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Summer Student Lectures, 1970

POLARISATION

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Introduction, Stoke's Parameters, and the P,χ representation

The radiation from most extragalactic radio sources is partially linearly polarised. This means that if we observe the source with a simple dipole feed, then the power that we receive changes as we rotate the feed, Fig. 1.

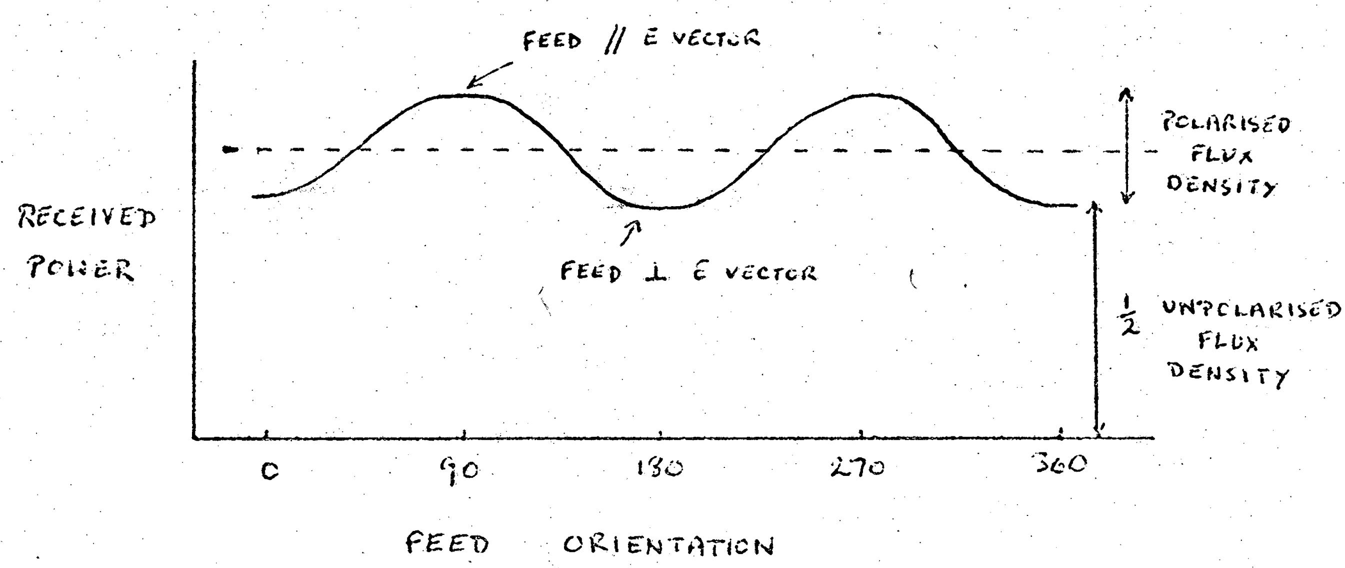

Fig 1

The orientation of the feed for which we receive maximum power gives the orientation, χ, of the electric vector of the polarised component of the incident radiation. The relative range of variation of the received power as we rotate the dipole gives the "degree of linear polarisation", m. Thus if we observe 100% polarised radiation with a feed aligned at right angles to the electric vector, then we will receive no power.

In the general case radiation is partially <u>elliptically</u> polarised. This means that it can be decomposed into three components: an unpolarised component (or more strictly, a randomly polarised component), a circularly polarised component, and a linearly polarised component. The formal treatment of this case (see, for instance, Kraus,1966, ch. 4), is done in terms of the Stoke's parameters, I, V, Q and U.

All four parameters have the dimension of <u>power</u>. I is the total power contained in the radiation. $|V|$ is the power in the circularly polarised component. If V is positive then we have lefthand circular polarisation, and if it is negative we have righthand circular polraisation. (This is purely conventional. Just for fun the optical astronomers use the opposite convention.)

The linearly polarised component is described by Q and U. Imagine that we observe this radiation with crossed dipoles, Fig. 2a.

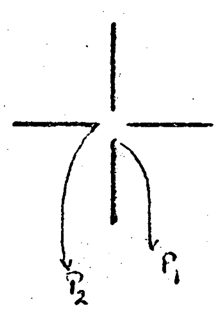

Fig 2a

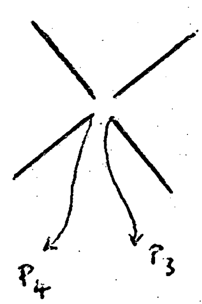

Fig 2b

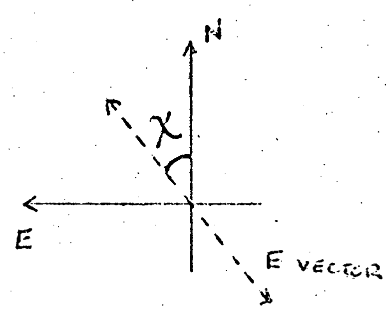

Fig 2c

If there is a linearly polarised component present, then in general the powers P_1 and P_2 received by the two dipoles will not be equal. We define

$$Q = P_1 - P_2$$

However in the case where the plane of the electric vector bisects the angle between the dipoles, then obviously $P_1 = P_2$, so we need to make another measurement. Rotate the dipoles through 45°, Fig. 2b, and measure the received powers again. We define

$$U = P_3 - P_4$$

This now pins down the linearly polarised component. The power in this component is $\sqrt{Q^2+U^2}$, so the degree of linear polarisation is

$$m = \sqrt{Q^2+U^2}\Big/ I$$

The orientation or "position angle" of the E vector, measured on the sky from North through East, Fig. 2c, is given by

$$\chi = \frac{1}{2}\tan^{-1} U/Q$$

The power in the unpolarised component of the radiation is simply $I - \sqrt{Q^2+U^2+V^2}$.

Stoke's parameters are a bit clumsy, and we don't use them if we can help it. However, they have the important property that since they are <u>powers</u> they are <u>additive</u>. That is, if we know the individual Stoke's parameters of each part of a source, then the Stoke's parameters of the combined radiation is simply the sum of the individual Stoke's parameters.

For synchrotron radiation there is (approximately) no circularly polarised component, i.e., $V = 0$, so we usually work in terms of the position angle of the electric vector, χ, and either the percentage polarisation, m, or the flux density of the polarised component, P. If the total flux of the source is S, then obviously $m = P/S$.

Although P has the units of flux density, we turn it into a vector, or a complex quantity $P \exp(2j\chi)$. In this way the contributions from different parts of the source add vectorially. Consider an extended radio source, and set up Cartesian coordinates in the sky. If an elementary area of the source emits a total flux $S(x,y)dxdy$, and a polarised flux $P(x,y)dxdy$ with position angle $\chi(x,y)$, then the integrated polarisation of the source (i.e., measured with the whole of the source inside the beam) is just given by

$$m\, e^{2j\chi} = \frac{\iint P(x,y)\; e^{2j\chi(x,y)}\, dxdy}{\iint S(x,y)\; dxdy}$$

The 2 in the exponent is a little puzzling at first, but it is simply a mathematical trick so that we can express the polarised radiation as a vector. The orientation of the E vector defines only a plane and not a direction. If we rotate χ through 180°, we come back to the same situation. However 2χ will have rotated through 360° before returning to the same situation, which is how a vector should behave.

Why is synchrotron radiation polarised?

Consider an electron moving in a circular orbit in a magnetic field, Fig. 3.

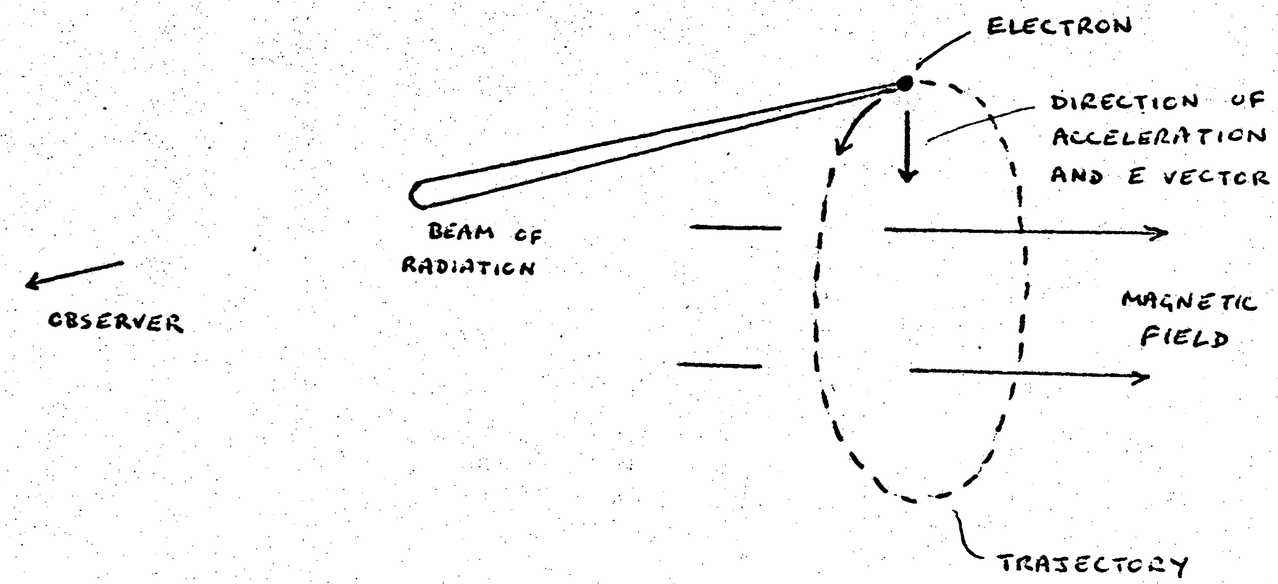

Fig 3

All the time the electron is being accelerated towards the center of its orbit. It therefore radiates continuously, and at any instant the electric vector of the radiation has the same direction as the acceleration. If the electron is not relativistic, then we can see it at all points on its orbit, so the E vector rotates with the electron, and the radiation is circularly polarised. This is called "cyclotron" or "gyro" radiation.

In the synchrotron case, the electron is relativistic, and all the emitted radiation is beamed into a narrow cone in the instantaneous direction of motion of the electron. Hence we only see the electron when it is moving directly towards us, i.e., when it is at the top of its orbit in Fig. 3. At this point the electric vector is perpendicular both to the line of sight and to the magnetic field. Hence synchrotron radiation is linearly polarised and the plane of polarisation is perpendicular to the magnetic field.

The above argument may suggest that synchrotron radiation should be 100% polarised. In fact the electron orbits are not all coplanar. Most electrons will have open orbits with a wide range of pitch angles. If we have an ensemble of electrons with an isotropic velocity distribution and whose energy spectrum is

$$N(E)\, dE \propto E^{-\gamma}\, dE$$

then Ginzburg and Syrovatskii (1965) show that

$$m = \frac{\gamma + 1}{\gamma + 7/3} \times 100\%$$

This is not very sensitive to the exact value of γ. If a radio source has a spectral index of -0.7, then $\gamma \sim 2.5$ and m is about 70%.

Measuring Polarisation

I shall not discuss in detail how you actually measure polarisation. In practice it is probably the most difficult measurement to make in all radio astronomy. This is because polarised flux densities are very small and instrumental effects are very large.

The most popular way of measuring polarisation with a single dish is to rotate a linearly polarised feed and see how the received power varies. This is discussed by Gardner and Davies (1970).

The best way to measure linear polarisation with an interferometer is to use a lefthand circularly polarised feed on one dish and a righthand circularly polarised feed on the other dish. This is what we do here, and it is discussed by Conway and Kronberg (1969).

The Astronomical Results

"There is a demand nowadays for the man who can make wrong appear right."

Publius Terentius Afer (190 - 159 BC)

Integrated Polarisation, Faraday Rotation, Depolarisation

If we measure m and χ for an extragalactic source at many different wavelengths, a typical result is shown in Fig. 4.

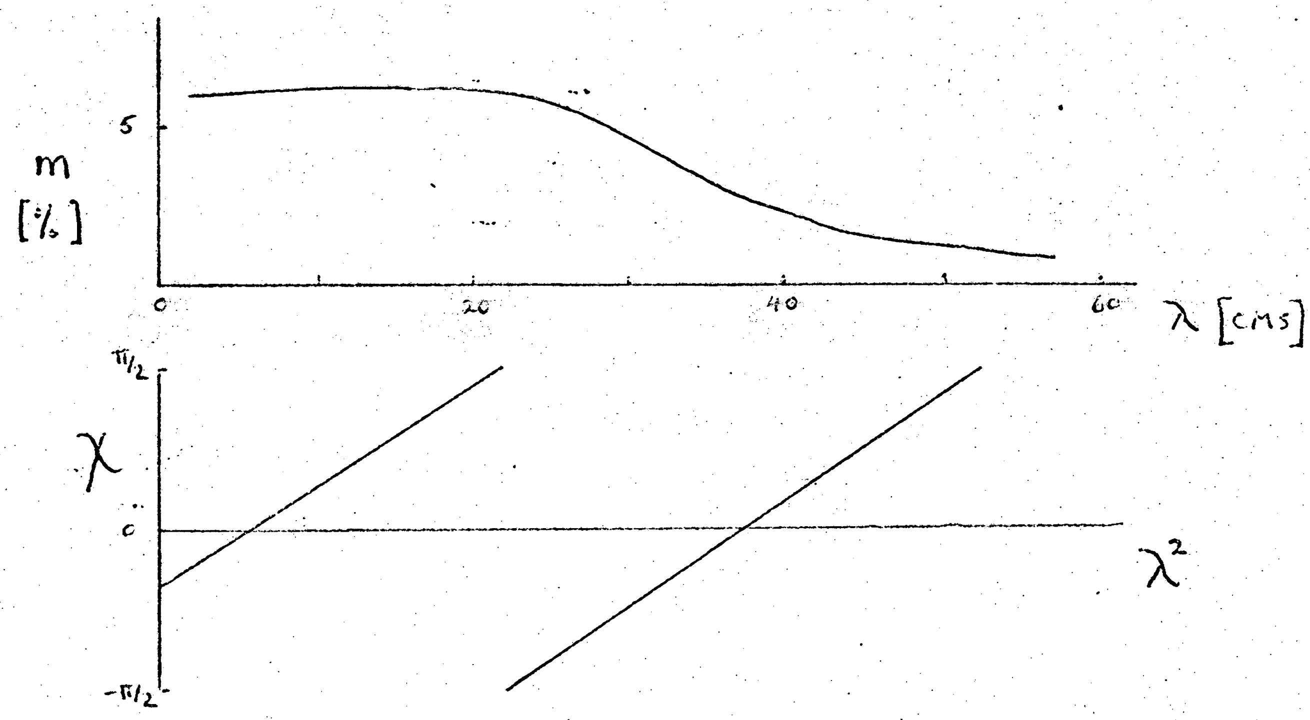

Fig 4

Notice three features.

(1) Even at short wavelengths m is much less than the 70% suggested by synchrotron theory. This is because the magnetic field has different orientations in different parts of the source, and the integrated polarisation is

the vector sum of the contributions from each part of the source. In the case where we have a magnetic field with random direction and strength H_r, superimposed on a uniform field of strength H_o, then the resultant polarisation is given roughly by (Burn, 1966)

$$m \sim \frac{\gamma + 1}{\gamma + 7/3} \times \frac{H_o^2}{H_o^2 + H_r^2}$$

Thus m serves as a measure of the degree of order in the magnetic field of the source.

(2) χ is not constant, but is often proportional to λ^2. This is due to Faraday rotation in a magnetoionic medium somewhere along the line of sight to the source, (or possibly inside the source itself). Thus χ often fits a formula like

$$\chi(\lambda) = \chi_o + K\lambda^2$$

χ_o is called the "intrinsic position angle" and tells us the orientation of the magnetic field in the source. K is called the "rotation measure" and tells us about the medium between us and the source. If the Faraday rotation occurs in a plane parallel slab of thickness L parsecs, containing free electrons with a density N cm^{-3} and a uniform magnetic field of B gauss inclined at an angle θ to the line of sight, then (Kraus, 1966, p 143)

$$K = 8.1 \times 10^5 \text{ NBL } \cos\theta \quad \text{radians/meter}^2.$$

K can be either positive or negative depending on whether the field is directed towards or away from the observer.

Most of the Faraday rotation that we observe is thought to be due to the interstellar medium in our own Galaxy, since

(a) K is largest ($\sim$200) for sources at low Galactic latitudes (when L, the path length through the Galactic disc, is large), and is negligible near the Galactic poles.

(b) The sign of K (giving the field direction) depends systematically on Galactic longitude. This dependence is consistent with a Galactic magnetic field consisting of a tightly wound helix along the local spiral arm, sheared by the differential rotation of the Galaxy. All the data are reviewed by Berge and Seielstad (1967).

(3) m usually drops to a small value at long wavelengths. This is called "depolarisation", and is a thorny problem. The "cutoff" wavelength varies widely from source to source, but is typically about 30 cms (1000 MHz). Remembering that the formula for m given by synchrotron theory is independent of λ, there are three obvious ways in which m can vary with λ.

(a) The source may consist of several components with various values of m and spectral index. Thus at different wavelengths, different components dominate. This would suggest that steep spectrum components have low polarisations. In fact the reverse appears to be true (see comments in the next section).

(b) Lines of sight through the Galaxy to different parts of the source may have different rotation measures. At long wavelengths different parts of the source will have different values of χ, and so the vector summation of all the polarised radiation will fall off.

This is just a random walk problem. Imagine there are irregularities in the interstellar medium, of size S parsecs and electron density N cm^{-3}, which contain a magnetic field of B gauss whose direction is random. If the path length of the line of sight through the Galaxy is D parsecs, then the standard deviation of the Faraday rotations along different lines of sight is given by (Bologna et al., 1969)

$$\sigma(\lambda) \sim 6 \cdot 10^5 \lambda^2 S^{1/2} D^{1/2} NB \quad \text{radians/meter}^2 .$$

m will fall to a small value when $\sigma \sim 1$. In fact Burn (1966) gives

$$m(\lambda) \sim m(o) \exp(-2\sigma^2).$$

There is some evidence that this process does in fact take place. Large angular diameter sources appear more likely to depolarise than small sources. Also, there are no sources at low Galactic latitude which are strongly polarised at long wavelengths. However, it is rather difficult to find reasonable values of S, N and B (see Bologna et al., 1969). The smallest reasonable value of S is about 1 parsec, so sources smaller than about 20" arc should not depolarise. Many of them do, however, so this is not the whole answer.

(c) A process like the one just described may take place inside or near the source itself. If emission and Faraday rotation occur in the same region then the situation becomes a bit complicated. This has been treated by Burn (1966), but only for very idealised models.

Probably processes (b) and (c) are both important, but no satisfactory treatment of depolarisation has yet been given.

It is worth pointing out that it is very difficult to measure polarisation at long wavelengths. The background radiation from our own Galaxy has a steep spectrum and is strongly polarised. Also the ionosphere introduces a fluctuating Faraday rotation of its own.

Polarisation, Structure, and Source Models

Few high resolution observations of the distribution of polarisation across a radio source have been made so far. The first sources to be studied were mainly very large and strong (e.g., Centaurus A, Virgo A, Cygnus A, Fornax A) and these are probably not very typical sources. Fanbeam observations with a resolution of 1' x 20' have been made with an E-W interferometer by Morris and Whiteoak (1968), and single dish observations with a resolution of 7.5' have been made by Davies and Gardner (1970). These show typical results.

Usually each component of a radio source is polarised, and sometimes there are relatively compact regions which are very highly polarised (20-30%), showing regions with highly ordered magnetic fields.

For a single component, the centroid of the polarised radiation is not usually coincident with the centroid of the unpolarised radiation. This is to be expected. Other things being equal (e.g. if the relativistic electron density is fairly constant over a large region) then the unpolarised radiation comes mainly from where the magnetic field is strongest, and the polarised radiation comes from where the field is most ordered. If the magnetic field is drawn out of the parent object by the ejected component, then the field will be disordered at the turbulent interface with the intergalactic medium, but more ordered at the trailing edge of the component where the field lines are stretched out. This agrees with observation.

Two features are clear from measurements of the integrated polarisation. (1) For double sources, there is a convincing relation between the intrinsic position angle, χ_o, and the orientation of the line joining the two components, θ.

In compact high brightness sources $\chi_o - \theta$ lies between 60° and 90°, showing the magnetic field is approximately parallel to the axis of the source. In extended low brightness sources, $\chi_o - \theta$ lies between 0° and 30°, and the magnetic field is nearly perpendicular to the axis of the source. Since these two source types are probably opposite ends of an evolutionary sequence, it is not clear how or when this change takes place. The orientation of the field in the high brightness case is consistent with the ejected plasmons dragging out the magnetic field from the parent object.

(2) If you divide all sources into two types--scintillators and non-scintillators (sources which scintillate must contain structure smaller than a few tenths of a second of arc), then there are clear differences in the median polarisations of the two groups. Gardner and Whiteoak (1969) give the following table.

	Quasars	Radio Galaxies	Unidentified
Scintillators	3.1 (29)	1.5 (15)	1.2 (17)
Non-scintillators	3.7 (17)	5.1 (52)	3.5 (22)
Total	3.5 (59)	3.3 (76)	2.6 (49)

The numbers in parentheses give the number of sources in each group, and the other numbers are the median polarisations in percent.

It is clear that for radio galaxies at least, very compact structure is not strongly polarised. This is understandable to some extent. Inside a compact "hot spot" there is a high density of very energetic electrons contained in a very small volume. The simplest sort of containment is some sort of "magnetic bottle", in which case we would expect all field directions to be present inside this small region and so the net polarisation will be low.

We get problems when we apply this idea to the compact quasars such as 3C 345 and 3C 279. These sources show negative depolarisation (Berge and Seielstad, 1969). That is, m decreases at short wavelengths. At the shortest wavelengths the radiation comes from an exceedingly small ($\sim$.001") region, and m is small as we would expect. (Compare the plots of $m(\lambda)$ given by Berge and Seielstad (1969), with the spectra of the total radiation given by Kellermann and Pauliny-Toth (1969)). However the radiation at intermediate wavelengths comes from similar but larger compact components which have expanded adiabatically, and m increases! Why does the field get more ordered with time?

Make what you will of the unidentified sources in the above table. Their median polarisation is near to that of the quasars, but the dependence on scintillation is similar to that of radio galaxies.

Synchrotron self-absorption, variable sources, and circular polarisation

Many compact sources show a turnover in their total spectrum at the long wavelength end. This is attributed to their absorbing their own radiation; i.e., becoming optically thick. According to synchrotron theory the expected polarisation changes (Pacholczyk and Swihart, 1967). The polarisation is now given by

$$m = \frac{3}{6\gamma + 13}$$

$$\sim 10\% \text{ for } \gamma \sim 2.5.$$

Also, the plane of polarisation rotates through 90°, and is parallel to the magnetic field. So far this effect has not been observed unambiguously, but it is probably important in sources which show variable polarisation.

Allen and Haddock (1967) have shown that several quasars, whose total fluxes fluctuate wildly at short wavelengths, also show variable polarisation (both m and χ) at 8000 MHz. The changes can be very fast ($\sim$ weeks) and often occur just before the unpolarised flux starts a rapid increase.

The variation in the total flux is attributed to a compact cloud which is initially optically thick. It expands adiabatically and soon becomes optically thin. (See Kellermann and Pauliny-Toth, 1968). Thus part of the changes in the polarisation may be due to the effects mentioned two paragraphs back. However the variations are very complex and no model has been devised which explains them in detail.

Under certain conditions synchrotron radiation can be circularly polarised. Cyclotron radiation is strongly circularly polarised, so if we look at electrons which are not too relativistic (i.e., at long wavelengths and/or in a source with a very strong magnetic field) we might expect to observe a small amount of circular polarisation.

The expected degree of circular polarisation, m_c, is given by Legg and Westfold (1968) as

$$m_c \sim \left(\frac{f_B \sin\theta}{f}\right)^{1/2} \cot\theta$$

where f is the observing frequency, f_B is the electron gyrofrequency and θ is the inclination of the field to the line of sight.

So far no circular polarisation has been detected convincingly. The most recent upper limits ($\leq$ 0.5%) on m_c for a large number of sources are given by Seaquist (1969). It is an important measurement to make since m_c

gives the magnetic field strength directly. The one source for which a positive detection of circular polarisation may have been made is the enigmatic BL lac, (Biraud and Véron, 1968). This source is so variable, however, that it is almost impossible to check other people's measurements.

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