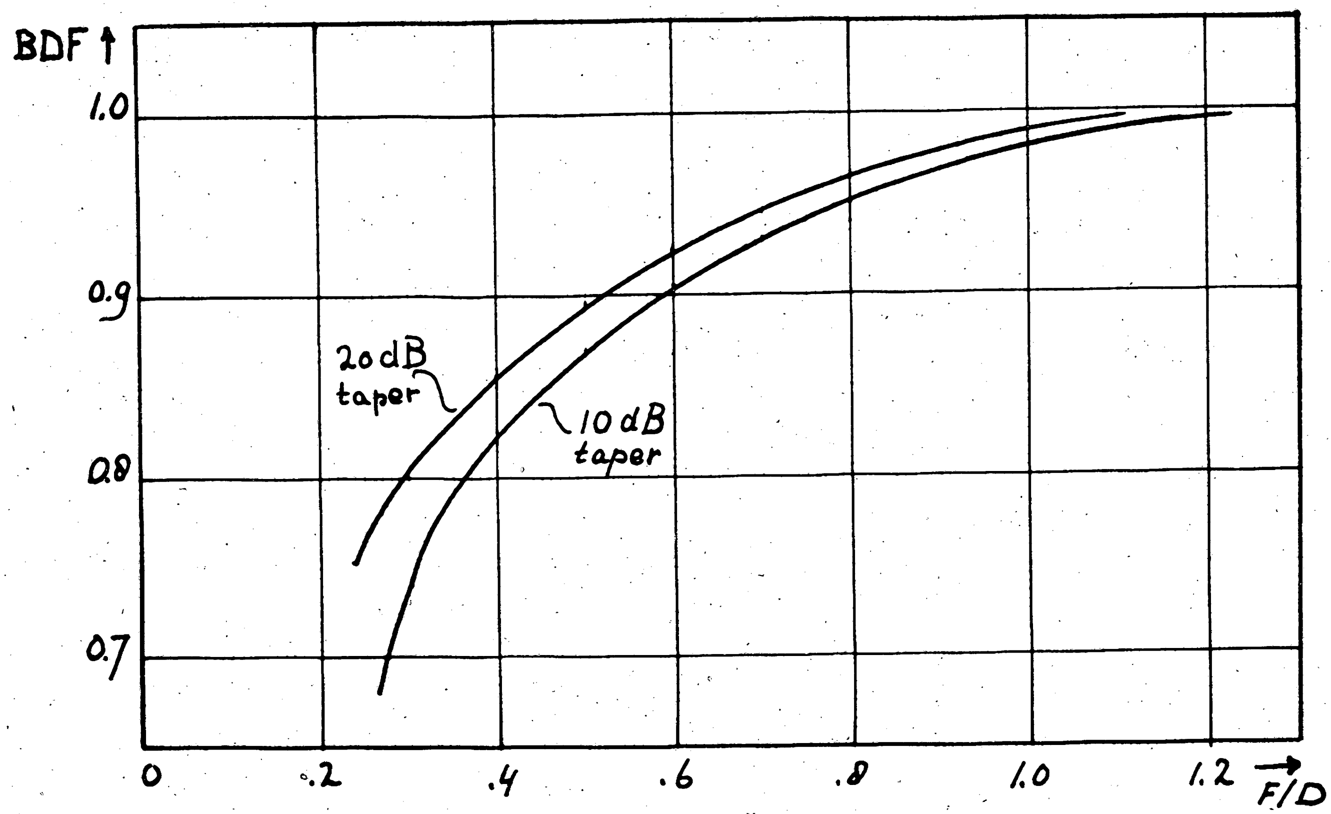

FIG. 6 BEAM DEVIATION FACTOR AS FUNCTION OF F/D RATIO FOR TWO ILLUMINATION TAPERS