US/GRBK/

GALACTIC DYNAMICS AND GALACTIC STRUCTURE

by S. Gottesman

I. GALACTIC ROTATION

The earliest evidence about the structure of our galaxy (the Milky Way system) comes from Shapley's work on globular clusters. This indicated that these objects were spherically distributed in space and that the sun was at the edge of this system. It was also found that these objects have velocities of the order of 250 km/s. If this system is at rest with the galaxy as a whole it implies that the sun rotates about the center of the galaxy, with a velocity of 250-300 km/s. Further, ignoring certain classes, most stars showed a velocity variation of ± 30 km/s about the sun. This implies that these stars share the sun's motion about the galactic center.

We consider a flat disk stellar system in circular rotation about an axis. The mass of the system is centrally concentrated. Stars will travel in almost Keplerian orbits about the galactic center. This form of motion where the velocity increases with decreasing radius is known as differential galactic rotation and implies the presence of shear forces.

Let us erect a coordinate system at the sun with the $-y$ axis in the direction of the galactic center and the $+x$ axis in the direction of the sun's motion. For differential rotation we expect more rapid circular velocities for $-90<\ell<90$ and less rapid circular velocities for $90<\ell<270$. At $\ell=0$, 90, 180, 270 we see no radial motion. Thus the expected plot of radial velocities would be $V_r \propto \sin 2\ell$. Tangential velocities or proper motion will also occur due to differential motion of stars at different distances from the galactic center.

r = radius of some point in the galaxy

R_o = radius of the local standard of rest (LSR)

Θ = linear rotation velocity at r

Θ_o = linear rotation velocity of the LSR

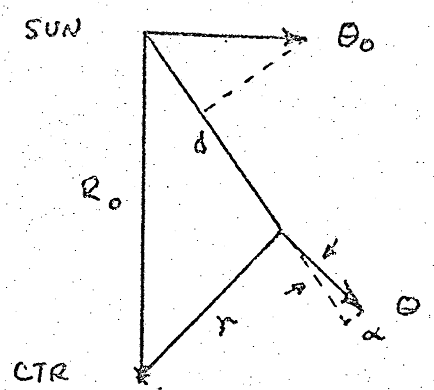
$\omega = \Theta/r$ = angular velocity at r about galactic center

$\omega_o = \Theta_o/R_o$ = angular velocity of the LSR about galactic center

d = distance from the sun to some point at radius r from galactic center

V_R = observed radial velocity relative to the sun

V_T = observed tangential velocity relative to the sun



$$V_R = \Theta \cos \alpha - \Theta_o \sin \ell$$

By the sine law

$$\frac{\sin \ell}{r} = \frac{\sin (90 + \alpha)}{R_o} = \frac{\cos \alpha}{R}$$

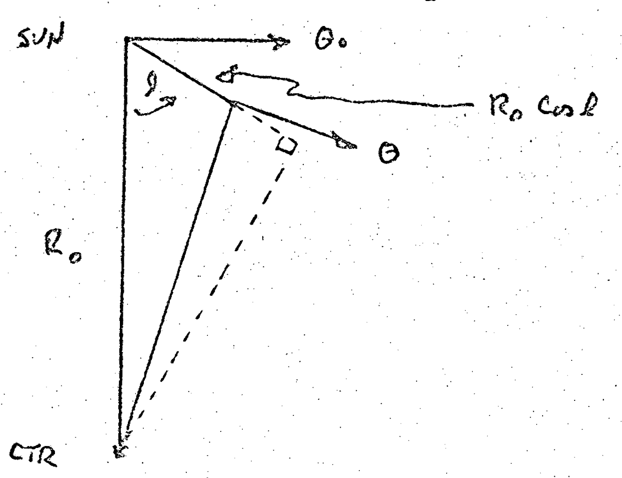
by substitution. The radial component

$$\underline{(1) \quad V_R = R_o (\omega - \omega_o) \sin \ell}$$

This is true only if particles move in circular orbits about the galactic center. The tangential or transverse component

$$V_T = \Theta \sin \alpha - \Theta_o \cos \ell$$

By convention, $V_T > 0$ for increasing longitude.

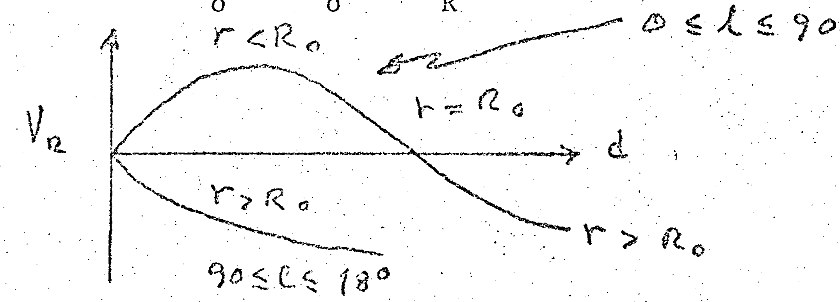


$$r \sin \alpha = R_o \cos \ell - d$$

or

$$\underline{(2) \quad V_T = R_o (\omega - \omega_o) \cos \ell - \omega d}$$

In the region $90 \leq \ell \leq 180$ if we hold ℓ constant $\omega < \omega_o$ and the difference increases as we increase r. For the region $0 \leq \ell \leq 90$ we first encounter regions of small r and large ω which reaches a maximum when r is normal to the line of sight ($r_{min} = R_o \sin \ell$ and $d = R_o \cos \ell$) as $r \to R_o$ $\omega \to \omega_o$ and $V_R \to 0$ and when $r > R_o$ $\omega < \omega_o$ and $V_R < 0$



In many cases observations are restricted to the region near the sun and we may simplify our formulae. To a first order

$$\frac{\omega - \omega_o}{r - R_o} = \left(\frac{d\omega}{dr}\right)_{R_o}$$

$$\frac{d\omega}{dr} = \frac{1}{r}\frac{d\Theta}{dr} - \frac{\Theta}{r^2}$$

Thus $$V_R = (r - R_o)\left[\left(\frac{d\Theta}{dr}\right)_{R_o} - \frac{\Theta_o}{R_o}\right] \sin \ell$$

In the case where $d \ll R_o$

$$R_o - r \simeq d \cos \ell$$

or $$V_R \simeq \left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta}{dr}\right)_{R_o}\right] \frac{1}{2} \sin 2\ell$$

We define the first of Oort's constants

$$(3) \qquad A \equiv \frac{1}{2}\left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta_o}{dr}\right)_{R_o}\right]$$

and therefore,

$$\text{(4)} \qquad V_R = A\, d \sin 2\,\ell$$

This equation is valid ONLY if $d << R_o$ and circular motion pertains.

With regard to the transverse component (Eq. 2)

$$\omega - \omega_o = \frac{r - R_o}{R_o} \left[\left(\frac{d\Theta}{dr}\right)_{R_o} - \frac{\Theta_o}{R_o} \right]$$

By use of a Taylor Expansion

$$\omega d = d \left[\omega_o + \left(\frac{d\omega}{dr}\right)_{R_o} (r - R_o) + \ldots \right]$$

If $d << R_o$ then $(r - R_o) \sim - d \cos \ell$

and $\omega d > \omega_o d - \left(\frac{d\omega}{dr}\right) d^2 \cos \ell$

and to a first order $\omega d = \omega_o d$ for material near the sun,

or

$$V_T \simeq \left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta}{dr}\right)_{R_o} \right] d^{\ell} \cos^2 \ell - \frac{\Theta_o}{R_o}\, d$$

which may be rewritten

$$V_T \simeq \frac{1}{2} \left[\frac{\Theta_o}{R_o} - \left(\frac{d\Theta}{dr}\right)_{R_o} \right] d \cos 2\ell - \frac{1}{2} \left[\frac{\Theta_o}{R_o} + \left(\frac{d\Theta_o}{dr}\right)_{R_o} \right] d$$

We define the second of Oort's constants

$$\text{(5)} \qquad B \equiv - \frac{1}{2} \left[\frac{\Theta_o}{R_o} + \left(\frac{d\Theta}{dr}\right)_{R_o} \right]$$

Thus

$$(6) \quad V_T = d\,(A \cos 2\ell + B)$$

From our definitions of A and B

$$(7) \quad \frac{\Theta_o}{R_o} = \omega_o = A - B$$

$$(8) \quad \left(\frac{d\Theta}{dr}\right)_{R_o} = -(A + B) \quad \text{or} \quad (8a) \quad A = -\frac{1}{2}\left(R\frac{d\omega}{dR}\right)_{R_o}$$

Observational proof of equation 4 was obtained from Cepheid variables in the late 1930's.

We wish to determine $\omega(r)$ but we can only readily observe $V_R(\ell)$. Thus if we can determine A, B, and R_o we may use equation 1 to establish $\omega(r)$.

Determination of A

(1) Measure V_R for stars near the sun for which d is known ($V_R = d\,A \sin 2\ell$).

(2) From the study of proper motions as a function of longitude.

(3) From the definition $A = -\frac{1}{2} R_o \left(\frac{d\omega}{dr}\right)_{R_o}$.

Methods (1) and (2) are clearly better than (3)

$$A = 15 \text{ km/s}|\text{kpc} \qquad (\text{known} \approx 10\%).$$

Determination of B

(1) From the study of proper motion.

(2) From other dynamical considerations yielding the ratio $-{}^{B}/_{A}$.

B is inherently more difficult to determine than A

$$B = -10 \text{ km/s}|\text{kpc} \qquad (\text{known} \approx 20\%).$$

Determination of R_o

(1) Directly obtained from variable stars about the galactic center.

(2) Measurement of the product AR_o.

At any longitude ω is maximum when r is minimum. Thus occurs $r = R_o$ sin

Hence $V_{max} = 2\,A\,R_o\,(1 - \sin \ell) \sin \ell$.

(3) Objects with $V_R=0$ must be at distance R_o from galactic center. At $r \sim R_o$, $d = 2 R_o \cos \ell$. Thus knowing d and ℓ yields R_o.

Methods (1) and (2) yield $R_o=10$ kpc (known $\simeq$ 10%).

Determination of Θ_o

(1) Velocity measured relative to extragalactic systems, $\Theta_o \sim 250$ km/s from the latest publication.

(2) Velocity measured relative to globular cluster subsystem, $\Theta_o \sim 200$ km/s. (As this subsystem probably relates we may say $\Theta_o \geq 200$ km/s.)

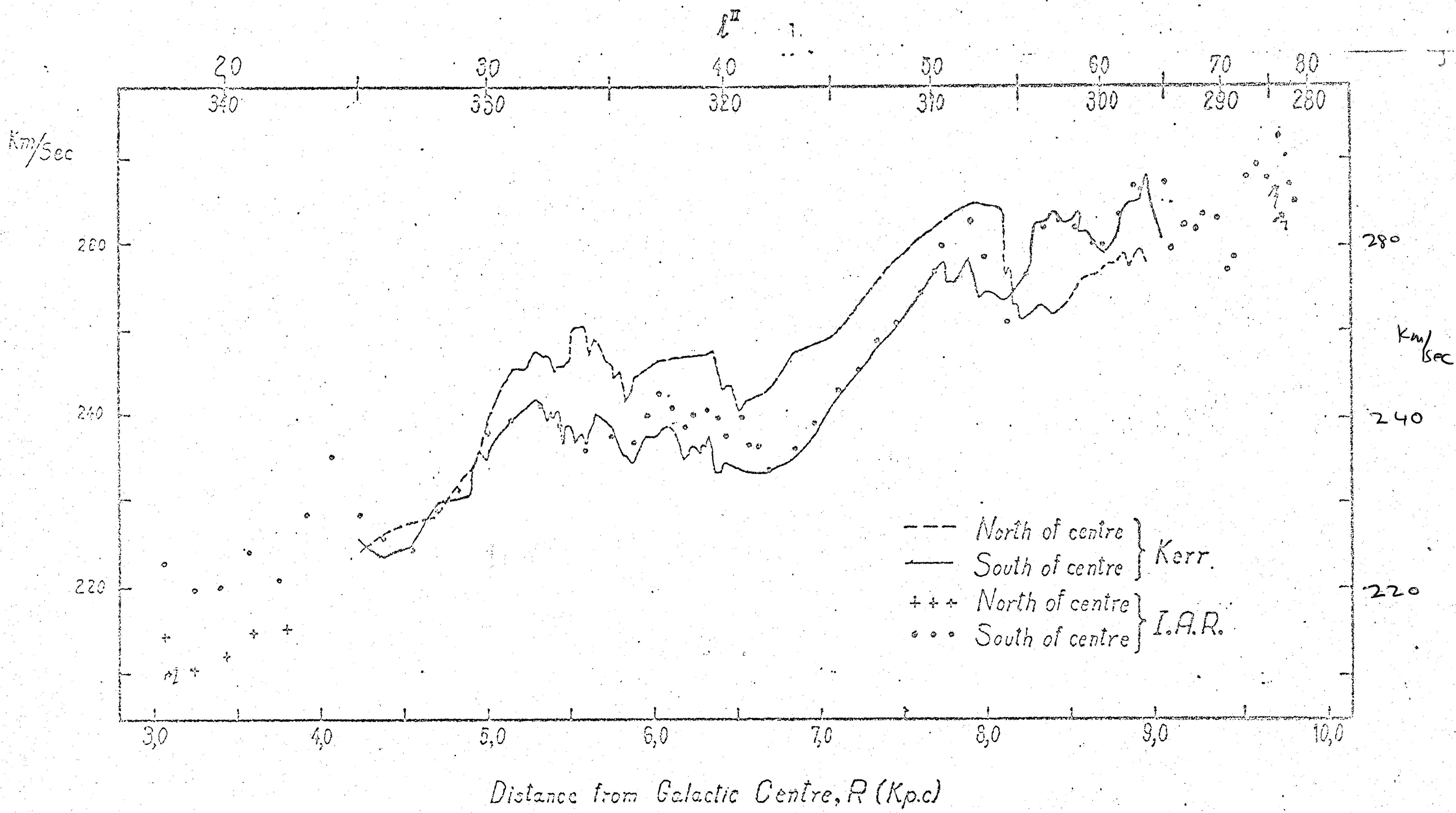
(3) From consideration of the escape velocity. (Clearly stars with enough energy will escape from the solar neighborhood of the galaxy. Relative to the LSR $V_{esc}{}^2 = \Pi^2 + (\Theta_o + \Theta^*)^2 + Z^2$ where Π = radial component, Z = component normal to galactic plane, Θ = component normal to Π, yields $\Theta_o = 276 \pm 26$ km/s, depends on validity of assumptions, particularly about mass gradients).

(4) $\Theta_o = (A-B)R_o$, $\Theta_o = 250$ km/s and solar period $= 2.45 \times 10^8$ yrs.

We may use these values and equation 1 to determine $\omega(r)$ or $\Theta(r)$. Radio studies of the neutral hydrogen spectral line at $\lambda = 21$ cm allow an investigation of the total galaxy to be made; obscuration is not a problem for long wavelengths.

The following table is indicative of galactic rotation.

r kpc	$\Theta(r)$ km/s	$R_o = 10$ kpc
0.32	220	
0.67	265	
3.53	206.4	
6.18	239.6	
7.74	248.5	
8.01	252.2	
10.00	250.0	



GALACTIC ROTATION CURVE for R_0=10

The most secure points of the observed rotation are determined from the point of maximum velocity which comes from $r_{min} = R_o \sin \ell$. At certain longitudes the line of sight is tangent to a spiral arm and the signal is a maximum at r_{min}. These longitudes are about 327°, 305°, 283°, 75°, 50°, 33°. For $r > R_o$ there will be no maximum V_r and thus $\omega(r)$ cannot be determined in this fashion and we must rely on stellar studies.

r kpc	Θ(r) km/s
11	244
12	236
13	227

The radio technique assumes circular motion to derive Θ(r). We therefore expect circular symmetry in the velocity field. The latest studies do not show this. No fully accepted explanation has been brought forth, although the "density wave" explanation looks most hopeful at this time.

II. THE DISTRIBUTION OF MASS IN THE GALAXY

In a spherical system with uniform density the force per unit mass outside sor radius r

$$F_r = -\frac{4}{3}\pi G\rho R.$$

This must be equivalent to the central force on the orbit

$$\omega^2 R = \frac{4}{3}\pi G\rho R$$

or

$$\frac{\partial\omega}{\partial R} = 0 \qquad A = 0.$$

Thus

$$B = \omega \quad \text{or} \quad B = \left(\frac{4}{3}\pi\rho G\right)^{1/2}.$$

Hence for this system there is NO differential rotation.

If all the mass is centrally concentrated the force per unit mass

$$F_r = -\frac{GM}{R^2}$$

$$\omega^2 R = \frac{GM}{R^2}$$

$$A = -\frac{1}{2} R \frac{d\omega}{dR} = \frac{3}{4} (GM)^{1/2} R^{-3/2} = \frac{3}{4} \omega$$

$$A - B = \omega \text{ or } A = \frac{3}{4} \omega \quad \text{and } B = -\frac{1}{4} \omega$$

Near the sun $A = -\frac{3}{2} B.$

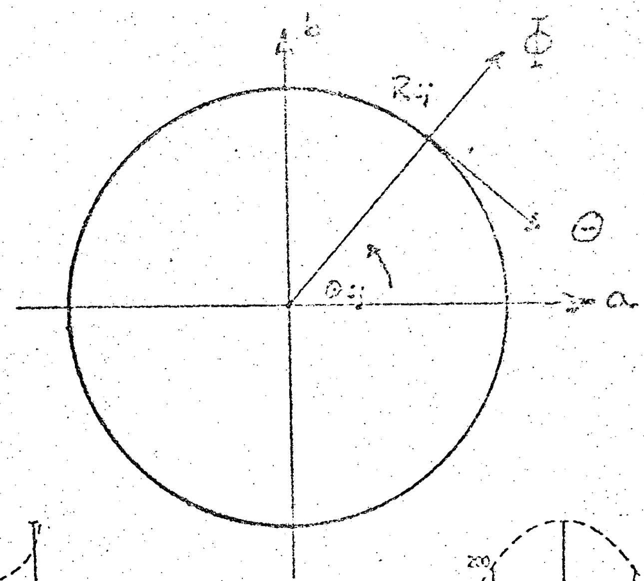
While for "solid body" rotation $A = 0$ and for a Keplerian system $A = -3B$. Thus for the Milky Way the mass is distributed between these extremes. The sun is far from the center, there must be a large density gradient towards the center and a significant fraction of the mass is distributed over the total galaxy.

The rotation curve of our galaxy is asymmetric showing the effects of non-circular motions. How general is this phenomena and do studies of extra-galactic systems give more detailed information?

Usually galaxies are inclined to the line of sight. We must therefore calculate the locus of constant velocity. We assume a smooth thin disk. At some radius R it has a circular velocity Θ and a radial velocity Φ. (See diagram.)

We tilt the disk about the a axis (the major axis). We then observe a projected value of the minor axis (b axis) velocity component. The observed radial velocity V_{ij}

$$V_{ij} = \Theta (R_{ij}) \cos \theta_{ij} \cdot \cos i + \Phi (R_{ij}) \sin \theta_{ij} \cdot \cos i + V_s$$



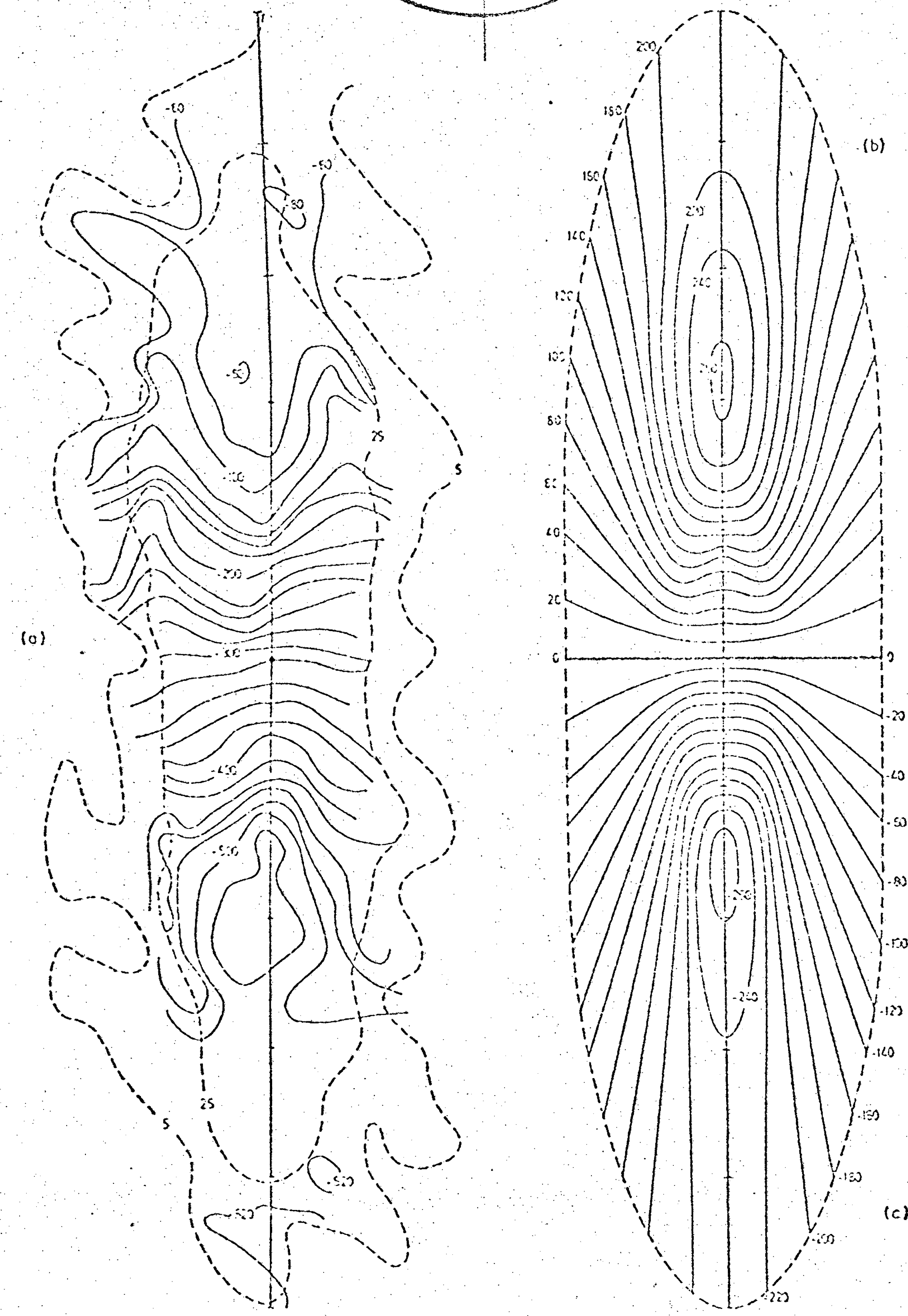


FIG. 5. (a) *Isovelocity map showing the distribution of median velocity over M 31. Contours of integrated neutral hydrogen emission are shown for 0·05 and 0·25 MHz °K of T_a.*
(b) *Isovelocity map expected for the NF major axis rotation curve.*

Where the angle i is the inclination of the plane to the line of sight, θ_i is the angle between the radius R_{ij} and the inclination of major axis, and V_s is the systemic velocity of the galaxy. The co-ordinates in the plane of the galaxy are related to the observed parameters by

$$a_i = a_i'$$

$$b_j = b_j'$$

$$\tan \theta_{ij} = \tan \theta'_{ij} \ \operatorname{cosec} i$$

Thus, knowing the major axis we can measure the radial velocity V and assuming that there are no radial motions and no motions out of the galactic plane, the radial velocity at any other point in the galaxy may be predicted.

In the case of M31 there is only general agreement. The symmetry expected about the major axis is not observed and the rotation curve is also asymmetrical. Similar circumstances exist for M33.

In the case of M31 we may be seeing the influence of either, or both, of the satellite galaxies M32 or NGC205. For M33 we may be seeing the gravitational effects of M31.

The galaxy NGC925 has a companion whose neutral hydrogen mass is ~ 3% that of the main galaxy. This might be the perturbing agent.

The evidence for a companion to the galaxy NGC300 is less clear but the velocity distortion is very large. The distortions observed in the galaxy NGC5236 are also very large and similar to those of NGC300. Both galaxies are barred spirals and near the center. The velocity contours are perpendicular to the bar. The position angle of the turn over points in the rotation curve is significantly different from that of bar. Perhaps this type of asymmetry is due to the bar?

A neutral hydrogen asymmetry exists for the galaxy M101 — perhaps caused by nearby dwarf companions. The rotation is certainly not symmetrical — and the velocity field shows non-circular effects that appear to correlate with the spiral structure.

We must conclude that dynamical asymmetries are a common galactic phenomena. The~~r~~e appear~~s~~ to be associated with mass asymmetries in the galaxy or nearby perturbing agents. Ultimately it is tempting to link these unbalanced forces with the spiral structure of the galaxies.