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[Lecture Notes - Summer 1970]

EXPERIMENTAL TESTS OF GENERAL RELATIVITY IN RADIO ASTRONOMY

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General Relativity Theory (GRT) is a geometrical theory of gravitation. Rather than describe gravity as a force acting on a particle, in GRT free falling particles are simply in natural motion in a curved space, i.e., they move along geodesics. It is the curvature of space rather than a force which brings bodies together.

The distribution of matter and energy in the universe determines the curvature of 4-dimensional space-time. These quantities are related by a set of non-linear differential equations called the Einstein Field Equations,

$$R_{uv} - 1/2\ g_{uv}\ R = T_{uv}$$

where T_{uv} = the stress energy tensor, which contains terms proportional to the energy and momentum densities and pressure in a region of space.

g_{uv} = the metric tensor defined by $ds^2 = g_{uv}\ dx^u dx^v$, which relates proper distance, ds (a physical quantity), to coordinate distance dx.

R_{uv} = Rici tensor

R = curvature scalar

These last two terms describe the curvature of space in terms of the first and second derivatives of g_{uv}.

The simplest was to solve the field equations, i.e., find the world line of a test particle, is

(1) assume a general form for the metric, g_{uv}

(2) take derivatives of g_{uv} and find R_{uv} and R

(3) from the field equations obtain coefficients of g_{uv}

(4) find the geodesics in the resulting space by solving the "geodesic equation" using the now known g_{uv}.

One of the first and most useful solutions is the Schwarzschild metric which exist for empty space outside a spherically symmetric body. It has the form

$$ds^2 = (1 - \frac{2m}{r})\, dt^2 - (1 - \frac{2m}{r})^{-1}\, dr^2 - r^2\, (d\theta^2 + \sin^2\, \theta d\phi^2)$$

where $m = \frac{GM}{c^2}$ and M = the mass of the spherical body.

Note the singularity at $r = 2m$. This is the "black hole" from which photons never emerge. For the earth $r \sim 1$ cm and for the sun $r \sim 3$ km.

In his original paper, Einstein proposed three test in which particle motion in GRT differed from Newtonian theory; these are

(a) the gravitational redshift

(b) the advance of the perihelion of an orbiting body

(c) the deflection of photons.

The redshift is given by

$$f(\infty)/f(r_o) = (1 - \frac{2m}{r_o})^{1/2}$$

where r_o is the radius of a massive body and $f(r)$ is the frequency at a given radius. For small shifts

$$\Delta f/f(r_o) = \frac{m}{r_o} \quad .$$

This has been observed astronomically in the spectral shift of lines from dense white dwarf stars, where $\Delta f/f \sim 2 \times 10^{-4}$. In a much more accurate laboratory experiment, a γ-ray falling in the earth's gravity field had a $\Delta f/f \sim 2 \times 10^{-15}$ which was measured with errors of less than 1%. Both tests agree with GRT.

The perihelion of an elliptical orbit will advance by $w = \frac{6\pi m}{a(1 - e^2)}$ radians/revolution, where a = the semimajor axis and e = the ellipticity of the orbit. For Mercury, w = 43".03/century which is the largest for any major planet. The measured perihelion advance is about 5600"/century, but all but 43".1 is explained by general percession of the celestial coordinate system and by perturbations due to the other planets. This difference is explained by GRT.

GRT predicts that photons will be deflected in a gravitational field by $\theta = \frac{2m}{r}$; for the sun $\theta = K/\rho$ where $\rho = r/r(\text{sun})$ and K = 1".75. This has been measured several times during solar eclipses when stars near the sun can be seen. However, it is a rather uncertain experiment with the measured values for K running between 1".2 to greater than 2". The deflection was also measured at radio frequencies by two groups of radio astronomers at CalTech in October 1969. Both groups used interferometers; one at the Owens Valley Radio Observatory and the other at the JPL Goldstone Tracking Station. They both followed the radio source 3C 279 as it was occulted by the sun and measured the phase change of the interferometer fringes as the source approached the solar disk. The interferometers used had the following parameters:

	OVRO	JPL
operating wavelength	3 cm	13 cm
baseline length (km)	1 km	21 km
" " (wavelengths)	3.4×10^{4}	1.6×10^{5}
fringe spacing	6".1	1".2

The relativistic deflection was accompanied by refraction in the solar corona. From previous radio and optical measurements, the electron density in the corona is approximately given by

$$N \sim 5\times10^{11}\rho^{-2} + 1.5\times10^{14}\,\rho^{-6}\ \mathrm{m}^{-3}$$

and from this the ray path is deflected by

$$-\theta_{ref} \sim 1.5\times10^{-2}\,\lambda^{2}\,\rho^{-2} + 8.5\,\lambda^{2}\,\rho^{-6}\ \text{arc sec}$$

where λ is the wavelength in centimeters. At $\rho = 4$ and $\lambda = 3$ cm, the GRT bending is 0".45 and $-\theta_{ref} \sim 0.01 + 0.02 = 0''.03$. At 13 cm $-\theta_{ref} = 0''.48$ and for accurate results the values of the electron density parameters had to be calculated from the interferometer data, relying on the different dependence on ρ of the GRT and corona bending to separate the two effects. The JPL group had the advantage of a smaller fringe spacing while the OVRO group had a higher frequency where the effects of the solor corona were less severe. Both experiments yielded results with about 10% accuracy with K = 1.77 for OVRO and K = 1.82 for the JPL group.

The radio experiment was one of the most accurate measurements of the photon deflection yet made, but it was still not good enough to distinguish between GRT and an alternative gravitational theory like that of Brans and Dicke. For this, experimental accuracy of a few percent will be needed. Such accuracy may be obtained by using a VLB interferometer to measure source positions near the sun and radio astronomers from MIT and NASA started such a project in 1969 and may have results soon.