SUMMER LECTURE NOTES 1970

WS/GR BR/

GALACTIC DYNAMICS AND GALACTIC STRUCTURE

by S. Gottesman

I. GALACTIC ROTATION

The earliest evidence about the structure of our galaxy (the Milky Way system) comes from Shapley's work on globular clusters. This indicated that these objects were spherically distributed in space and that the sun was at the edge of this system. It was also found that these objects have velocities of the order of 250 km/s. If this system is at rest with the galaxy as a whole it implies that the sun rotates about the center of the galaxy, with a velocity of 250-300 km/s. Further, ignoring certain classes most stars showed a velocity variation of ±30 km/s about the sun. This implies that these stars share the sun's motion about the galactic center.

We consider a flat disk stellar system in circular rotation about an axis. The mass of the system is centrally concentrated. Stars will travel in almost Keplerian orbits about the galactic center. This form of motion where the velocity increases with decreasing radius is known as differential galactic rotation and implies the presence of shear forces.

Let us erect a coordinate system at the sun with the -y axis in the direction of the galactic center and the +x axis in the direction of the sun's motion. For differential rotation we expect more rapid circular velocities for $-90<\ell<90$ and less rapid circular velocities for $90<\ell<270$. At $\ell=0$, 90, 180, 270 we see no radial motion. Thus the expected plot of radial velocities would be $V_r \propto \sin 2\ell$. Tangential velocities or proper motion will also occur due to differential motion of stars at different distances from the galactic center.

r = radius of some point in the galaxy

R_o = radius of the local standard of rest (LSR)

Θ = linear rotation velocity at r

Θ_o = linear rotation velocity of the LSR

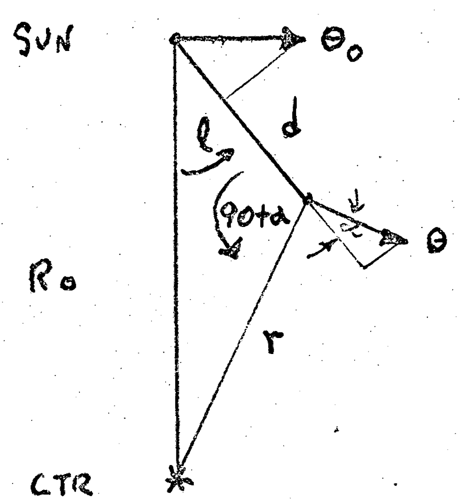
ω = Θ/r = angular velocity at r about galactic center

ω_o = Θ_o/R_o = angular velocity of the LSR about galactic center

d = distance from the sun to some point at radius r from galactic center

V_R = observed radial velocity relative to the sun

V_T = observed tangential velocity relative to the sun



$$V_R = \Theta \cos \alpha - \Theta_o \sin \ell$$

By the sine law

$$\frac{\sin \ell}{r} = \frac{\sin (90 + \alpha)}{R_o} = \frac{\cos \alpha}{R}$$

by substitution. The radial component

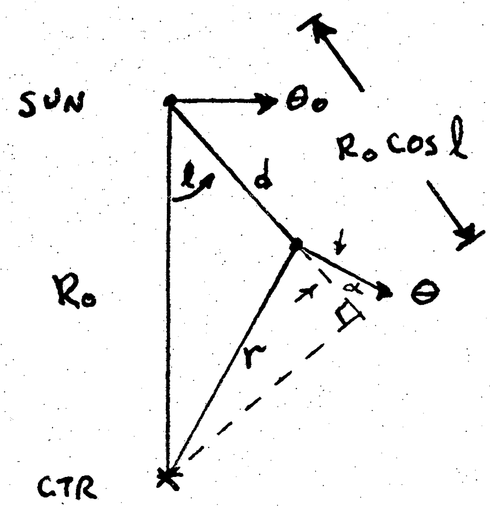
$$(1) \quad V_R = R_o (\omega - \omega_o) \sin \ell$$

This is true only if particles move in circular orbits about the galactic center.

The tangential or transverse component

$$V_T = \Theta \sin \alpha - \Theta_o \cos \ell$$

By convention, $V_T > 0$ for increasing longitude.

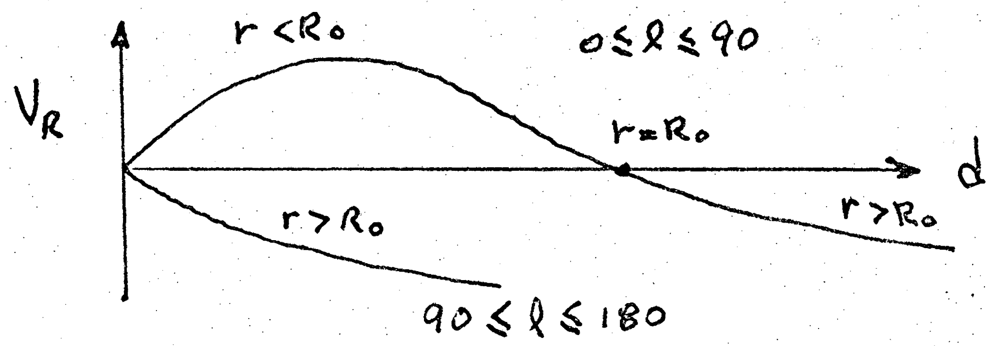


$$r \sin \alpha = R_o \cos \ell - d$$

or

$$(2) \quad V_T = R_o (\omega - \omega_o) \cos \ell - \omega d$$

In the region $90 \leq \ell \leq 180$ if we hold ℓ constant $\omega < \omega_o$ and the difference increases as we increase r. For the region $0 \leq \ell \leq 90$ we first encounter regions of small r and large ω which reaches a maximum when r is normal to the line of sight ($r_{min} = R_o \sin \ell$ and $d = R_o \cos \ell$) as $r \to R_o$ $\omega \to \omega_o$ and $V_R \to 0$ and when $r > R_o$ $\omega < \omega_o$ and $V_R < 0$



In many cases observations are restricted to the region near the sun and we may simplify our formulae. To a first order

$$\frac{\omega - \omega_o}{r - R_o} = \left(\frac{d\omega}{dr} \right)_{R_o}$$

$$\frac{d\omega}{dr} = \frac{1}{r} \frac{d\Theta}{dr} - \frac{\Theta}{r^2}$$

Thus $$V_R = (r - R_o) \left[\left(\frac{d\Theta}{dr}\right)_{R_o} - \frac{\Theta_o}{R_o} \right] \sin \ell$$

In the case where $d \ll R_o$

$$R_o - r \simeq d \cos \ell$$

or $$V_R \simeq \left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta}{dr} \right)_{R_o} \right] \frac{1}{2} \sin 2 \ell$$

subscript

We define the first of Oort's constants

$$\text{(3)} \qquad A \equiv \frac{1}{2}\left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta_o}{dr}\right) R_o\right]$$

and therefore,

$$\text{(4)} \qquad V_R = A\ d \sin 2\ \ell$$

This equation is valid <u>ONLY</u> if $d \ll R_o$ and circular motion pertains.

With regard to the transverse component (Eq. 2)

subscript

$$\omega - \omega_o = \frac{r - R_o}{R_o}\left[\left(\frac{d\Theta}{dr}\right) R_o - \frac{\Theta_o}{R_o}\right]$$

By use of a Taylor Expansion

$$\omega d = d\left[\omega_o + \left(\frac{d\omega}{dr}\right) R_o\ (r - R_o) + \ldots\right]$$

If $d < < R_o$ then $(r - R_o) \sim - d \cos \ell$

and $\omega d > \omega_o d - \left(\frac{d\omega}{dr}\right) d^2 \cos \ell$

and to a first order $\omega d = \omega_o d$ for material <u>near</u> the sun,

or

$$V_T \simeq \left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta}{dr}\right) R_o\right] d^2 \cos^2 \ell - \frac{\Theta_o}{R_o}\ d$$

which may be rewritten

$$V_T \simeq \frac{1}{2}\left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta}{dr}\right) R_o\right] d \cos 2\ell - \frac{1}{2}\left[\frac{\Theta_o}{R_o} + \left(\frac{d\Theta_o}{dr}\right) R_o\right] d$$

We define the second of Oort's constants

$$\text{(5)} \qquad B \equiv -\frac{1}{2}\left[\frac{\Theta_o}{R_o} + \left(\frac{d\Theta}{dr}\right) R_o\right]$$

Thus

$$\text{(6)} \qquad V_T = d\ (A \cos 2\ell + B)$$

From our definitions of A and B

(7) $$\frac{\Theta_o}{R_o} = \omega_o = A - B$$

(8) $$\left(\frac{d\Theta}{dr}\right)_{R_o} = -(A + B)$$ or (8a) $$A = -\frac{1}{2}\left(R \frac{d\omega}{dR}\right)_{R_o}$$

Observational proof of equation 4 was obtained from Cepheid variables in the late 1930's.

We wish to determine $\omega(r)$ but we can only readily observe $V_R(\ell)$. Thus if we can determine A, B, and R_o we may use equation 1 to establish $\omega(r)$.

Determination of A

(1) Measure V_R for stars near the sun for which d is known ($V_R = d\ A \sin 2\ \ell$).
(2) From the study of proper motions as a function of longitude.
(3) From the definition $A = -\frac{1}{2} R_o \left(\frac{d\omega}{dr}\right)_{R_o}$.

Methods (1) and (2) are clearly better than (3)

$$A = 15 \text{ km/s}|\text{kpc} \qquad (\text{known} \approx 10\%).$$

Determination of B

(1) From the study of proper motion.
(2) From other dynamical considerations yielding the ratio $-B/A$.

B is inherently more difficult to determine than A

$$B = -10 \text{ km/s}|\text{kpc} \qquad (\text{known} \approx 20\%).$$

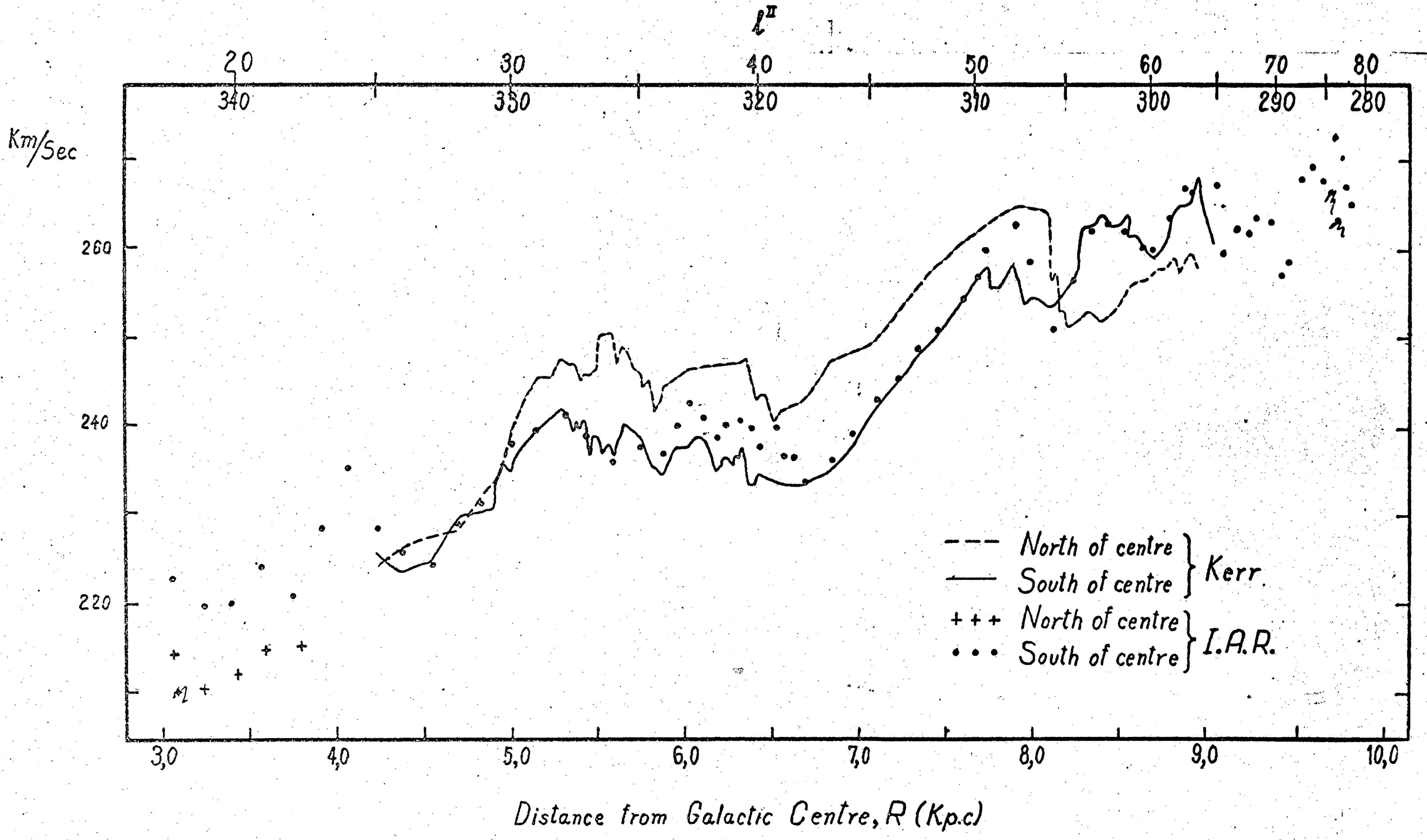
Determination of R_o

(1) Directly obtained from variable stars about the galactic center.
(2) Measurement of the product AR_o.
At any longitude ω is maximum when r is minimum. Thus occurs $r = R_o \sin \ell$
Hence $V_{max} = 2\ A\ R_o\ (1 - \sin \ell) \sin \ell$.
(3) Objects with V_R=0 must be at distance R_o from galactic center.
At $r \sim R_o$, $d = 2\ R_o \cos \ell$. Thus knowing d and ℓ yields R_o.

Methods (1) and (2) yield R_o=10 kpc (known $\approx$ 10%).

Determination of Θ_o

(1) Velocity measured relative to extragalactic systems, $\Theta_o \sim 250$ km/s from the latest publication.
(2) Velocity measured relative to globular cluster subsystem, $\Theta_o \sim 200$ km/s. (As this subsystem probably relates we may say $\Theta_o \gtrsim 200$ km/s.)
(3) From consideration of the escape velocity.
(Clearly stars with enough energy will escape from the solar neighborhood of the galaxy. Relative to the LSR $V_{esc}^2 = \Pi^2 + (\Theta_o + \Theta^*)^2 + Z^2$ where Π = radial component, Z = component normal to galactic plane, Θ = component normal to Π, yields Θ_o= 276 ± 26 km/s, depends on validity of assumptions, particularly about mass gradients).



GALACTIC ROTATION CURVE for R_0=10

(4) $\Theta_o = (A-B)R_o$, $\Theta_o = 250$ km/s and solar period = 2.45×10^8 yrs.

We may use these values and equation 1 to determine $\omega(r)$ or $\Theta(r)$. Radio studies of the neutral hydrogen spectral line at $\lambda = 21$ cm allow an investigation of the total galaxy to be made; obscuration is not a problem for long wavelengths.

The following table is indicative of galactic rotation.

r kpc	$\Theta(r)$ km/s	$R_o = 10$ kpc
0.32	220	
0.67	265	
3.53	206.4	
6.18	239.6	
7.74	248.5	
8.01	252.2	
10.00	250.0	

The most secure points of the observed rotation are determined from the point of maximum velocity which comes from $r_{min} = R_o \sin \ell$. At certain longitudes the line of sight is tangent to a spiral arm and the signal is a maximum at r_{min}. These longitudes are about 53.°4, 50.°9, 38.°4 and 20.°9. For $r > R_o$ there will be no maximum V_r and thus $\omega(r)$ can not be determined and we must rely on stellar studies.

r kpc	$\Theta(r)$ km/s
11	244
12	236
13	227

The radio technique assumes circular motion to derive $\Theta(r)$. We therefore expect circular symmetry in the velocity field. The latest studies do not show this. No fully accepted explanation has been brought forth.

II. THE DISTRIBUTION OF MASS IN THE GALAXY

In a spherical system with uniform density the force per unit mass outside some radius r

$$F_r = -\frac{4}{3}\pi G\rho R.$$

This must be equivalent to the central force on the orbit

$$\omega^2 R = \frac{4}{3}\pi G\rho R$$

or

$$\frac{\partial\omega}{\partial R} = 0 \qquad A = 0.$$

Thus

$$B = \omega \quad \text{or} \quad B = \left(\frac{4}{3}\pi\rho G\right)^{1/2}.$$

Hence for this system there is NO differential rotation.

If all the mass is centrally concentrated the force per unit mass

$$F_r = - \frac{GM}{R^2}$$

and

$$\omega^2 R = \frac{GM}{R^2}$$

$$A = - \frac{1}{2} R \frac{d\omega}{dR} = \frac{3}{4} (GM)^{1/2} R^{-3/2} = \frac{3}{4} \omega$$

$$A - B = \omega \quad \text{or} \quad A = \frac{3}{4} \omega \qquad \text{and} \quad B = - \frac{1}{4} \omega \; .$$

Near the sun $A = - \frac{3}{2} B$.

While the "solid body" rotation $A = 0$ and for a Keplerian system $A = - 3B$. Thus for the Milky Way the mass is distributed between these extremes. The sun is far from the center, there must be a large density gradient towards the center and a significant fraction of the mass is distributed over the total galaxy.

III. SPIRAL STRUCTURE OF THE GALAXY

It became clar that northern and southern observations did not demonstrate the expected circular symmetry. The first explanation involved radial motions of the gas in the plane of the galaxy.

A Russian model of the galaxy was proposed in 1964 (Kardashev, N.S., Lozinskaya, T.A., and Sleptsova, N.F., (1965), Soviet A.T., 8, 479) which made use of the idea of expansion to derive a picture of the spiral structure of the Milky Way. The information used:

(1) Galactic spectral observations at 21 cm.
(2) Thermal "spiral arms" from radio observations
(3) Clustering of radio sources as indicators of spiral arms
(4) Non-thermal emission as indicative of spiral arm structure
(5) Optical photometry of the galaxy.

Spiral features are identified with intensity maxima on 21 cm spectra. A plot of the velocity-longitude distribution of these maxima gives information about the distribution of the arms. The information from the center is confused by expansion. However, there appears to be arms whose tangential points are seen at $\ell = 35°$, 55°, 328°, 310-320°, 300-290°. The other non-spectral information may be used to gain information. All the data appears to give tangential points at $\ell = 14°$, 25°, 35°, 50°, 346°, 338°, 329°, 310°, 286°.

Assume a double branched logarithmic spiral structure

$$R = A \, e^{\psi \cos \alpha} \, ,$$

The tangents to the line of sight give a torsion angle $\alpha = 84°$. We assume that α is constant over the galaxy. We wish to determine the rotation law. We assume the observed radial velocity has two components (a) circular and (b) radial, where this term is $\varepsilon (R) = \frac{1}{R} \times \frac{dR}{dt}$.

Then in the galactic plane (b = 0)

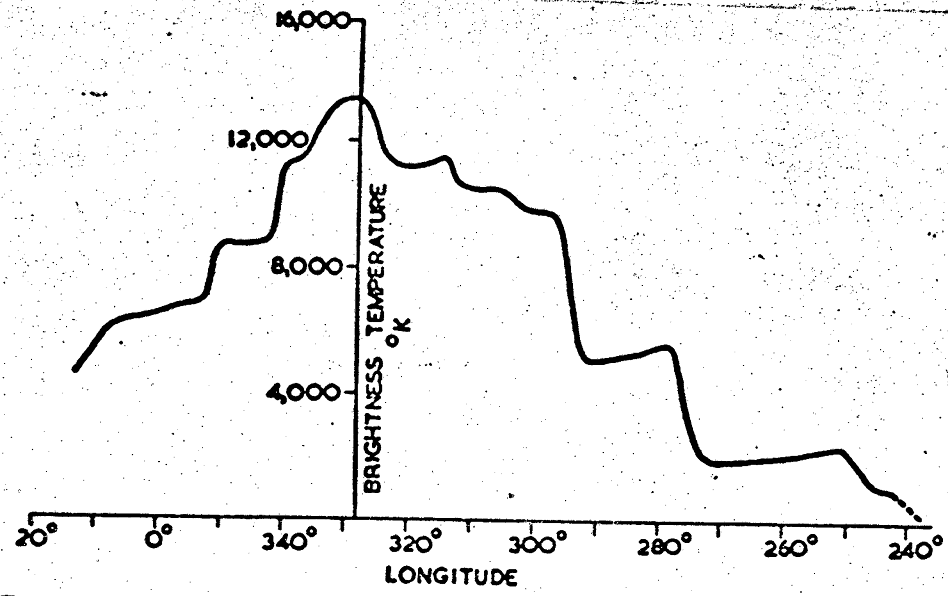


Fig. 9. The longitude distribution of the disk component after subtracting the recognizable sources and the coronal and extragalactic components.

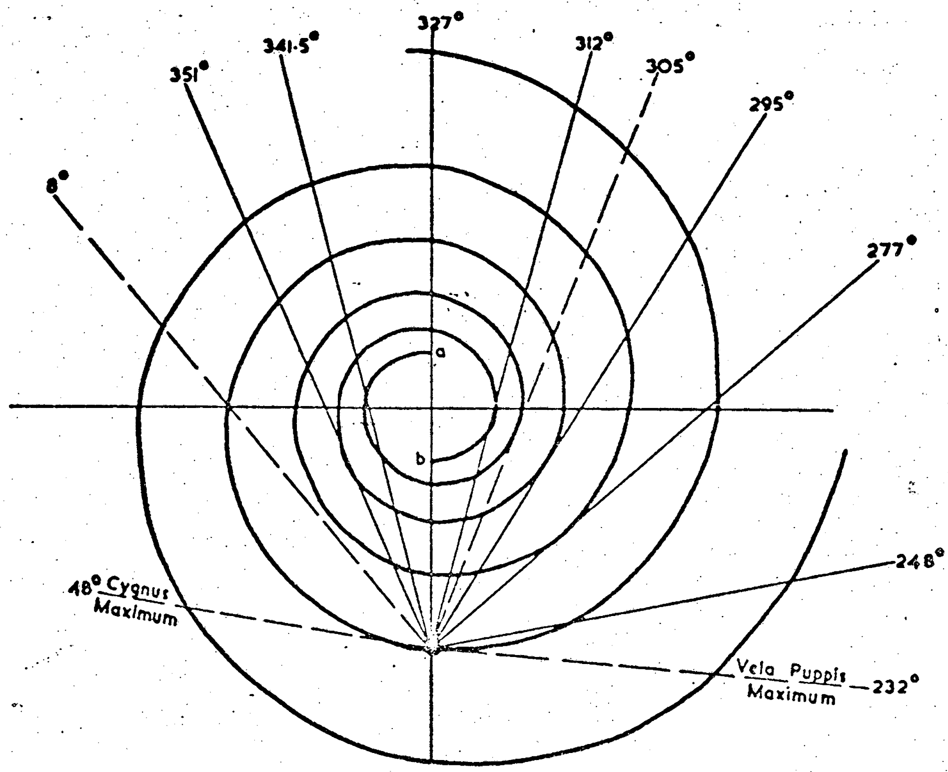


Fig. 11. The spiral pattern of the Galaxy as defined by "steps" in the longitude distribution of the disk component. The directions of the steps are indicated by the lines radiating from the solar position. When the line is dotted, the step is not very pronounced. The directions of the two pronounced maxima in Cygnus and Vela are also shown by dotted lines.

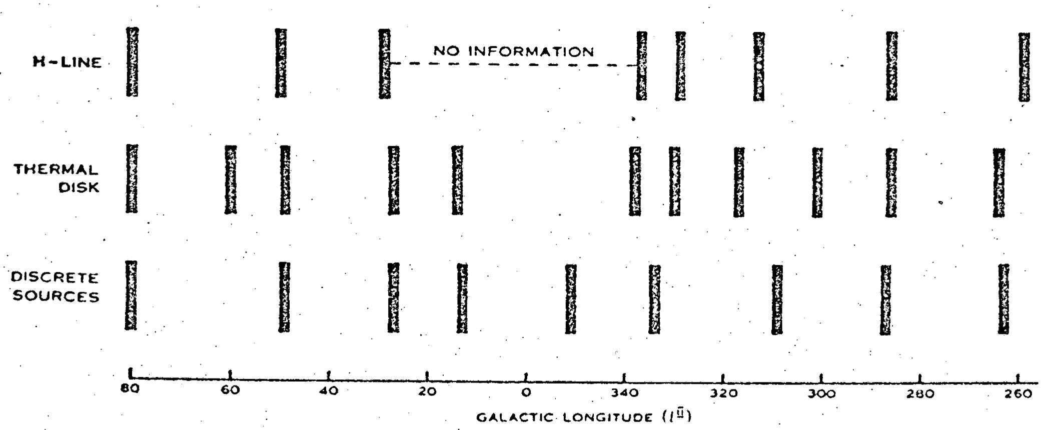


Fig. 3.—Comparison of the galactic longitudes along the galactic plane at which concentrations of neutral hydrogen are expected from H-line studies (top) to those where concentrations of ionized hydrogen are predicted from the present analysis of the thermal disk component (centre). At the bottom are shown the longitudes at which the discrete sources tend to cluster (Fig. 4).

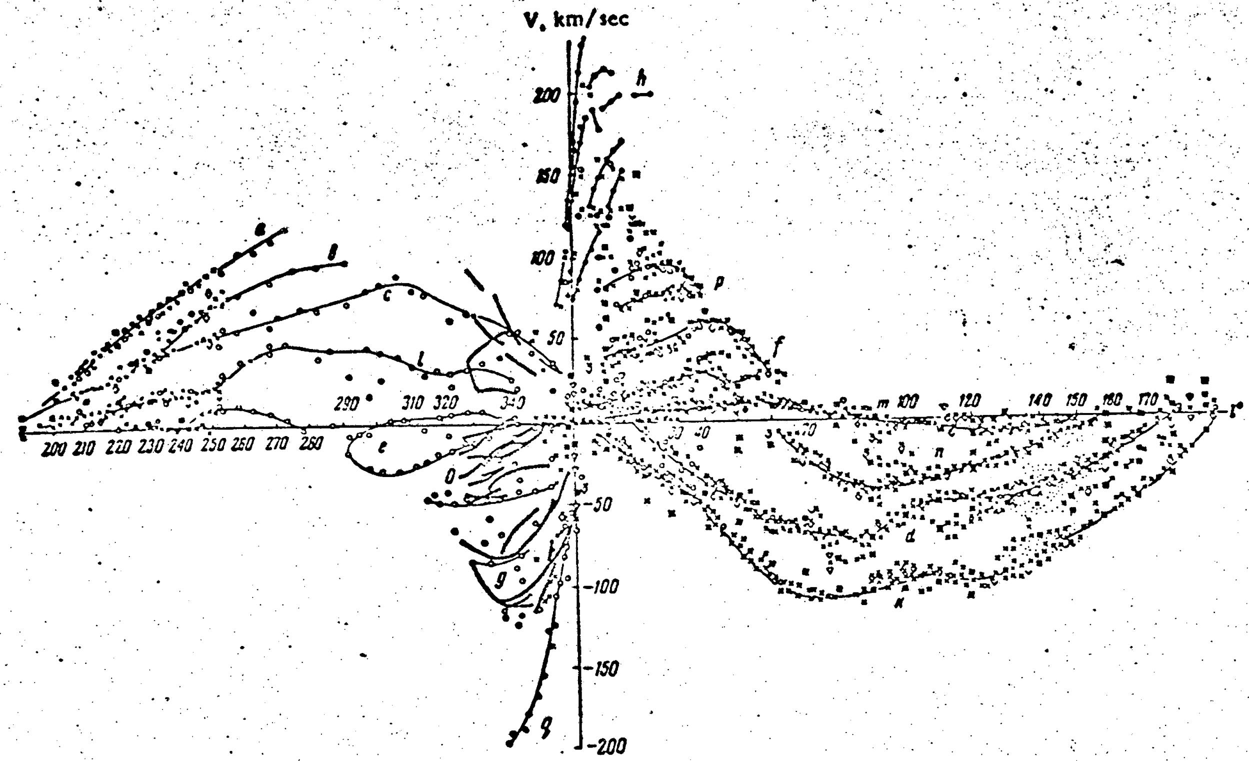
$$V_R = R_o\ [\omega(R)-\omega_o]\ \sin \ell - R_o\ [\varepsilon(R) - \varepsilon_o]\ \cos \ell + \varepsilon d$$

where d is the distance from the sun to the object observed. $\omega(R)$ and $\varepsilon(R)$ may be expanded as a power series in $(R - R_o)$ and the constants solved for by use of the observations. The resulting curve only shows the effects of the expansion term for $R < 3$ kpc and $R > 15$ kpc (using a distance scale which places the sun ~8 kpc from the galactic center). These authors thus claim that spiral structure and circular motion over most of the galaxy are the norm.

Let us do the inverse and investigate the angle of torsion α for the galaxy; we assume circular motion. We are looking for a structure of the form

$$\ln \frac{R}{R_o} = \ln \frac{A}{R_o} + \psi \cos \alpha \cdot$$

We may plot the distribution of $\ln \frac{R}{R_o}$ against ψ employing our rotation law to determine R for every point observed. The information shows a variation from $\alpha = 83°$ in the inner region to $\alpha = 85°$ in the outer. There is a little ambiguity about some of the outer arms. The value of $\frac{A}{R_o}$ on this distance scale gives the distance to the nearest outer arm as 800 pc while that of the inner is 900 pc. Criticism may be made about some of the observations on which this model is based and it shows a very strange structure in the solar neighborhood.


V, km/sec
200
150
100
50
-50
-100
-150
-200
200 210 220 230 240 250 260 270 280
290 310 320 340
100 120 140 150 160 170
h
p
f
m
d
n
κ
c
l
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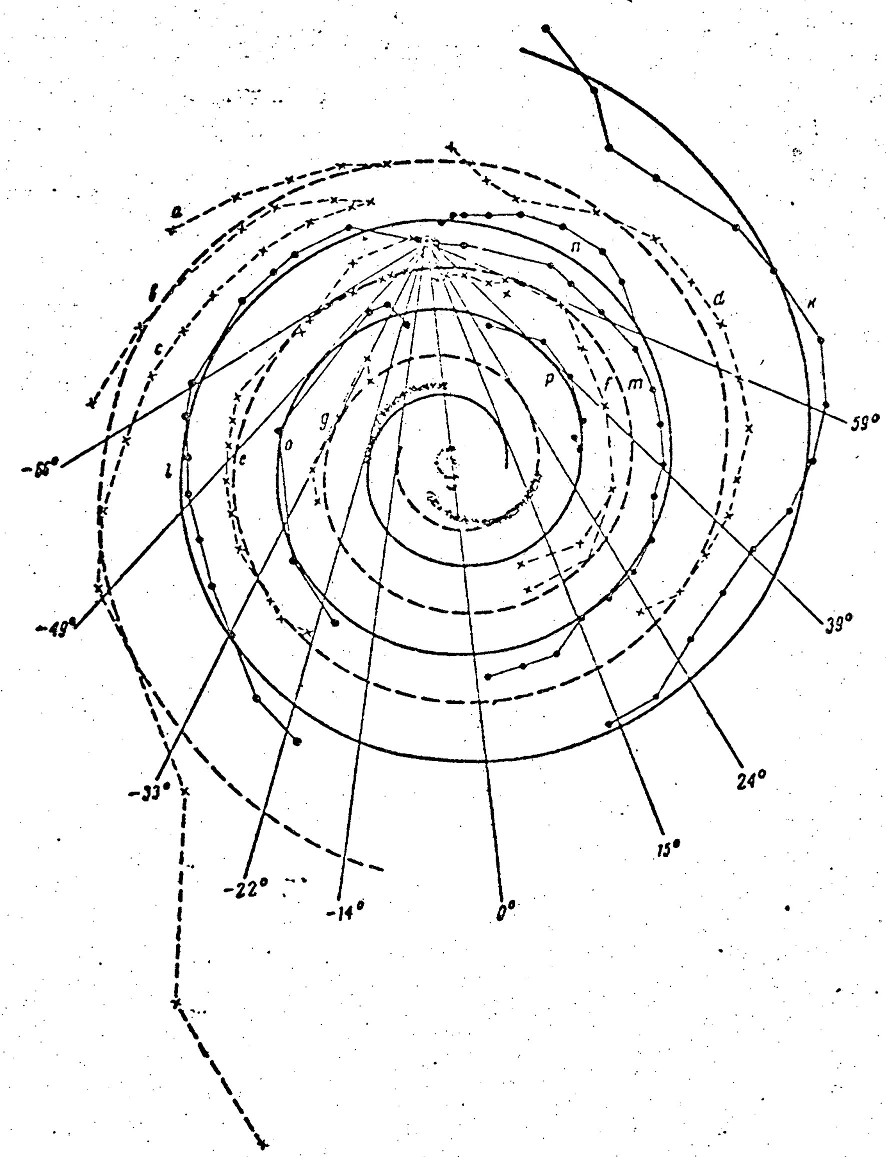
o
g
q
l°

Fig. 1.


a
b
c
l
e
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f
m
n
d
κ
59°
39°
24°
15°
0°
-14°
-22°
-33°
-49°
-65°

Fig. 5.

Some Notes on Galactic dynamics

From a consideration of the velocity residuals about the solar motion we can distinguish two extreme stellar groups; one confined to the disc of the Galaxy and executing circular motion about the centre, the second showing an extremely large dispersion in its Z velocity component and a small θ velocity component. This second group clearly follows very eccentric orbits about the Galactic centre and does not participate significantly in the differential Galactic rotation. It may be described as the halo population of the Galaxy. Spectroscopically it is found that these stars have a very low metal content relative to the sun. This implies that these are amongst the oldest objects in the Galaxy. Assume that initially there is a large contracting cloud of gas. Some stars and globular clusters form very early in the life of this system. These objects will have radial trajectories and highly eccentric orbits and, as they form out of the primeval Galactic material, they will have a low metal content. The gas continues to collapse losing energy through collisions and radiation and ultimately forms a disc-like structure. The newest stars are formed here from this metal enriched medium.

Very crudely, from conservation principles

$$T_o + \Omega_o = T_p + \Omega_p$$

where T_o, Ω_o are the initial kinetic and potential energies of the system; T_p and Ω_p are the present values of these terms.

If the virial theorem is satisfied

$$T_p = -\tfrac{1}{2}\Omega_p$$

and initially as the cloud is at rest

$$T_o = 0 .$$

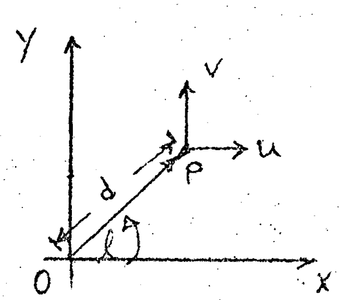
Thus $$\Omega_p = 2\Omega_o$$

If $\Omega \propto 1/R$ where R is some characteristic radius then

$$R_p = \tfrac{1}{2}R_o$$

This must be a crude examination for we have not considered the effects of spiral structure, magnetic fields or of the large, massive, central, nuclear bulge.

Let us now consider matter in the plane of the Galaxy perpendicular to the axis of rotation. We may treat the medium as an incompressible fluid - e.g. div($\underline{v}$) = 0. We erect a coordinate system with the sun at the origin and consider a star at a point (P), a distance (d) from the sun.



We have two velocity components

$u = u(x,y)$

$v = v(x,y)$

The difference in velocity components at the sun and (P)

$$\Delta u = \frac{\partial u}{\partial x}\,dx + \frac{\partial u}{\partial y}\,dy$$

$$= d\frac{\partial u}{\partial x}\cos \ell + d\frac{\partial u}{\partial y}\sin \ell$$

Similarly $$\Delta v = d\frac{\partial v}{\partial x}\cos \ell + d\frac{\partial v}{\partial y}\sin \ell$$

The radial velocity of (P) relative to the sun

$$V_r = \Delta u \cos \ell + \Delta v \sin \ell$$

$$= x(\cos \ell\,\frac{\partial u}{\partial x} + \sin^2 \ell\,\frac{\partial v}{\partial x}) + y(\cos \ell\,\frac{\partial u}{\partial y} + \sin \ell\,\frac{\partial v}{\partial y})$$

or $$V_r = d(\cos^2 \ell\,\frac{\partial u}{\partial x} + \sin^2 \ell\frac{\partial v}{\partial y} + \tfrac{1}{2}\sin 2\ell(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})$$

We choose our axes so that at the sun ($x = 0$, $y = 0$), $\frac{\partial u}{\partial x} = 0$, then from the equation of continuity $\frac{\partial v}{\partial y} = 0$. Hence

$$V_r = d.\tfrac{1}{2}.\sin 2\ell\ \ (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})$$

We define the first of Oort's constants

$$A \equiv \tfrac{1}{2}(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})$$

$$\therefore\ V_r = dA \sin 2\ell$$

We examine the transverse velocity component

$$V_T = \Delta v \cos \ell - \Delta u \sin \ell$$

$$= x(\cos \ell\,\frac{\partial v}{\partial x} - \sin \ell\,\frac{\partial u}{\partial x}) + y(\cos \ell\,\frac{\partial v}{\partial y} - \sin \ell\,\frac{\partial u}{\partial y})$$

$$= d(\cos^2 \ell\,\frac{\partial v}{\partial x} - \sin^2\frac{\partial u}{\partial y})$$

for by the equation of continuity terms in $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ are zero. This equation may be re-written

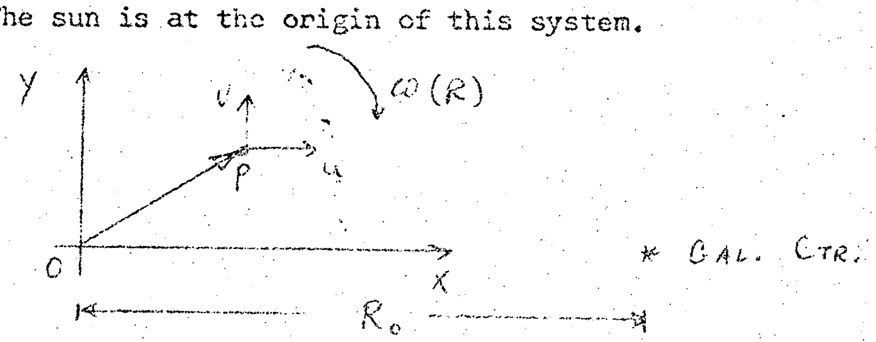
$$V_T = \{(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})\cos 2\ell + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\}\,\frac{d}{2}$$

We define the second of Oort's constants

$$B \equiv \tfrac{1}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})$$

Thus $$V_T = d(A \cos 2\ell + B)$$

Let us now consider the angular velocity of the system as observed from the sun. We erect our coordinate system with the x-axis in the direction of the Galactic centre and consider a star near to the sun at point (P). The sun is at the origin of this system.



Remembering that $\theta_p = \omega(P).R_p$

we may say

$$u = \omega(P).y$$

Thus for very small distances, $u \approx \omega(R_o).y$

for $\omega(P) \approx \omega(R_o)$

where the angular velocity $\omega(R_o)$ is that of the sun

$$V = \omega(P)(R_o - x) - \omega(R_o).R_o$$

The distances from the Galactic centre to (P)

$$R_p = \left[(R_o - x)^2 + y^2\right]^{\frac{1}{2}} \approx (R_o - x)$$

This is only true, of course, for points near the sun. Thus

$$V = \omega(R_o - x).(R_o - x) - \omega(R_o).R_o$$

By expanding in a Taylor series we can show

$$-x\frac{d}{dR}\left[R.\omega(R)\right] = \omega(R_o - x).(R_o - x) - \omega(R_o).R_o$$

From our definitions

$$\frac{\partial u}{\partial y} \approx \omega(R_o)$$

now

$$A \equiv \tfrac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -\tfrac{1}{2}\left(R\frac{d\omega}{dR}\right)_{R_o}$$

and

$$B \equiv \tfrac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\omega(R_o) - \tfrac{1}{2}\left(R\frac{d\omega}{dR}\right)_{R_o}$$

Therefore we may conclude that at the sun

$$\omega(R_o) = A - B$$

and

$$\left(\frac{\partial\omega}{\partial R}\right)_{R_o} = -\frac{2A}{R_o}$$

By the latest analyses of the available data

$$A = 15\text{km.sec}^{-1}\text{kpc}^{-1}$$
$$B = -10\text{km.sec}^{-1}\text{kpc}^{-1}$$
$$R_o = 10\text{kpc}$$

Therefore $\omega(R_o) = 25\text{km.sec}^{-1}\text{kpc}^{-1}$

$$\left(\frac{\partial\omega}{\partial R}\right)_{R_o} = -3\text{km.sec}^{-1}\text{pkc}^{-2}$$

We may draw two immediate conclusions:

(a) the fact that $\omega(R)$ is positive implies that the system rotates in the direction of decreasing longitude,

and (b) as $\partial\omega/\partial R$ is negative the absolute value of $\omega(R)$ decreases with increasing radial distance from the Galactic centre.

Hence, for that material partaking in differential Galactic rotation the inner parts of the Galaxy will complete their orbits in shorter periods. The medium for which this is true is confined closely to the Galactic plane. By use of material near the sun we can establish $\omega(R_o)$ and R_o. We can then use the general expression for the observed radial velocity of material in the Galactic plane

$$V_r = R_o\left[\omega(R) - \omega(R_o)\right] \sin \ell$$

to establish the rotation law $\omega(R)$ for the Galaxy.

Articles and Books which pursue these topics in greater depth are:

R.J. Trumpler and H.F. Weaver, "Statistical Astronomy", 1962, Dover Publication Inc., New York.

D. Mihalas, "Galactic Astronomy", 1968, W.H. Freeman & Co., London.

Stars and Stellar Systems, Vol. V - "Galactic Structure", ed. by A. Blaauw and M. Schmidt, 1965, The University of Chicago Press, London.

"Radio Astronomy and the Galactic System", ed. by H. van Woerden, 1967, Academic Press, London. (in particular, papers 24, 25, & 51).

Encyclopedia of Physics, Vol. 53, "Stellar Systems", ed. by S. Flügge, 1959, Springer-Verlag, Berlin.

Articles: "Galactic Dynamics" B. Lindblad

"Radio Frequency Studies of Galactic Structure", J.H. Oort.

Kerr, F.J. (1962), Mon. Not. Roy. Astr. Soc., 123, 327.

Lin, C.C., and Shu, F.H., (1966), Proc. Nat. Acad. Sci., 55, 229.