[Lecture Notes - Summer 1971]

EXPERIMENTAL TESTS OF GENERAL RELATIVITY IN RADIO ASTRONOMY

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General Relativity Theory (GRT) is a geometrical theory of gravitation. Rather than describe gravity as a force acting on a particle, in GRT free falling particles are simply in natural motion in a curved space, i.e., they move along geodesics. It is the curvature of space rather than a force which brings bodies together.

The distribution of matter and energy in the universe determines the curvature of 4-dimensional space-time. These quantities are related by a set of non-linear differential equations called the Einstein Field Equations,

$$R_{uv} - 1/2\ g_{uv}\ R = T_{uv}$$

where T_{uv} = the stress energy tensor, which contains terms proportional to the energy and momentum densities and pressure in a region of space.

g_{uv} = the metric tensor defined by $ds^2 = g_{uv}\ dx^u dx^v$, which relates proper distance, ds (a physical quantity), to coordinate distance dx.

R_{uv} = Rici tensor

R = curvature scalar

These last two terms describe the curvature of space in terms of the first and second derivatives of g_{uv}.

The simplest way to solve the field equations, i.e., find the world line of a test particle, is

(1) assume a general form for the metric, g_{uv}

(2) take derivatives of g_{uv} and find R_{uv} and R

(3) from the field equations obtain coefficients of g_{uv}

(4) find the geodesics in the resulting space by solving the "geodesic equation" using the now known g_{uv}.

One of the first and most useful solutions is the Schwarzschild metric which exist for empty space outside a spherically symmetric body. It has the form

$$ds^2 = (1 - \frac{2m}{r})\, dt^2 - (1 - \frac{2m}{r})^{-1}\, dr^2 - r^2\,(d\theta^2 + \sin^2\theta d\phi^2)$$

where $m = \frac{GM}{c^2}$ and M = the mass of the spherical body.

Note the singularity at r = 2m. This is the "black hole" from which photons never emerge. For the earth $r \sim 1$ cm and for the sun $r \sim 3$ km.

In his original paper, Einstein proposed three test in which particle motion in GRT differed from Newtonian theory; these are

(a) the gravitational redshift

(b) the advance of the perihelion of an orbiting body

(c) the deflection of photons.

The redshift is given by

$$f(\infty)/f(r_o) = (1 - \frac{2m}{r_o})^{1/2}$$

where r_o is the radius of a massive body ard $f(r)$ is the frequency at a given radius. For small shifts

$$\Delta f/f(r_o) = \frac{m}{r_o} \quad .$$

This has been observed astronomically in the spectral shift of lines from dense white dwarf stars, where $\Delta f/f \sim 2 \times 10^{-4}$. In a much more accurate laboratory experiment, a γ-ray falling in the earth's gravity field had a $\Delta f/f \sim 2 \times 10^{-15}$ which was measured with errors of less than 1%. Both tests agree with GRT.

The perihelion of an elliptical orbit will advance by $w = \frac{6\pi m}{a(1 - e^2)}$ radians/revolution, where a = the semimajor axis and e = the ellipticity of the orbit. For Mercury, w = 43".03/century which is the largest for any major planet. The measured perihelion advance is about 5600"/century, but all but 43".1 is explained by general percession of the celestial coordinate system and by perturbations due to the other planets. This difference is explained by GRT.

GRT predicts that photons will be deflected in a gravitational field by $\theta = \frac{2m}{r}$; for the sun $\theta = K/\rho$ where $\rho = r/r(\text{sun})$ and K = 1".75. This has been measured several times during solar eclipses when stars near the sun can be seen. However, it is a rather uncertain experiment with the measured values for K running between 1".2 to greater than 2". The deflection was also measured at radio frequencies by monitoring the position of the radio source 3C279 as it was occulted by the sun. This occultation occurs each year in early October. In 1969 two separate groups at Cal Tech used interferometers at the Owens Valley Radio Observatory and the JPL Goldstone Tracking Station to perform the experiment. In 1970 the deflection was measured with the NRAO interferometer. The parameters of the interferometers and the values obtained for K are:

	OVRO	JPL	NRAO
operating wavelength	3 cm	13 cm	11 cm and 4 cm
baseline length (km)	1 km	21 km	2.7 km
" " (wavelengths)	3.4×10^4	1.6×10^5	2.4 and 7.3×10^4
fringe spacing	6".1	1".2	8".7 and 2".9
gravitational defl. (K)	1".77±0.20	$1''.82^{+.24}_{-.17}$	1".57 ± 0.08

The relativistic deflection was accompanied by refraction in the solar corona. From previous radio and optical measurements, the electron density in the corona is approximately given by

$$N \sim 5 \times 10^{11} \rho^{-2} + 1.5 \times 10^{14} \rho^{-6} m^{-3}$$

and from this the ray path is deflected by

$$-\theta_{ref} \sim 1.5 \times 10^{-2} \lambda^2 \rho^{-2} + 8.5 \lambda^2 \rho^{-6} \text{ arc sec}$$

where λ is the wavelength in centimeters. At $\rho = 4$ and $\lambda = 3$ cm, the GRT bending is 0".45 and $-\theta_{ref} \sim 0.01 + 0.02 = 0''.03$. At 13 cm $-\theta_{ref} = 0''.48$. In the OVRO and JPL experiments the uncertainty in K was primarily due to the uncertainty in the coronal refraction. However, in the NRAO experiment, the observations were made simultaneously at two wavelengths, This gave an accurate measurement of the refraction which was independent of the gravitational deflection. The uncertainty in K for this experiment was primarily due to phase noise in the interferometer.

During the next few years more accurate deflection measurements will probably be made using longer baselines such as available with the NRAO radio-link interferometer (30 km) or a VLB interferometer (3000 km). The VLB experiment is being developed by groups at both NRAO and MIT.

A forth test of GRT, first suggested by Irwin Shapiro in 1964, involves measuring an increase in the time of flight of a photon to a target when that photon has to traverse a gravitational field. This test has been performed by timing radar signals bouncing off Mercury and Venus (MIT group) and by timing signals relayed from a Mariner spacecraft near the sun. Preliminary results from both of these experiments agree with the predictions of GRT to within a few percent.