US/GR BK/

Summer Student Lecture Notes -- 1970

RADIO RECOMBINATION LINES AND H II REGIONS

M. A. Gordon

I. THE NATURE OF H II REGIONS

As everyone knows, stars condense from the interstellar medium--a gas of unknown thermodynamic characteristics, polluted with helium, strange molecules, and dust, but consisting mainly of hydrogen. Observations of the 21-cm line of neutral hydrogen tell us that the gas is inhomogeneous and turbulent. Presumably, the right conditions can exist where stars can form. Formation proceeds rapidly, and with respect to other time scales in the interstellar gas, we can imagine the star simply to "turn on".

The young stars emit considerable ultraviolet (UV) radiation. That component of the radiation having wavelengths shorter than 912A ionizes the surrounding hydrogen atoms, thereby creating hydrogen ions which Sharpless named H II to distinguish it from neutral hydrogen, known as H I. Some of the H II immediately recombines with the nearest available free electron, and consequently emits a photon. If the wavelength of this photon is less than 912A, it too will cause an ionization when it collides with the nearest neutral hydrogen atom. It is important to note that whereas the initial photon came from the direction of the star, this secondary photon can be re-emitted into any direction. Thus the UV radiation field from the star is scattered and diluted. In general, though, this ionization front moves radially outward from the star, leaving behind it an ionization zone or "H II Region".

Some of the UV radiation from the star is lost. If the hydrogen ion and the free electron recombine into an upper energy state of the hydrogen atom, no UV photon is produced. The electron subsequently cascades downward from state to state, emitting a series of lines known as <u>recombination lines</u> as it tumbles down this quantum stairway. Another method of losing energy from the UV radiation field is by collisions. In general, the ionized atoms obtain kinetic energy from the UV photons, since the energy absorbed by the atom is usually greater than that needed for ionization. Binary collisions with other ionized components quickly redistributes this kinetic energy so as to raise the temperature of the gas from, say, the 100° K of the neutral gas to approximately 10,000° K. The exact temperature depends upon the available infrared transitions which cool the gas by radiation.

Because of the high temperature, the pressure of the H II regions exceeds that of the surrounding cold neutral gas. Thus the region also expands mechanically, sometimes at supersonic velocities so as to cause shock waves to develop within the H II region. For most H II regions, we expect considerable variations in density (and hence temperature).

Because of their association with young stars and hence clumps of neutral hydrogen, H II regions are apt to lie in the spiral arms of the galaxy. They are difficult to observe optically owing to the large amounts of gas and dust which obscure them from us. Notable H II regions lying nearby are the Orion Nebula, the Omega or Horseshoe Nebula, and the Rosette Nebula. The visible radiation from these nebulae is largely composed of radiation from oxygen atoms, known as "nebular lines", and of course, Hα.

II. RADIO RECOMBINATION LINES

One method of piercing the surrounding interstellar gas and dust to study these regions is to use radio waves. At frequencies above 100 MHz, the interstellar medium effects little absorption. Using discrete radio lines, we can observe Doppler shifts and thus position the H II regions with respect to the galactic rotation system by their velocities.

The frequency ν of any transition between two principal quantum states of the hydrogen atom may be calculated from the Rydberg formula,

$$\nu = R_c z^2 \left[\frac{1}{n^2} - \frac{1}{(n + \Delta n)^2}\right]$$

where R is the Rydberg constant, c the speed of light, z the nuclear charge and Δn the change in quantum number n. The Rydberg constant varies with mass

$$R = R_\infty(1 - \frac{m_e}{M})$$

(a) Radio Transitions

Consider transitions where $\Delta n << n$. Then

$$\nu = \frac{R_c z^2}{n^2} \left[1 - \frac{1}{(1 + \frac{\Delta n}{n})^2}\right]$$

$$\approx 2R_c z^2 \frac{\Delta n}{n^3}$$

after expansion. Furthermore, we can calculate the separation between these lines

$$\Delta\nu = 2R_c z^2 \, \Delta n \left[\frac{1}{n^3} - \frac{1}{(n+1)^3} \right]$$

$$\approx 6R_c z^2 \frac{\Delta n}{n^4} = \frac{3\nu}{n}$$

(b) Table

We may use the above equations to produce the following table

ν(MHz)	λ	n	$\Delta\nu$(MHz)
$3\mathrm{x}10^6$	100μ	13	690 000
$3\mathrm{x}10^5$	1 mm	28	32 000
$3\mathrm{x}10^4$	1 cm	60	1 500
$3\mathrm{x}10^3$	10 cm	130	70
$3\mathrm{x}10^2$	1 m	280	3.2
3x10	10 m	600	0.15

III. LINE FORMATION

(a) Line Shape

To be seen, these lines must radiate adequate power. From our qualitative discussion on the nature of the H II region, we should expect the ionized gas to thermalize quickly, that is, achieve a velocity distribution known as Maxwellian. In fact, the number of atoms with velocity components along the line of sight is

$$dN(v_x) = N \sqrt{\frac{m}{2\pi kT}} \quad \exp\left[- \frac{m v_x^2}{2kT} \right] dv_x$$

Owing to the Doppler effect, each atom will radiate at a frequency ν compared to the line rest frequency, ν_o.

$$\nu = \nu_o(1 - \frac{v_x}{c})$$

or

$$v_x = c(1 - \frac{\nu}{\nu_o})$$

or

$$dv_x = -c \frac{d\nu}{\nu_o}$$

Now the total number of emitters within a velocity interval control the intensity of the line within a frequency interval,

$$dI(\nu) \propto dN(v_x)$$

and

$$I \propto N$$

Substitution gives the line shape to be

$$dI(\nu) = I\sqrt{\frac{m}{2\pi kT}} \cdot \frac{c}{\nu_o} \exp\left[-\frac{mc^2}{2kT}\left(\frac{\nu_o-\nu}{\nu_o}\right)^2\right] d\nu$$

which is the equation for a gaussian line. We can make this into a more convenient equation by rewriting the equation in terms of the total width at half-intensity, $\Delta\alpha$.

$$dI = I \sqrt{\frac{4\ell n2}{\pi}} \; \frac{1}{\Delta\nu} \; \exp\left[-4\ell n2 \left(\frac{\nu_o - \nu}{\Delta\nu} \right)^2 \right]$$

and if we define the line shape function f(ν) to be

$$dI(\nu) = I \cdot f(\nu) d\nu$$

then

$$f(\nu) = \sqrt{\frac{4\ell n2}{\pi}} \; \frac{1}{\Delta\nu} \; e^{-4\ell n2 \left(\frac{\nu_o - \nu}{\Delta\nu} \right)^2}$$

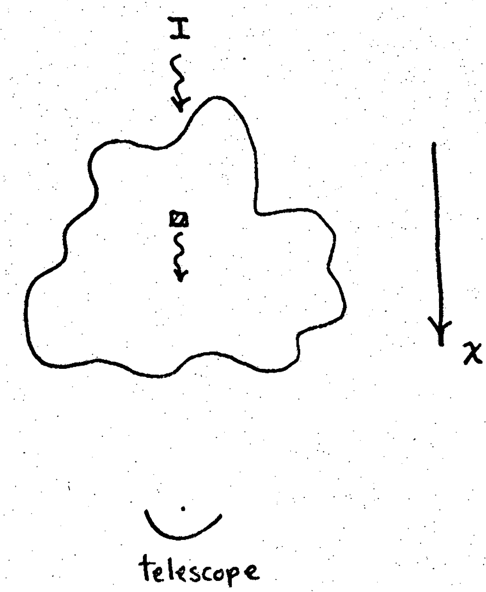
which described the expected shape of the recombination line, where the square of the total width at half-intensity is

$$(\Delta\nu)^2 = 4\ell n2 \, \frac{2kT}{Mc^2} \; \nu_o^{\,2}$$

in the case of pure thermal broadening. If there are turbulent cells within the beamwidth, then

$$(\Delta\nu)^2 = 4\ell n2 \; \frac{\nu_o^{\,2}}{c^2} \left(\frac{2kT}{M} + \frac{2}{3} \langle v_t^2 \rangle \right)$$

where $\langle v_t^2 \rangle^{1/2}$ is the most probable velocity of the turbulence.



(b) Line Intensities.

The differential intensity contributed from an elemental volume of length dx, in the direction of the observer, is

$$dI = \underbrace{-Ikdx}_{\text{absorption}} + \underbrace{jdx}_{\text{emission}}$$

which interprets to

$$I(L) = \underbrace{e^{-\tau(L)} \int_0^{\tau(L)} Se^{\tau}\, d\tau}_{\text{emitted}} + \underbrace{I(0)e^{-\tau(L)}}_{\text{background}}$$

when the source function S is defined as

$$S \equiv \frac{j}{k}$$

and the optical depth τ is

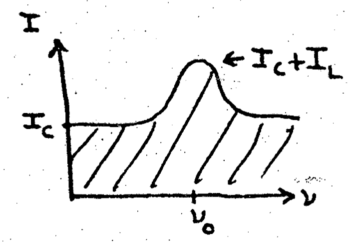
$$\tau(L) = \int_0^L kdx$$

If S is a constant, and $k \neq k(x)$, then

$$\boxed{I(L) = \frac{j}{k}\left(1 - e^{-kL}\right) + I(0)e^{-kL}}$$

and the problem is now to find the emission and absorption coefficients.

Consider the following picture



$$I_L + I_c = I(L) = \frac{j}{k}\left(1 - e^{-(k_L+k_c)L}\right)$$

$$I_c = \frac{j}{k}\left(1 - e^{-k_cL}\right)$$

and at line center, we calculate the ratio of line to continuum antenna temperatures to be

$$\frac{T_L}{T_c} = \frac{(I_L + I_c) - I_c}{I_c} = \frac{1 - e^{-(k_L+k_c)L}}{1 - e^{-k_cL}} - 1$$

and if the gas is optically thin,

$$(k_L + k_c)\, L << 1$$

$$\boxed{\frac{T_L}{T_c} = \frac{k_L}{k_c}}$$

which is just the ratio of the absorption coefficients. Substituting for k_L and k_c, we obtain

$$\frac{T_L}{T_c} = 1.2 \times 10^5 \; \frac{\frac{\Delta n}{n} f_{n'n} \cdot \nu^{2.1}}{T_e^{1.15} \; \Delta\nu}$$

IV. USE OF RECOMBINATION LINES

The above equation can be used to measure the temperature of the gas T_e. All we have to do is to observe the lines. Let's put in a proposal to Bill Howard for the λ3-cm receiver and the 140-foot telescope so as to observe the 86→85 transition at 10.522 GHz, and the 108→106 transition at 10.738 MHz.

After making the observations of the Orion Nebula, we calculate T_e from the data to get

Line	T_e
H85α	8000° K
H106β	9500° K

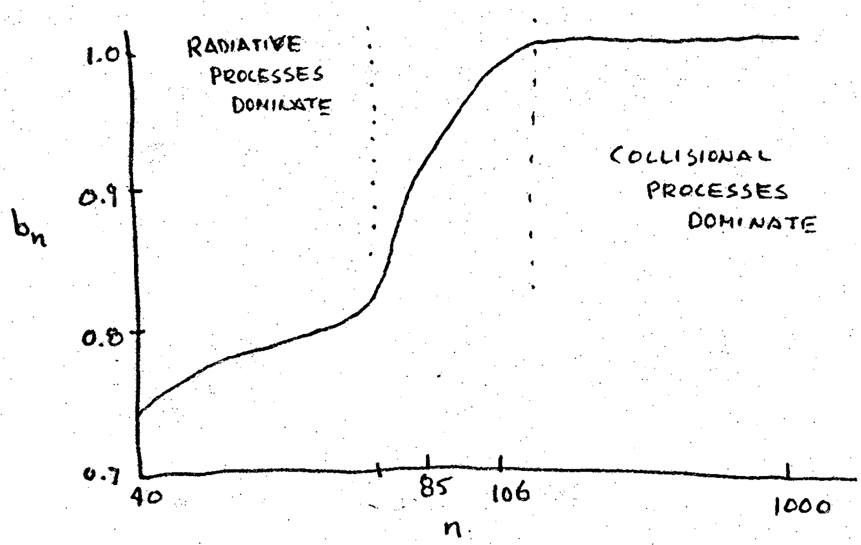
and discover that we get different temperatures for each line. Why?

The reason is that we assumed that all the level populations of the hydrogen can be described by a single value of temperature. In other words, we assumed that these levels interacted with the gas by means of collisions to the same degree. This assumption is often called thermodynamic equilibrium (TE).

(a) Departures from TE

Consider the relative size of the hydrogen atom as a function of quantum number. The area varies with n^2. So the atom is $\left(\frac{106}{85}\right)^2 = 1.6$ times larger for the β line than for the α line, and the 108th quantum level is more apt to interact with the kinetic field of the gas than the 85^{th} quantum level simply because its "target area" is larger.

If we balance the number of ways into a quantum level with the number of ways out of a level, one can calculate the number of atoms actually in a level with the number of atoms which should be in the level -- the factor being known as the



b_n-factor. This assumption is known as statistical equilibrium, and it implies that the level populations are statistically invariant over time scales long compared to the microprocesses.

The figure shows that at large n, where the target area is large, level populations are controlled by the kinetic temperature. Thus our b_n-factor is one. At small quantum numbers, the atom interacts poorly with the kinetic field. These levels are controlled by radiative processes.

In the intermediate region, the upper levels of any transition are slightly overpopulated with respect to the lower levels, that is

$$b_{n+\Delta n} > b_n$$

and a slight maser effect occurs that causes the line intensity to increase and correspondingly, the derived kinetic temperature to be erroneously low.

In fact, for frequencies where the gas is optically thin, and where maser effects are significant, the apparent (excitation) temperature relates to the true kinetic temperature by

$$T_{apparent} = T_{rue}/(b_n + \text{const.} \frac{E \cdot T_e}{\nu} \; b_n \frac{d\ell n b_n}{dn} \Delta n)^{0.87}$$

In the bracket, the first term corrects for the number of atoms in the upper level, the second term for the "maser" amplification. Note that the emission measure is a co-factor of the b_n-slope. For H II regions with low emission measures $[\int n_e^2 d\ell]$, maser effects will be small.

V. USE OF RECOMBINATION LINES

It is possible to use line intensities to find the kinetic temperature by a number of techniques. One is to make a general non-TE solution to observations of many lines. Another is to make observations of α, β, γ, etc. lines at a single frequency (constant $k_c L$). Here each higher-order transition involves a larger n, and we explore the b_n-curve experimentally.

In any case, the lines can be used to measure the velocity of the gas with respect to the observer because of Doppler effects. After correction for thermal broadening, the line widths reflect the dispersion of turbulent velocities within the beam. It is even possible that our detection systems will become sensitive enough to detect Stark broadening, and we can get a direct measurement of electron density.