COSMOLOGY

Sebastian von Hoerner

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13.0.0. Introduction

Cosmology tries to describe the universe as a whole (assuming this to be a meaningful concept). The single objects, from atoms to stars to galaxies and their clusters, are used as sources of observational information, and cosmology must also provide a proper frame for them, which enables them their formation and evolution and finally us their description. But the main emphasis in cosmology is mostly not on the objects. It is on the metric of space and time; and on the average density of matter, radiation and energy, on its change with time and maybe its spatial fluctuations.

Unfortunately, cosmology has up to now been mostly theory and only very little observational information; many radio observations obtain their cosmological relevance only in connection with some optical observations; and any "latest news" in cosmology have invariably turned out to be wrong. These three facts will be reflected in the contents of this chapter, which then will be more "textbook-like" than the others. The style of writing will be effected, too, by trying to compress much information into few pages; but the less we understand something, the more information we need for its description. Some effort has been spent in providing an extensive (hopefully useful) list of references.

Much more emphasis than usual will be put on problems, oddities and uncertainties, since these seem to be, after all, very essential features of this fascinating field of study.

13.1. General Problems

13.1.1. Limited Experience

We want to describe the whole universe, but our range of experience is badly limited. (a) Our telescopes reach only to a certain distance; (b) The human time scale is very short as compared to cosmological changes; (c) Our laws of physics are derived from moderate ranges of density and temperature, wereas all big-bang models begin with a singularity. (d) The following objects are known and studied: galaxies since about 40 years, clusters about 30, superclusters still undecided; quasars 10 years (but distance still undecided), and background radiation since 7 years. Theoreticians suggest the existence of "black holes" (remnants from gravitational collapse) and "white holes" (delayed little-bangs), and antimatter should be just as frequent as matter but is not seen. Finally, the "hidden mass" problem (Section 13.4.1.) indicates that all visible matter is maybe only 1/100 of the total. Question: how complete, or at least how representative and informative is this list of known objects? (e) A similar question concerns the observables: we have observed light thousands of years, but radio waves only 40 years; we just started with X-rays and γ-rays, and maybe we observe neutrinos and gravitational waves; but what else are we missing? (f) In addition, most world models have a horizon (Section 13.2.3.), a principal limit to any observation.

Whether we regard this as a rather hopeless situation or as an exciting challenge, is completely up to us. But even deciding for the latter we will frequently feel pushed to the former and then should honestly say so.

13.1.2. Entanglement

a. The Problem. We would like to deal separately with questions concerning space and time of world models, and evolution of the observed objects. We need "evolution-free model tests" and "model-free evolution tests"; the first ones to be divided into measurements of spatial curvature and isotropy, and independent measurements of time-dependent things like expansion and deceleration. In the actual observations, however, all three items are completely tangled up; disentangling them is most urgent and difficult (and completely unsolved in most cases).

b. Distance = Past. The further we look out into space, the further we look back in time, because of the finite speed of light. Only in steady-state theory is it of no concern. But in all big-bang models we see the more distant parts of the universe in earlier phases, all the way back to time zero if we could look out to infinite redshift. See Figure 13.1, calculated with H_o = 100 (km/sec)/Mpc.

c. Objects vs. World. We see only objects, but neither space nor time. Objects are formed and evolve, they have a history of their own. We must distinguish between their individual evolution and class evolution:

evolution	matters if objects have a	which holds for	
individual	life-time > 10^{10} years	optical galaxies	(13.1)
class	life-time << 10^{10} years	quasars + radio galaxies	

Class evolution means that certain average properties of the objects (like creation frequency, luminosity, diameter, life-time) may be functions of time-dependent parameters (like surrounding density, chemical composition, radiation temperature).

For disentangling, we need a theory of the objects. This does exist for optical galaxies (approximately at present, but improvable); it is completely missing for quasars and radio galaxies. If such theory is missing, then the observational data must be solved for an additional number of unknowns (evolution parameters in addition to model parameters). With enough evolution parameters, any set of observations then can give a good fit to any given world model; this is our present situation with number counts.

13.1.3. Observables and Their Standards

Most observables are useful for cosmology only if we know standards. For using the observed radio flux S or optical magnitude m, we must know the absolute luminosity L of the source ("standard candle"); for using the angular diameter θ, we need the linear diameter D ("standard rod"); and for any number count, n(S) or n(z, S), we need the luminosity function $\phi(L)$.

Optical and radio luminosities, of galaxies as well as of quasars, have a very large range, more than a factor 100 in L. Fortunately, the optical luminosity function of galaxies drops very steeply at the bright end, and many galaxies occur in clusters; thus, the brightest galaxy in a rich cluster is a fairly good standard candle, with a scatter in L of about a factor 1.3 ($\pm$ .25 mag). But this is not so for quasars nor radio galaxies, where we are left with the full range of L.

Optical diameters of rich clusters may become useful in the near future. Radio diameters of galaxies and quasars (or separation between doubles) have a tremendous range, a factor 10^7 in D, but it seems that their upper limits of about 300 kpc can be used. Our knowledge of the luminosity function is also very poor, see Figure 13.8, with an uncertainty of $\phi(L)$ of at least a factor 4 (up and down) over most of its range, and at least a factor 10 at its ends.

13.1.4. How Distant are Quasars?

Appreciable differences between world models show up only for redshifts above $z \geq 1$. The observed galaxies and clusters have $z \leq .25$, with only one exception at $z = .46$; while quasars are observed within $.16 \leq z \leq 2.88$. About half of the known radio sources are quasars. Thus, quasars are the most (maybe the only) promising objects of cosmology; if they are not at cosmological distance, then even the N(S) plot of radio sources is useless since 1/2 are quasars. But whether or not they are distant is still undecided. For summaries, see M. Burbidge (1967) and Schmidt (1969, 1971) in general, and Cohen (1969) for structure. Some arguments, against and in favor of their cosmological distance (CD), are discussed below.

a. Against CD. The first objection raised regarded the large amount of energy (up to 10^{61} erg) and mass (up to $10^{8}\ M_{\odot}$) following from CD, confined to an extremely small volume (.01 pc = 1 light-week diameter) following from the fast variability of many quasars. This argument does not count any more, since massive small objects can be obtained by gravitational collapse (Texas Symposia), or by stellar-dynamical evolution up to stellar collisions (von Hoerner 1968), or a combination of both (Spitzer and Saslaw 1966). Second, the redshift distribution n(z) showed a high and narrow maximum at $z = 1.95$ (G. Burbidge 1967), and some periodicities (Burbidge 1968, Cowan 1968). But both effects have completely disappeared with a larger number of data (208 quasars, Wills 1971).

Third, one should expect quasars (just like radio galaxies) to be a certain type or a special phase of galaxies. Optical and radio galaxies occur preferentially in clusters, but quasars don't. Five cases were once claimed by Bahcall et al. (1969), but questioned by Arp (1970). Fourth, Arp (1967, 1968, 1971) and Weedman (1970) find several cases of close companionship with large differences Δz in redshift; like a quasar or Seyfert nucleus sitting in the spiral arm of a near-by galaxy, or connected to it by a bridge. The observed examples seem to be too numerous and too striking for just a chance projection. If confirmed and real, they would give evidence for non-CD large redshifts of unexplained origin (which could effect galaxies as well as quasars).

Fifth, VLB observations show fast lateral expansion for two quasars (Whitney et al. 1971; Cohen et al. 1971; Gubbay et al. 1969; Moffet et al. 1971). If at CD, the lateral velocity V of expansion (or separation of doubles) would exceed the speed of light: $V/c \gtrsim 2$ for 3C273, and $V/c \gtrsim 3$ for 3C279. Either we drop CD, or we have a choice between several possible but somewhat artificial geometrical explanations, and a relativistic explanation going back to Rees (1967): if a source shoots off a companion almost at us (angle β to line-of-sight) with a velocity v almost the speed of light, then the apparent lateral speed V is maximum for $\cos \beta = v/c$, in which case $V = \gamma v$ and $\Delta z = \gamma - 1$, with $\gamma = (1 - v^2/c^2)^{-1/2}$. For example, $v/c = .95$ needs $\beta = 18°$, and yields $V/c = 3.05$ and $\Delta z = 2.2$. This explanation seems to save CD and might even help for some companions with large Δz; but I should mention that shooting off a cloud of restmass m with speed $v \gtrsim c$ needs the energy

$$E = (\gamma - 1)\, mc^2, \tag{13.2}$$

or $E = 2.2\, mc^2$ in our example. Since nuclear fusion gives 1% of mc^2, we must burn up a very large mass, of 220 m, and funnel all that energy just into the kinetic energy of the small cloud m, without destroying the cloud. This sounds very difficult.

b. <u>In Favor of CD</u>. First of all, wishful thinking, of course, since not much observational cosmology is left without the quasars. Second, this has been done all the time for all distant galaxies; should the whole Hubble relation be rediscussed? Third, nothing else seems to work: (a) Gravitational redshifts would give much broader lines (or ridiculously small distances). (b) To overcome this difficulty, Hoyle and Fowler (1967) suggested a cluster of many collapsed objects, with an emitting cloud at its center; but Zapolski (1968) found too short a lift-time plus other difficulties. (c) Shooting off clouds at high speed from many galaxies should give blueshifts as well (Faulkner et al. 1966). (d) Terrell (1967) thus assumes our Galaxy as the sole origin of these explosions; but this needs 10^{63} erg of kinetic energy (or burning up to 10^{11} $M_{\odot}$) for 10^{6} quasars, and each single shot would also lead to the problem mentioned with equation (13.2).

Fourth, a nice continuity (and some overlapping) between radio galaxies and quasars. Heeschen (1966) plotted radio luminosity versus surface brightness, which was extended and confirmed by Braccesi and Erculiani (1967). Meanwhile, many other plots of various quantities gave similar results; even the radio luminosity function, Section 13.5.1. Most of this would be mere coincidence without assuming CD. A strong similarity between radio galaxies and quasars is also shown by spectra, structure, and variability. Fifth, without CD it would again be mere coincidence that the stellar collision model (Dyson 1968, von Hoerner 1968) gives about the right values for mass, radius, luminosity, and variability.

Sixth, the angular sizes of quasars as a function of redshift, $\theta(z)$, not only continue nicely the radio galaxies, but fall off with $\theta \sim 1/z$ just as they should (Legg 1970, Miley 1971). Some astronomers consider this as the strongest argument for CD; but actually it is very odd that $\theta \sim 1/z$ continues further down than any world model would allow (Section 13.5.2.). The m(z) and S(z) relations are mostly, but not completely, blurred by the large scatter of L, and will be discussed in Section 13.5.2.

In summary, quasars are the most important but most uncertain objects for cosmology. Most promising for the future seems to be:

A. More VLB work.

(a) Fast geometrical changes (against CD).

(b) Continue $\theta(z)$ relation (in favor of CD).

(c) Proper motions? (300 km/sec at 6 Mpc distance gives 10^{-4} arcsec in 10 years.)

B. Further examples and details about close odd companions of galaxies.

C. A medium-sized optical telescope in space.

(a) Are quasars galactic nuclei?

(b) Do they occur in clusters of galaxies?

(c) Optical diameters.

13.2. Basic Theory

This section treats concepts and formulas which are more basic and general than the various theories and models treated later.

13.2.1. Space, Time, References

Time is usually considered as just a fourth coordinate (Minkowski). This may be used as a convenience, but there is a fundamental difference (von Weizsäcker: "the past is factual, the future is possible") meaning that time has an arrow while space has not. Space has three dimensions, and in our normal experience space is "flat" or Euclidean, meaning that parallel lines keep their distance constant. But space could as well be curved, as already discussed by Gauss, and only observations can tell.

Absolute vs. Relative. An absolute frame of rest and even its origin was defined for the ancient Greeks by the center of the Earth, see Table 13.1. Galilei and Newton "relativated" location and velocity, whereas unaccelerated (inertial) motion, and the absence of rotation, still kept an "absolute" meaning. Mach's principle, about 1893, postulates everything to be relative, or more exactly, to be (somehow) defined by the total masses of the universe.

Special relativity draws the line where Newton did. With general relativity, Einstein wanted originally to go further and to fulfill (and specify) Mach's principle, but actually went one step back by permitting a curved empty space (which defines a frame of rest as can be shown, although I have never seen it printed).

Table 13.1. Various Degrees of Relativity.

The vertical line connected with each theory separates the relative quantities (left) from those which have absolute standards (right).

	ancient Greeks (Earth)	general relativity, any curved empty space	Galilei, Newton, special relativity	Mach's principle
Location	x (origin	$\dot{x}$ (rest)	$\ddot{x}$ (acceleration)	
Angle	α (direction)	$\dot{\alpha}$ (rotation)	$\ddot{\alpha}$ (torque)	

13.2.2. Metric

A "metric" is the generalization of Pythagoras' law, including time and with general metric coefficients $g_{\mu\nu}$, but restricted to small distances ds:

$$ds^2 = \sum_{\mu=1}^{4} \sum_{\nu=1}^{4} g_{\mu\nu} \, dx^\mu dx^\nu. \tag{13.3}$$

A metric is said to be Riemannian if it has the quadratic form of equation (13.3), and if the coefficients depend on coordinates only (space and time) but not, for example, on their derivatives.

The universe is mostly imagined as being filled with a "substratum" (evenly smeared-out matter and radiation) expanding with the universe but without peculiar motion. A "fundamental observer" is at rest in the substratum. The expansion is thrown into the $g_{\mu\nu}$ which makes the space coordinates "co-moving". For simplification and in accordance with our limited observations, the universe is mostly postulated to be homogeneous and isotropic. Schur's theorem says that isotropic means always homogeneous, too, but not vice versa. Homogeneous plus isotropic is frequently called "uniform".

Weyl's postulate says "world lines of fundamental observers do not intersect (except maybe at the origin)", If it holds, time in (13.3) is orthogonal on space and a "cosmic time" can be established, the same for all fundamental observers. (Counter-example: two satellites in different orbits, meeting each other sometimes, do not fulfill the postulate and generally keep different times.)

Under the three assumptions of Riemannian metric, Weyl's postulate, and uniformity, equation (13.3) reduces to the Robertson-Walker metric:

$$ds^2 = c^2\,dt^2 - R^2(t)\,\frac{dx^2 + dy^2 + dz^2}{(1 + kr^2/4)^2}, \quad \text{with} \quad r^2 = x^2 + y^2 + z^2. \tag{13.4}$$

Here, t = cosmic time; R(t) = radius of curvature of 3-space if $k \neq 0$, and R(t) = distance between any two fundamental observers if k = 0; space may be closed (k = +1), flat (k = 0), or hyperbolic (k = -1). The x, y, z are <u>some</u> comoving metric space coordinates but can be transformed into any other form. For example, the transformation $\bar{r} = r/(1 + kr^2/4)$, with polar coordinates, yields

$$ds^2 = c^2\,dt^2 - R^2(t)\left\{\frac{d\bar{r}^2}{1 - k\,\bar{r}^2} + \bar{r}^2\,(d\theta^2 + \sin^2\theta\,d\phi^2)\right\}. \tag{13.5}$$

But both r and $\bar{r}$ are somewhat confusing, see Table 13.2. A better metric distance, called u by Sandage but ω by McVittie, is obtained by the transformation $u = \arcsin\bar{r} = 2\arctan(r/2)$ for k = +1, $u = \bar{r} = r$ for k = 0, and $u = \operatorname{arcsinh}\bar{r} = 2\operatorname{arctanh}(r/2)$ for k = -1. This leads directly to a physically useful distance ℓ:

$$\ell = uR = \text{proper distance} = \text{rigid-rod distance} = \text{radar distance}. \tag{13.6}$$

Defining a function

$$\mathcal{S}(u) = \bar{r} = \begin{cases} \sin u & \text{for } k = +1 \\ u & 0 \\ \sinh u & -1 \end{cases} \qquad (13.7)$$

then yields the metric as

$$ds^2 = c^2\, dt^2 - R^2(u) \left\{ du^2 + \mathcal{S}^2(u)\ [d\theta^2 + \sin^2\theta\ d\phi^2] \right\} . \qquad (13.8)$$

Table 13.2 compares r, $\bar{r}$ and u (for k = -1, an "equator" has been formally defined by u = $\pi/2$, too). The behavior at the antipole and at infinity shows clearly why we called u better and less confusing than r or $\bar{r}$.

Table 13.2. Three Types of Metric Distances

metric distance	k = +1		k = -1	
	equator	antipole	"equator"	$\ell = \infty$
r	2	∞	1.312	2
$\bar{r}$	1	0	2.30	∞
u	$\pi/2$	π	$\pi/2$	∞

The following lists a few properties of a sphere of radius ℓ = uR in curved uniform 3-space, centered at the observer; derivations are omitted here but can easily be obtained from metric (13.8):

	element	whole sphere	
circumference	$dC = R\mathcal{S}d\phi$	$C = 2\pi\ R\mathcal{S}$	(13.9)
surface area	$dA = R^2\ \mathcal{S}^2\ \sin\theta\ d\theta\ d\phi$	$A = 4\pi\ R^2\ \mathcal{S}^2$	(13.10)
volume	$dV = 4\ \ R^3\ \mathcal{S}^2\ du$	$V = \frac{4\pi}{3}\ R^3 \mathcal{R}$	(13.11)

with

$$\mathcal{R}(u) = 3\int_0^u \mathcal{Y}^2(u)\,du = \begin{cases} \frac{3}{2}\,[u - \frac{1}{2}\sin(2u)] = u^3(1 - u^2/5 \pm \ldots) & k = +1 \\ u^3 & 0 \\ \frac{3}{2}\,[\frac{1}{2}\sinh(2u) - u] = u^3(1 + u^2/5 \pm \ldots) & -1 \end{cases} \qquad (13.12)$$

Finally, for $k = +1$, the whole universe has the following total values:

Circumference (origin-antipole-origin)	$C = 2\pi R$,	(13.13)
Area of plane (to antipole, all directions)	$A = 4\pi R^2$,	(13.14)
Volume (total of all space)	$V = 2\pi^2 R^3$.	(13.15)

13.2.3. Horizons

a. Particle Horizon (or just "horizon") is a sphere in 3-space, of metric radius u_{ph} where objects would be seen with infinite redshift and at age zero. Particles within this sphere are observable, those outside are not. Since $u_{ph}(t)$ increases monotoneously, more particles get observable all the time, and none of them can ever leave the horizon again. See Rindler (1956,b).

A particle horizon means that the whole history of the universe is observable, to age zero, but only a limited part of all space (6000 Mpc in Fig. 13.1). A given world model has a particle horizon if the integral exists:

$$u_{ph} = \int_0^t \frac{dt}{R(t)} \; . \qquad (13.16)$$

b. Event Horizon. If $\Lambda > 0$, the expansion of the universe may finally be so much accelerated that some distant photon coming our way will never reach us. Photons just reaching us at $t = \infty$ define the event horizon; it sets an upper limit to the age at which we can see a given object, and it exists if the integral exists:

$$u_{eh} = c\int_{t}^{\infty} \frac{dt}{R(t)} . \qquad (13.17)$$

c. If Both Horizons exist, some distant object is first not observable. Second, it enters at some time given by (13.16) our particle horizon with infinite redshift and age zero. Third, it gets older and its redshift decreases. Fourth, the redshift goes through a minimum and increases again, while the object seems to age more slowly. Fifth, if we observe it infinitely long, the redshift goes to infinity, but the age at which we see the object approaches a finite age given by (13.17).

In four-dimensional space-time, both horizons are light-cones. Our particle horizon is our forward light-cone emitted by us at $t = 0$; our event horizon is our backward light-cone reaching us at $t = \infty$.

13.2.4. Observables

The following formulas are derived from (13.8) to (13.12). They assume nothing else except Riemannian metric, Weyl's postulate, and uniformity; R(t) and k are left unspecified. Any special cosmological theory then will provide the dynamics, a differential equation for R(t), mostly in the form $\dot{R} = \dot{R}(R)$. And a special world model then will have selected values for constants of integration, k, and other parameters.

The indices mean: o = present, r = received, e = emitted; bol = bolometric, ν = certain frequency and limited bandwidth. The spectrum index α is defined by $L_\nu \sim \nu^{+\alpha}$, spectrum curvature is neglected. First and second derivative of R(t) are frequently used as

Hubble parameter $H_o = \dot{R}_o/R_o$, (13.18)

Deceleration parameter $q_o = -R_o\ddot{R}_o/\dot{R}_o^2$. (13.19)

a. Redshift z.

$$1 + z = R_o/R_e; \qquad z = \tau H_o + (1 + q_o/2)\, \tau^2 H_o^2 \pm \ldots \tag{13.20}$$

The metric distance u is derived from (13.8), with ds = dθ = dϕ = 0, as

$$u = c \int_{t_e}^{t_o} \frac{dt}{R(t)} = c \int_{R_e}^{R_o} \frac{dR}{R\,\dot{R}(R)} . \tag{13.21}$$

With (13.20) we then obtain the following formula which describes how a special theory, via dynamics, enters the formulas connecting observables and redshift:

$$u(z) = c \int_{R_o/(1+z)}^{R_o} \frac{dR}{R\,\dot{R}(R)} \tag{13.22}$$

or approximately

$$u(z) = \frac{c\,z}{R_o H_o} \left\{ 1 - \frac{q_o + 1}{2}\, z \pm \ldots . \right\} . \tag{13.23}$$

Sandage (1962) detected the possibility of a true "evolution-free model test": you observe a distant object during a long time, and find its change of redshift. Unfortunately, dz/dt is of the order of H_o; measuring z with an accuracy of 10^{-4}, say, then needs observation during 10^6 years.

b. Flux S (or magnitude m). The flux can be written as

$$S_{bol} = \frac{L_{bol}}{4\pi \ell_{bol}^2} \quad \text{or} \quad S_\nu = \frac{L_\nu}{4\pi \ell_\nu^2} \tag{13.24}$$

with a

$$\text{luminosity distance} \qquad \ell_{bol} = R_o \mathcal{Y}(u)\ (1+z). \qquad (13.25)$$

The flux observed at frequency ν in a given bandwidth is emitted first at a higher frequency and second within a broader band, which together gives a factor $(1+z)^{1+\alpha}$ for the observed flux. Thus

$$\ell_\nu = R_o \mathcal{Y}(u)\ (1+z)^{(1-\alpha)/2} = \ell_{bol}\ (1+z)^{-(1+\alpha)/2} \qquad (13.26)$$

and approximately

$$S_\nu = \frac{L_\nu}{4\pi} (H_o/cz)^2 \left\{ 1 + (q_o+\alpha)z \pm \ldots \right\} . \qquad (13.27)$$

Note: for flat space $\ell_{bol} = (1+z)\ell$, which means that $S \sim \ell^{-2}$ not in "Euclidean space" (as sometimes stated) but only in "static Euclidean space" where all $z = 0$.

In optical astronomy, the transition from (13.25) to (13.26) is much more complicated (strong lines, curved spectrum, wide band) and is called the "K-correction". The conversion from optical magnitude m into flux S, in flux units, is done by

$$S(m) = 10^{a-0.4m} \quad \text{with} \quad a = \begin{cases} 3.258 & \text{for U} \\ 3.621 & \text{B} \\ 3.584 & \text{V} \end{cases} \qquad (13.28)$$

c. Angular size θ (D = linear size).

$$\theta = \frac{D\ (1+z)}{R_o \mathcal{Y}(u)} = \frac{D\ (1+z)^2}{\ell_{bol}} . \qquad (13.29)$$

d. Surface Brightness b (B = nearby value). From (13.24) and (13.29) we find

$$b_{bol} = \frac{B_{bol}}{(1+z)^4} \quad \text{and} \quad b_\nu = \frac{B_\nu}{(1+z)^{3-\alpha}} . \tag{13.30 a,b}$$

These formulas do not depend on world models; if they are not fulfilled, the reason can only be evolution (class or individual, of the sources). Thus (13.30) is a true "model-free evolution test"; or would be, if we had a standard for B which we don't. But with more data, the following limit could be used. Kellermann and Pauliny-Toth (1969) give a theoretical upper limit for the brightness temperature, $T \leq 10^{12}$ °K, close to which are several of the variable maxima of quasars and galactic nuclei. Since $\alpha = 0$ at the maximum, and $T \sim b\lambda^2$ which means $T_r \sim T_e(1+z)^2$, equation (13.30,b) predicts an observed z-dependence of

$$T_{max} = \frac{10^{12}\ °K}{(1+z)} . \tag{13.31}$$

e. Parallax Distance. Call α and γ the angles from the end points of a (perpendicular) baseline D to some distant object; then its parallax distance can be defined as $\ell_{par} = D/\beta$, with $\beta = \pi - (\alpha+\gamma)$. Call θ the angle under which D is seen from the object; then in flat space $\beta = \theta$. But in general:

$$\ell_{par} = \frac{R_o \mathcal{S}(u)}{\sqrt{1 - k\mathcal{S}^2(u)}} = \begin{cases} R_o \tan u & k = +1 \\ R_o\, u & 0 \\ R_o \tanh u & -1 \end{cases} \tag{13.32}$$

and

$$\beta/\theta = \sqrt{1 - k\mathcal{S}^2(u)} = \begin{cases} \cos u & k = +1 \\ 1 & 0 \\ \cosh u & -1 \end{cases} \tag{13.33}$$

With u(z) from equation (13.22), equation (13.32) then gives $\ell_{par} = \ell_{par}(z)$, which is a true "evolution-free model test". Or would be, if we could measure angles with an accuracy of 10^{-10} arcsec; for D = Earth orbit, and $H_o = 100$, we have approximately

$$\beta = 3 \times 10^{-10} \text{ arcsec } \frac{1}{z} \left(1 - \frac{q_o+1}{2} z \pm \ldots \right). \qquad (13.34)$$

Weinberg (1970) introduced the parallax distance for a different purpose:

$$k/R_o^2 = (1+z)^2 \ell_{bol}^{-2} - \ell_{par}^{-2} \qquad (13.35)$$

yields a direct measure of the curvature and thus is a "dynamic-free curvature test" but unfortunately contains evolution (L_{bol} for ℓ_{bol}); whereas (13.32) contains dynamics, u(z), but no evolution.

f. <u>Number counts</u>. With all sources of same L, the cumulative count would be

$$N(S) = \frac{1}{4\pi} Q \mathcal{R}(u) = \text{number/steradian with flux} \geq S \qquad (13.36)$$

where $Q = (4\pi/3)\, \rho_o\, R_o^3 = (4\pi/3)\, \rho\, R^3$ = constant, neglecting source evolution, and for all theories except steady-state; ρ is the number of sources/volume; $\mathcal{R}(u)$ is defined in (13.12); u(S) is obtained from (13.24) and (13.26), with dynamics (13.22) for a special theory. Approximately,

$$N(S) = \frac{1}{3} \rho_o (L/4\pi S)^{3/2} \left\{ 1 - \frac{3}{2}(1-\alpha) \frac{H_o}{c} (L/4\pi S)^{1/2} \pm \ldots \right\} \qquad (13.37)$$

and the famous slope of the log N - log S plot is

$$\beta = \frac{d \log N}{d \log S} = -\frac{3}{2} + \frac{3}{4}(1-\alpha) \frac{H_o}{c} (L/4\pi S)^{1/2} \pm \ldots \qquad (13.38)$$

with

$$\frac{H_o}{c}\,(L/4\pi S)^{1/2} \approx z\ . \qquad (13.39)$$

Thus, the slope is appreciably less steep than -3/2 already for small z. For $\alpha = -0.8$

z	.05	.10	.15	.20
$-\beta$	1.43	1.36	1.30	1.23

(13.40)

Observationally, the slope β should be calculated from the data by the maximum-likelihood method of Crawford et al. (1970).

Instead of the cumulative count N(S) one should rather plot the differential count $n(S) = -dN/dS$, where $n(S)dS$ is the number/steradian within S ... S+dS, because:

1. The n(S) points are statistically independent of each other and give an honest picture; whereas the N(S) points contain the same (strong) sources again and again, feigning more accuracy than they have (Jauncey 1967).
2. Any details are shown sharper in the n(S) plot, they are more smeared out and propagated to fainter fluxes in the N(S) plot. For a good example, see Bridle et al. (1972).

Approximately,

$$n(S) = \frac{1}{2}\,\rho_o (L/4\pi)^{3/2}\, S^{-5/2}\left\{1 - 2(1-\alpha)\underbrace{\frac{H_o}{c}\left(\frac{L}{4\pi S}\right)^{1/2}}_{\approx z} \pm \dots\right\}. \qquad (13.41)$$

Because of the wide spread of L, one must know (or pretend to know) the luminosity function $\phi(L)$. The previous formulas then should be integrated, $\int \dots \phi(L)dL$; or, one may introduce the redshift z and count $n(S,z)\,dS\,dz$, with

$$n(S,\ z) = 3\ Qc\ \frac{R_o^2 \psi^4(u)\ \phi(L)}{\dot{R}(z)\ (1+z)^{\alpha}} \tag{13.42}$$

where all functions on the right-hand side can be expressed in terms of S and z by using (13.24) and 13.26), and dynamics (13.22). In addition, it turns out that evolution must be introduced, too, which will be discussed in connection with the observations, Section 13.5.4.c.

Equations (13.37), (13.38), and (13.41) show that the flatness of the bright end, as calculated in (13.40), is the <u>same</u> for <u>all</u> models (except steady-state where it is still flatter). It depends only on the spectral index α, but not on model parameters like q_o or k. Differences between world models can only show up at the fainter part of the plot, from terms of higher order. Thus, the brighter part of the log N/log S plot can only yield a model-free evolution test.

13.2.5. <u>Matter, Antimatter, and Radiation</u>

For the dynamics, one needs an equation of state, $p = p(\rho,\ T)$. But the pressure is significant only in the early phases of big-bang models; thus the following applies only to those models. On the other side, all big-bang models get more and more similar to each other the further we go back in time; thus the following applies to all big-bang models in about the same way (almost model-independent).

a. <u>Comparison</u>. In general, we have

$$p = p_m + p_r \tag{13.43}$$

$$\rho = \rho_m + \rho_r \tag{13.44}$$

m = matter (nucleons, electrons)
r = radiation (photons, neutrinos)
o = present value

At present,

$$3p_{mo}/c^2 = \rho_{mo}(w/c)^2 = 10^{-6}\ \rho_{mo} = 3 \times 10^{-36\pm1}\ \mathrm{g/cm^3}\ (w = 300\ \mathrm{km/sec}), \qquad (13.45)$$

$$3p_{ro}/c^2 = 6.8 \times 10^{-34}\ \mathrm{g/cm^3} \quad (3\ ^\circ\mathrm{K\ background\ radiation}), \qquad (13.46)$$

$$\rho_{mo} = 3 \times 10^{-30\pm1}\ \mathrm{g/cm^3}\ (\text{visible vs. hidden matter, Section 13.4.1}), \qquad (13.47)$$

$$\rho_{ro} = 3p_{ro}/c^2 = 6.8 \times 10^{-34}\ \mathrm{g/cm^3}. \qquad (13.48)$$

Thus at present, to a good approximation,

$$p = 0; \qquad \rho = \rho_m. \qquad (13.49)$$

Going backwards in time, we have, if matter and radiation do not interfere (decoupled):

$$\rho_m \sim R^{-3}; \quad \text{conservation of mass}; \quad d(\rho R^3) = 0. \qquad (13.50)$$

$$\rho_r \sim R^{-4}; \quad \text{conservation of energy}; \quad dE + pdV = 0;\ T_r \sim R^{-1}. \qquad (13.51)$$

The densities of matter and of radiation then were equal when

$$\rho_r = \rho_m \quad \text{when} \quad 1+z = R_o/R = 5 \times 10^2 \ \ldots\ 5 \times 10^4 \qquad (13.52)$$

$$\text{and} \qquad T_r = 10^3 \ \ldots\ 10^5\ ^\circ\mathrm{K} \qquad (13.53)$$

for visible ... hidden matter.

b. Equation of State. Instead of deriving $p(\rho,T)$ from physics, one mostly just defines $\varepsilon = p/(\rho c^2)$ and makes simplifying assumptions about $\varepsilon(t)$; see Chernin (1966), McIntosh (1968), Zeldovich (1970). The range is $0 \leq \varepsilon \leq 1/3$, between a dust universe (matter only) and a universe containing radiation only.

In general, from $dE + pdV = 0$,

$$\rho \sim R^{-3(1+\varepsilon)} . \tag{13.54}$$

The very early phase of a big-bang universe is the hadronic state (Hagedorn 1965, 1970; Moellenhoff 1970). All surplus energy goes into pair-creation of heavy and super-heavy particles (hadrons), without further increase of the temperature. This leads for $t \to 0$ to $\rho \to \infty$ and $p \to \infty$, but $\varepsilon \to 0$ and $T \to T_h$; densities are above 10^{15} g/cm^3, and

$$T \leq T_h = 1.86 \times 10^{12}\ {}^\circ K = 160 \text{ MeV}. \tag{13.55}$$

This is followed by a phase of dominating radiation, up to limit (13.52), followed by our present phase of dominating matter. Since the dominance is always strong, except for short transitions, a fairly good approximation for $\varepsilon(t)$ is just a step-function:

$$\left.\begin{array}{llll} 1. & \text{Hadronic state} & \varepsilon = 0 & \text{for } T \geq 10^{11}\ {}^\circ K,\ \ t \leq 10^{-4} \text{ sec} \\ 2. & \text{Radiation universe} & \varepsilon = 1/3 & \text{before limit (13.52)} \\ 3. & \text{Dust universe} & \varepsilon = 0 & \text{after limit (13.52)} \end{array}\right\} \tag{13.56}$$

How far back in time may we trust our physics? Except for a more general feeling of distrusting all extremes, nobody has come up with any well-founded limitation. Frantschi et al. (1971) find that quarks will be present in ultra-dense matter, but will not change the equation of state; Misner (1969,a) finds that quantization gives no change for at least

$$R \geq (G\, h/c^3)^{1/2} = 10^{-33} \text{ cm}. \tag{13.57}$$

c. Decoupling, Viscosity, Relics. Some agent is said to decouple (from the rest of the world) when its collision time gets larger than the Hubble time (H_o^{-1}), or when its mean free path gets larger than the particle horizon, whichever comes first. Some equilibrium then is terminated. Two such agents are important: neutrinos and photons.

Surrounding the time of decoupling, this agent may yield a large viscosity; whereas earlier the range of interaction was too small, and later there is no interaction. A large viscosity may smear-out primeval finite-size inhomogeneities and anisotropies (fluctuations, turbulence, condensations); it increases uniformity or keeps it up.

Decoupling also leaves (non-equilibrium) relics. Neutrino decoupling at 10^{10} °K means the termination of nucleon pair-creation, which means a frozen-in neutron/proton ratio, which finally defines the helium/hydrogen ratio Y. The helium is made at $10^8 - 10^9$ °K, when deuterium gets no longer thermally disintegrated while neutrons are still not decayed. One finds Y = .30, almost model-independent; except that large fluctuations of T would decrease Y (Silk and Shapiro, 1971).

The observed 3 °K background radiation is (most probably) the relic of photon decoupling which happened at about 3000 °K when hydrogen recombination suddenly decreased the opacity. Predicted by Gamow (1956), Alpher and Herman (1948); forgotten and repredicted by Dicke (1964); found independently by Penzias and Wilson (1964). With expansion, $T_r \sim R^{-1}$, which means we see these photons now with a redshift $z \approx 1000$.

After photon decoupling comes probably a time of Jeans instabilities ($z \approx 100$) leading to condensations of matter, decoupling from each other, with galaxies and cluster as relics. But there are some serious problems, and it seems we do not yet have a satisfactory theory of galaxy formation.

d. Matter and Antimatter. In our experiments there is always pair-creation and pair-annihilation; for heavy particles (conservation of baryon-number) as well as for light ones (conservation of lepton-number). And in our theories, this particle-antiparticle symmetry is one of the "cornerstones" of quantum physics. The meaning of this conservation law is direct and exact (as opposed to statistical): one particle and one antiparticle of a pair are created at the same instant and the same spot.

Thus the creation of matter, either 10^{10} years ago in a big-bang or all the time in steady-state, should give equal amounts of matter and anti-matter, well mixed. But we see no antimatter nor any sign of annihilation. For a good summary of this problem, see Steigman (1969, 1971).

A possible solution is given by Omnes (1969, 1970), supported and explained by Kundt (1971) in a good summary of the early phases. At the end of the hadron state happens a phase-transition with demixing, yielding one-kind droplets of 10^5 gram; stopped by lack of time from world expansion. Next comes a state of diffusion and annihilation along the droplet boundaries; stopped again by lack of time. Finally comes a state of coalescence where surface tension makes the droplets merge into larger and larger ones; stopped by condensation of matter. The largest droplet size then is about 10^{46} g, corresponding to clusters of galaxies.

This theory works for big-bang models only (if it works at all). It results in a universe divided into alternating cells of cluster size, of matter only or of antimatter only. Which, at present, is neither contradicted nor supported by any observation. It could be decided in the future if a gamma-ray background would be observed, of the right intensity and spectrum, as predicted from the annihilation along the droplet boundaries.

13.3. World Models

13.3.1. Newtonian Cosmology

Between 1874 and 1896, Neumann and von Seliger applied Newton's law of gravitation to an infinite, Euclidean, uniform universe. They found no static universe which was considered an obvious demand at that time. They solved the problem by inventing a repulsive force increasing with distance, very similar to the cosmological constant Λ of Einstein. But all this did not find much favor and became forgotten.

After general relativity was introduced by Einstein and the expansion of the universe found by Hubble, Milne and McCrea showed in 1934 the striking similarity between Newtonian and relativistic cosmology. See Heckmann (1942, 1968), Bondi (1950, 1960), and McVittie (1956, 1965).

Newtonian world dynamics can be properly derived, see the last quotations. For a sloppy derivation, see Fig. 13.2a. Select an arbitrary origin, and an arbitrary particle at some distance R, and consider the particle as being attracted to the origin by the gravitation from the sphere of radius R about the origin. Call $\mathcal{M} = \frac{4\pi}{3} R_o^3 \rho_o = \frac{4\pi}{3} R^3 \rho$ = constant. The differential equation of the dynamics then is

$$\ddot{R} = -\frac{G\mathcal{M}}{R^2} \tag{13.58}$$

or, integrated once, and representing the conservation of energy:

$$\dot{R}^2 = \frac{2G\mathcal{M}}{R} + 2E \tag{13.59}$$

where E = constant of integration = total energy per mass (kinetic plus potential). Equation (13.59) can be integrated analytically yielding t(R), while R(t) cannot be written analytically except for E = 0 where

$$R(t) = (\frac{9}{2} GM)^{1/3} t^{2/3} = R_o (6\pi G \rho_o)^{1/3} t^{2/3}, \quad \text{for } E = 0. \qquad (13.60)$$

There are three types of expansion, see Figure 2b. They merge together at the beginning:

$$R(t) \sim t^{2/3} \quad \text{for} \quad t \to 0, \quad \text{for any E}. \qquad (13.61)$$

The three items: dynamics $R(t)$, world age $t_o(H_o, q_o)$, and traveling time of light $\tau(z)$, of Newtonian cosmology, are all three identical with those of general relativity for $p = \Lambda = 0$. Most observables, however, depend on space curvature and are identical only for the parabolic case $E = 0$ ($q_o = 1/2$).

The elliptical case, $E < 0$ of Newtonian as well as of relativistic cosmology, is frequently called an oscillating universe, although Hawking and Ellis (1968) have proven that no "bouncing" is possible. This is one of the completely unsolved problems, regarding universes as well as any massive black hole, for a comoving observer:

What comes after a gravitational collapse? (13.62)

13.3.2. <u>General Relativity</u> (GR)

a. <u>Early History</u>. Special relativity was founded by Einstein in 1905, but did not contain accelerations or gravitation. GR followed in 1915; it is basically a theory of gravitation, while cosmology is just one of its fields of application. First, Einstein was only interested in solutions of a static universe.

In 1922 Friedmann suggested pressure-free dynamic big-bang solutions, justified in 1929 when Hubble observed the expansion of the universe. Lemaitre further studied the big-bang models in 1931, also with pressure; and the dynamical models evolving out of an almost static case or staying close to one for a long time.

Between 1948 and 1953, Gamow, Alpher, Herman, Hayashi and others predicted from big-bang models a present background radiation of 4 - 6°K, and a primordial helium abundance of Y = .29. For a good and critical early summary on optical observables and our instrumental limits of observation, see Sandage (1961 a,b).

b. Basic Concepts.

1) Mass = Energy ($E = mc^2$). A system of restmass m_o has the total (inertial = gravitational) mass

$$m = E/c^2 = m_o + (E_{kin} + E_{pot} + E_{rad} + \dots)/c^2. \qquad (13.63)$$

Photons and neutrinos have energy and thus have mass, but do not have any restmass.

2) Principle of Covariance. Laws of nature are independent of our choice of coordinates, including curved ones. Leading to tensor calculus.

Additional demand: space-time metric shall be Riemannian, equation (13.3).

3) Principle of Equivalence. No difference, locally, between a free fall in a gravitational potential and an unaccelerated flight. Potentials can be reduced to zero by transformations to proper coordinates; leading to curved space-time. (Sometimes called "geometrization of physics.")

Then: a) Trajectories of particles, s = min. (geodesics, shortest way);

b) Trajectories of photons, $s = 0$ (null-geodesics).

4) Field Equations. They express the equivalence, in covariant form, by equating the (physical) energy-momentum tensor with some (geometric) tensor built from the metric $g_{\mu\nu}$ and its first and second derivatives. Symmetry leaves ten independent equations. For uniform models (homogeneous and isotropic), this reduces to only two differential equations for R(t):

$$8\pi G\,\rho = -\Lambda + 3\,\frac{kc^2}{R^2} + 3\left(\frac{\dot{R}}{R}\right)^2, \tag{13.64}$$

$$\frac{8\pi G}{c^2}\,p = +\Lambda - \frac{kc^2}{R^2} - \left(\frac{\dot{R}}{R}\right)^2 - 2\,\frac{\ddot{R}}{R}. \tag{13.65}$$

This yields the following combination, identical with (13.58) for $p = \Lambda = 0$,

$$\ddot{R} = -\frac{4\pi G}{3}\,(\rho + 3p/c^2)\,R + \frac{1}{3}\,\Lambda R. \tag{13.66}$$

Most models start with a singularity at $t = 0$, with $\rho = \infty$:

$$\text{Big-bang,}\quad R \sim \begin{cases} t^{1/2} & \text{radiation only, } \varepsilon = 1/3, \\ t^{2/3} & \text{matter only, } \varepsilon = 0. \end{cases} \tag{13.67}$$

5) Cosmological Constant Λ. See McCrea (1971) for a good discussion. In the literature one finds three main versions:

a) The geometric tensor of the field equations must have zero divergence for yielding conservation laws. This is a differential equation of first order, having Λ as its constant of integration, whose value must be found from observation.

b) Einstein introduced Λ for enabling a static universe. After Hubble's discovery, Λ is not needed ($\Lambda = 0$).

c) Einstein introduced Λ for fulfilling Mach's principle in a finite static world, but abandoned it ($\Lambda = 0$) when De Sitter showed that non-Machian empty universes still are possible.

Personally, I think that Λ must be found from observation. Since the only theoretical reason for $\Lambda = 0$ is simplicity, one could as well demand $E = 0$ ($q_o = 1/2$) for simplicity and forget about observation altogether. (Furthermore, the simplest universe is an empty one!)

c. Tests of General Relativity. (All agree, but within large errors.)

1) Gravitational Redshift. From a stellar surface to infinity, $z = GM/rc^2$; or cz = 0.635 km/sec for the Sun and about 50 km/sec for white dwarfs. Observations agree with theory, within their mean errors of ± 5% for the Sun (Brault 1963), and ± 15% for white dwarfs (Greenstein + Trimble, 1967).

Two clocks at different height h in the same building keep different time, with $z = gh/c^2 = 1.09 \times 10^{-16}$ h/meter. This was measured with the Mösbauer effect by Pound + Snider (1965) who found agreement within a mean error of ± 1.0%.

2) Perihel Advance. For Mercury actually observed 5596"/century; subtract 5025 for precession, and 528 for perturbations from other planets; there remains a residual of 43"/century, first found by Leverrier in 1859. From GR follows a value of 43.03 and the best observations give 43.1 ± 0.5. Agreement is also obtained for Venus, Earth and Icarus (Shapiro et al. 1968, a), but with larger errors. According to Dicke, 20% of Mercury's advance are due to an oblateness of the Sun; a future decision is possible with artificial solar satellites of different eccentricities.

3) Light Deflection at Sun's Rim. General relativity demands 1.75, Brans-Dicke only 1.63 arcsec. Optical measurements, from eclipses during

the last 50 years, are summarized by von Klüber (1960) and seem 30% too high but very uncertain. Radio interferometers give

Seielstad et al. (1970)	1.77 ± 0.20	arcsec.	
Muhleman et al. (1970)	1.82	.26	
Hill (1971)	1.87	.33	(13.68)
Sramek (1971)	1.57	.08	

4) Light Delay at Sun's Rim. Suggested by Shapiro in 1964: for radar reflected by Mercury or Venus beyond Sun, GR demands a delay of 0.2 milliseconds. This can easily be measured, but the orbits are not well enough known, and one must solve for a total of 24 parameters. Measurements (Shapiro et al. 1968, b) agreed within errors of ± 20%.

5) PPN-approximation (parametrized post-Newtonian; Thorne + Will 1971, Will 1971). A minimum of theoretical assumptions gives 9 open parameters to be found by observation: solar satellites with gyro, enclosed in self-correcting sphere for shielding.

13.3.3. GR, Pressure-Free Uniform Models

a. Formulas, Calculations. In addition to H_o and q_o from (13.18) and (13.19), we define three dimensionless parameters:

density parameter $\quad \sigma_o = 4\pi G\rho_o/3H_o^{\,2}$ (13.69)

cosmol. constant, normalized $\quad \lambda_o = \Lambda/3H_o^{\,2}$ (13.70)

curvature parameter $\quad \kappa_o = k(c/H_oR_o)^2 = k\,(c/\dot{R}_o)^2.$ (13.71)

The two differential equations of GR, (13.64) and (13.65), then yield for the present and $p = 0$:

$$\lambda_o = \sigma_o - q_o \qquad (13.72)$$

$$\kappa_o = 3\sigma_o + q_o - 1. \qquad (13.73)$$

This would be the way to obtain Λ, R_o and k from observation, if the problem of the hidden mass could be settled.

The pressure-free uniform models are, basically, a two-parameter family: once σ_o and q_o are chosen, λ_o and κ_o follow from (13.72) and (13.73); while H_o does not describe a model but tells only its present age.

The differential equation for R(t) is equation (13.64) with $\rho = \rho_o(R_o/R)^3$. This is easily solved for $\rho_o = 0$ and/or $\lambda_o = 0$, and some results are given in Table 13.3. For the general case, one uses best the tables of Refsdal, Stabell and de Lange (1967) who calculated numerically 101 different models and printed practically all needed properties as functions of z, including a large number of useful graphs.

b. Special Features. There is some confusion regarding the words "elliptical" and "hyperbolical." The same word applies to both space curvature and expansion type only for $\lambda_o = 0$ and a small surrounding. Most models have elliptical (closed) space but hyperbolical (never stopping) expansion, or hyperbolical (open) space but elliptical (through maximum to collapse) expansion. Note: Euclidean (flat) space also is "open."

With increasing distance, z goes through a minimum for some models, S has a minimum for some more, and angles θ have a minimum for most of all models. In some models, sources close to the antipole would show two images, separated by 180°.

Table 13.3. Various distances; in general, and in four simple world models.

Name	Defined	Calculated	Approximation
metric distance	$u = \int_0^{\bar{r}} \frac{d\bar{r}}{\sqrt{1 - k\bar{r}^2}}$	$u = c \int_{t_e}^{t_o} \frac{dt}{R(t)}$	$= \frac{cz}{R_o H_o} \left(1 - \frac{1+q_o}{2} z \pm \ldots\right)$
radar distance = rigid-rod distance	light travel time = ℓ_{rad}/c	$\ell_{rad} = R_o u$	$= \frac{cz}{H_o} \left(1 - \frac{1+q_o}{2} z \pm \ldots\right)$
luminosity distance (bolometric)	$S_{bol} = \frac{L_{bol}}{4\pi \ell_{bol}^2}$	$\ell_{bol} = R_o \mathcal{T}(u)\ (1+z)$	$= \frac{cz}{H_o} \left(1 + \frac{1-q_o}{2} z \pm \ldots\right)$
parallax distance	$\Delta\phi$ = (base-line)/ℓ_{par}	$\ell_{par} = \frac{R_o \mathcal{T}}{\sqrt{1 - k\mathcal{T}^2}}$	$= \frac{cz}{H_o} \left(1 - \frac{1+q_o}{2} z \pm \ldots\right)$
volume distance	$V = \frac{4\pi}{3} \ell_{vol}^3$	$\ell_{vol} = R_o \left\{3 \int_0^u \mathcal{T}^2(u)\, du\right\}^{1/3}$	$= \frac{cz}{H_o} \left(1 - \frac{1+q_o}{2} z \pm \ldots\right)$
diameter distance	$\phi = D/\ell_\phi$	$\ell_\phi = \ell_{bol}\ (1+z)^{-2}$	$= \frac{cz}{H_o} \left(1 - \frac{3+q_o}{2} z \pm \ldots\right)$

q_o	σ_o	k	λ_o	Name	$\frac{H_o R_o}{c}$	$t_o H_o$	$\frac{H_o}{c} \ell_{rad}$	$\frac{H_o}{c} \ell_{bol}$	$\frac{H_o}{c} \ell_{par}$	$\frac{H_o}{c} \ell_{vol}$
1	1	+1	0	/	1	.571	$\arcsin \frac{z}{1+z}$	z	$\frac{z}{\sqrt{1+2z}}$	$\left[\frac{3}{2}\left\{\arcsin \frac{z}{1+z} - \frac{z}{1+z}\sqrt{1-\left(\frac{z}{1+z}\right)^2}\right\}\right]^{1/3}$
$\frac{1}{2}$	$\frac{1}{2}$	0	0	Einst.-de Sit.	/	.667	$2\left\{1 - \frac{1}{\sqrt{1+z}}\right\}$	$2\left\{1+z - \sqrt{1+z}\right\}$	$2\left\{1 - \frac{1}{\sqrt{1+z}}\right\}$	$2\left\{1 - \frac{1}{\sqrt{1+z}}\right\}$
0	0	-1	0	Milne	1	1	$\ln(1+z)$	$z(1 + \frac{1}{2} z)$	$z \frac{1 + \frac{1}{2} z}{1+z + \frac{1}{2} z^2}$	$\left[\frac{3}{4}\left\{(1+z)^2 - (1+z)^{-2} - 2 \ln(1+z)\right\}\right]^{1/3}$
-1	0	0	1	de Sitter (steady-st.)	/	∞	z	$z(1+z)$	z	z

c. Classification, see Stabell + Refsdal (1966). A short summary is compressed into Figure 13.3 and Table 13.4. There, de Sitter means $R(t) = R_o \exp (tH_o \sqrt{\lambda_o})$; Einstein - de Sitter is $R(t) = R_o(\frac{3}{2} tH_o)^{2/3}$ = Newton; Milne is $R(t) = ct$; static is the original Einstein solution with $R = R_E = c^2/\Lambda$ with $\Lambda = 4\pi G \rho_o$. The big-bang singularity ($R = 0$) is either "strong" ($\dot{R} = \infty$), or "mild" ($\dot{R} = c$).

Table 13.4. Expansion Types, Singularities, and Horizons

Fig. 13.3; q_o, σ_o plane	expansion type	singularity	horizon
left of A_2	reversing	none	event
on A_2	from static to de Sitter	none	event
between A_2 and A_1	big-bang to de Sitter	$\sigma = 0$: mild $\sigma > 0$: strong	event both
on A_1	big-bang to static	strong	particle
right of A_1	big-bang to collapse	$\sigma = 0$: mild $\sigma > 0$: strong	none particle

13.3.4. GR, Early Phases of Big-Bang Models

a. Uniform Models. A good summary is given by Kundt (1971) from which Figure 13.4 is taken. The model used is $q_o = \sigma_o = 1/2$ ($k = \Lambda = 0$) with approximation (13.56), but the early phases are almost model-independent.

b. Fluctuations. Primordial fluctuations (of temperature, velocity, density ...) may decrease the helium production considerably, see Silk + Shapiro (1971); for example an rms $(\Delta T/T) = 0.5$ reduces the helium fraction by a factor 0.1 in the hot spots and by a factor 0.6 in the average. Fluctuations probably play a crucial role in the formation of galaxies (which we omit because of too many unsettled problems). Small primordial anisotropies will be smoothed-out to = 0.03% by the high viscosities from neutrino and photon decoupling (Misner 1968) but not the larger anisotropies (Stewart 1968).

The present fluctuations (galaxies, clusters) or any larger anisotropies cause a distortion of light-rays resulting in (a) apparent ellipticity (Kristian + Sachs 1966, Kristian 1967); (b) errors of angular measurements (Gunn 1967) and of magnitude (Kantowski 1969); and (c) splitting-up of a strong source into many faint ones (Refsdal 1970). All these effects are negligible for $z \leq 1$, but some may be large for $z > 2$.

c. The Mixmaster Problem. All non-empty big-bang models have a strong singularity and thus a particle horizon. From its definition in (13.16) it follows that $u_{ph} \to 0$ for $t \to 0$. This means there was no interaction in the beginning. How can the universe then look as homogeneous as we see it? If the last chance for homogenizing was at photon decoupling, $z \approx 1000$, we may calculate the particle horizon for that time, and we then would expect large inhomogeneities and anisotropies beyond this horizon, which means today for distances ≥ 100 Mpc and for angles $\geq 3°$, which both is not the case. But the problem is more basic than that, and I think it is much more severe than most people realize (or admit); I would like to formulate:

All non-empty uniform big-bang models assert a "common but unrelated" origin of all things. (13.74)

As a solution, Misner (1969,b) suggested the "mixmaster universe" with slightly anisotropic expansion. For $t \to 0$ it has an infinite series of partial singularities, leaving always one non-singular direction for mixing, with changing directions, which hopefully would give enough primordial mixing for the uniformity observed today. There are three objections: (a) It sounds awfully complicated; (b) According to some people (Brighton meeting), the mixing occurs only in thin tubes of decreasing length and thus covers only a small fraction of space. (c) This mixmaster phenomenon occurs even in empty space, thus once more emphasizing the "physical reality of empty space" which sounds very odd.

Being unsatisfied and going back to the roots, we find that it all comes from the fact that general relativity asserts velocities > c; actuall $v \rightarrow \infty$ with $\dot{R} \rightarrow \infty$ for $t \rightarrow 0$. (In GR, special relativity holds only locally but breaks down globally.) The mixmaster problem does not occur in Milne's universe. Maybe we need some change of GR which prevents any v or $\dot{R} \geq c$.

13.3.5. Other Theories

a. Steady-State theory was introduced in 1948 by Bondi, Gold and Hoyle; see Bondi (1960). The old (weak) cosmological principle of uniformity, where all fundamental observers see the same picture at any place and in any direction, is now extrapolated to the perfect (strong) cosmological principle, including "at any time."

It follows that the expansion is exponential, $R(t) = R_o e^{Ht}$, where R is an arbitrary scale factor since the curvature is zero, $k = 0$; further $q_o = -1$ and $\lambda_o = \sigma_o = +1$ (no free parameters). Expansion plus steady density needs continuous creation of matter, of $3\rho_o H_o = 1$ atom $(\text{year})^{-1}$ $(\text{km})^{-3}$. Metric, dynamics and observables are all identical with de Sitter, except for number counts (flatter than any GR). The average age of matter is only $H_o^{-1}/3$.

Originally, it was clearly said that steady-state is easy to disprove because it has no free parameters and no evolution. Then came two objections: the N/S plot was too steep, demanding evolution, and the 3 °K background asks for a dense, hot beginning. This disproves the original theory; but then Hoyle and Narlikar introduced a fluctuating steady-state with evolving irregularities, maybe local little-bangs, where the universe is steady only over very large ranges of space and time. In this way, the theory can (just barely) be saved but has lost all its beauty. For comparisons with observations see Burbidge (1971) and Brecher, Burbidge + Strittmatter (1971).

Steady-state needs continuous creation of matter (just as unexplained as a primordial creation of the universe); it would need additional creation of 3 °K background radiation, and maybe that of helium (again unexplained, while big-bang predicts both); a steep source count would need a local hole or local evolution (a nuisance but possible); it violates the conservation of baryon and lepton number (if Omnes' theory could be proven, this would be the strongest argument for big-bang).

b. Brans-Dicke (1961) add, to the tensor field of GR, a scalar field $\phi(r,t)$ and a coupling constant ω, with a non-constant $G(r,t)$ of gravitation. This is basically a theory of gravitation, will best be checked by local PPN-tests, but does not make much difference for cosmology (Greenstein 1968), Roeder 1967, Dicke 1968).

c. Hierarchy; a very attractive idea, suggested by Charlier in 1908 (for avoiding Olbers' paradox which now is irrelevant), splendidly revived by de Vaucouleurs (1970): atoms are clustered in stars, stars in stellar clusters, these are clustered in galaxies, followed by clusters of galaxies, clusters of clusters, and so on to infinity (or to $2\pi R_o$ if $k = +1$). For simplification, assume each supercluster consisting of N clusters occupying the fraction β of its volume; then the density $\rho(r)$, averaged over a sphere of radius r, is

$$\rho(r) \sim r^{-\theta}, \quad \text{with} \quad \theta = \frac{3}{1 + \dfrac{\ln N}{\ln(1/\beta)}} \tag{13.75}$$

and $\rho \to 0$ for $r \to \infty$ (if $\theta > 0$). A large-scale hierarchy of objects could well be the result and the left-over from a primordial hierarchical turbulence. And maybe it could even come close enough to an empty universe for avoiding a strong singularity and the mixmaster problem.

d. Kinematic Relativity, Milne (1935, 1948); for a more critical summary see Heckmann (1968), for a more positive one see Bondi (1960). Before any laws of physics are introduced, Milne postulates exact uniformity; from this he derives the Lorentz transformation, to be valid not only locally (as in GR) but also globally. As for gravitation, he first found G(t) but later tried to keep G = const.

The connection to GR was worked out by Robertson and Walker in 1935-37, see Rindler (1956,a). The metric and dynamic, $R(t) = ct$, are identical with the pressure-free GR model of $q_o = \Lambda = 0$, $k = -1$, which in GR is empty but now contains matter. Remarks: (a) looks very promising because of avoiding the mixmaster problem without being empty; (b) how on Earth can a universe contain matter without having any deceleration, $q_o = q(t) = 0$, for all time?

e. Dirac-Jordan claim that the three dimensionless large numbers, coulomb/gravity $\approx R_o$/electron radius $\approx$ (total particle number)$^{1/2} \approx 10^{40}$ are identical and constant. It follows that $G(t) \sim t^{-1}$, $R \sim t^{1/3}$, $q_o = \sigma_o = 2$, $\Lambda = 0$. Found not much favor. Alfven-Klein suggest a reversing model with strong annihilation at minimum R (Alfven 1965, 1971).

13.4. Optical Observations

13.4.1. Hubble Parameter, Density, Age

a. The Hubble Parameter H_o gives the linear increase of velocity (redshift) with distance, $v = cz = H_o \ell$. Since $H = \dot{R}/R = H(t)$ it is no constant, and H_o is just its present value. Distance determinations are still very uncertain, see Sandage (1970). Hubble's original value in 1936 was $H_o =$ 560 (km/sec)/Mpc, Baade reduced it in 1950 to $H_o = 290$, the present range of uncertainty is 50 ... 130, and for simplicity one mostly uses

$$H_o = 100 \text{ (km/sec)/Mpc}; \qquad H_o^{-1} = 10^{10} \text{ years.} \qquad (13.76)$$

b. The Density ρ_o is extremely uncertain because of the "hidden mass" problem of groups and clusters of galaxies. First, one obtains the visible mass M_g of all galaxies in a cluster from their number and type (single masses calibrated with nearby galaxies from rotation curves); second, one obtains the virial mass M_v needed to keep the cluster gravitationally stable against the measured scatter of velocities; then one should have $\mu = M_v/M_g = 1$, but one actually finds μ up to 2000 with a median of $\mu = 30$ (Rood, Rothman, Turnrose 1970). Thus only 1/30 of the matter is visible, the rest is hidden and might be left-over from galaxy formation (Oort 1970). It is a severe problem that we do not observe this hidden mass or its radiation, although many estimates say that we should (Turnrose, Rood 1970). Ambartsumian suggested that most of the clusters and groups with $\mu >> 1$ actually are unstable and flying apart, but this would give a very young age for most of the objects.

The visible matter of the universe was estimated with 3×10^{-31} g/cm^3 by Oort (1958), and 6×10^{-31} by van den Bergh (1961). If the hidden matter were stars, Peebles and Partridge (1967) find an upper limit of 4×10^{-30}g/cm^3 from measuring the background sky brightness and subtracting zodiacal light and faint stars. And for (13.69) we have

$$\sigma_o = \rho_o/\rho_c \quad \text{with a density unit of} \quad \rho_c = \frac{3H_o^2}{4\pi G} = 4 \times 10^{-29} g/cm^3. \qquad (13.77)$$

In summary:

visible matter	$\sigma_o = .01$	
sky background, stars	$\leq .10$	(13.78)
hidden mass with $\mu = 30$	.30	
Einstein - de Sitter, Newton	.50	

I think it is worthwhile to be quite amazed by the close agreement between visible plus hidden mass and the simplest of all non-empty world models ($k = \Lambda = 0$, zero energy). Even the visible matter alone is not off by a large factor. I do not know any a-priori reason why ρ_o should be comparable to H_o^2/G.

c. The Age t_o of the universe seemed for a long time to be less (factor 2 - 3) than that of the oldest objects. Present values give good agreement; but see de Vaucouleurs (1970) for some nicely formulated doubt about the finality of our present values.

Almost all big-bang models give ages somewhat less than H_o^{-1} (except very close to the Lemaitre limit A_2 in Fig. 13.3); for $\Lambda = 0$, we have $t_oH_o = 0.571$ for $q_o = 1$, and $t_oH_o = 2/3$ for $q_o = 1/2$. The last one then gives $t_o = (7\pm2) \times 10^9$ years, with $H_o = (100 \pm 25)$(km/sec)/Mpc. As to the objects, the oldest globular clusters give $t_o = (9 \pm 3) \times 10^9$ years according to Iben and Faulkner (1968); and the age of radioactive elements like uranium can be found from the estimated original, and the observed present, abundance ratios with $t_o = (7.0 \pm .7) \times 10^9$ years according to Dicke (1969).

globular clusters	$t_o = (9 \pm 3) \times 10^9$ years	
radioactive elements	7 ± 1	(13.79)
Einstein - de Sitter, Newton	7 ± 2	

13.4.2. Redshift - Magnitude Relation

Since the redshift increases with distance (Hubble), the flux of a source is $S \sim z^{-2}$ for small z and depends on world models for large z; see equations (13.24) and (13.28), and Sandage (1961,a). As a standard candle one mostly uses the brightest galaxy in rich clusters; quasars are visible at much larger redshifts, but their luminosities scatter too much (Solheim 1966).

There are several corrections to the luminosity. (a) The K-correction discussed before equation (13.28), see Solheim (1966) and Oke and Sandage (1968). (b) A richness correction, if the brightest galaxy of a rich cluster is brighter than that of a poor one, to be neglected if $N \geq 30$ (Sandage 1961,a). (c) Evolution of luminosity, plus traveling time of light. See Sandage (1961,b, 1968), Solheim (1966), Tinsley (1968), Peach (1970). Still very uncertain, but improvable. (d) Thomson scatter from intergalactic electrons may increase q_o by 15% (Peach 1970). (e) Distortion effect from local inhomogeneities (Kantowski 1969) may increase q_o from 1.5 to 2.7 (Peach 1970). Figures 13.5 and 13.6 show some recent results, and illustrate their uncertainties.

13.4.3. Number Counts

a. n(z,m) of quasars, or luminosity-volume test, will be discussed in Section 13.5.3.

b. n(z) of quasars, the odd bumps which disappeared, has been treated in Section 13.1.4.

c. N(m) of galaxies: no use according to Sandage (1961,a); maybe outside atmosphere.

d. N(m) of optical QSO, see Braccesi + Formiggini (1969); 300 objects were selected optically for UV and IR excess, and a number of 195 objects are complete to $m_b = 19.4$. These give a N(S) slope of $\beta = -1.74$. A fraction of 20% are estimated to be white dwarfs, which gives a correction resulting in

$$\text{N(S) slope:} \quad \beta = -1.80 \pm 0.15. \tag{13.80}$$

Spectra are known for 27 of these objects, with redshifts between 0.5 and 2.1. For the model $q_o = \sigma_o = 1$, the slope should be $\beta = -1.1$ without evolution, but $\beta = -2.0$ with the evolution $(1+z)^5$ found by Schmidt (1968). The authors conclude that evolution is definitely needed, and that optical and radio evolution are about the same. The latter agrees with Golden (1971), but not with Arakelian (1970).

13.4.4. Angular Diameters

In all non-empty expanding models, the angular diameter $\theta(z)$ drops to a minimum and then increases again for increasing z. This makes the diameter a very promising observable; values for the minimum are shown in Figure 13.7. They require $z > 1$.

Single galaxies are just marginal, reaching only to $z \leq 0.5$ where all reasonable models are still too similar and, for 10 kpc, give $\theta = 2 - 3$ arcsec which is too small for accurate measurements; maybe there is a chance from outside the atmosphere. Peach and Beard (1969) investigate 646 Abell clusters with diameters from Zwicky but, unfortunately, find some very strong systematic effect; if it could be removed, an accuracy of $q_o \pm 0.2$ would be possible.

From diameter and mass of a cluster, one can calculate its relaxation time, t_r. For those clusters where $3t_r \leq 10^{10}$ years, we do not expect (much) evolution of the linear diameter, and angular diameters thus could yield an (almost) evolution-free model test.

13.5. Radio Observations

In this whole section, we make the (helpful but unproven) assumption that quasars are at cosmological distance; see Section 1.4.

13.5.1. Radio Sources for Cosmology

a. Types and Numbers. In the 3CR there are about 40% galaxies, 30% quasars, and 30% empty fields (unidentified but tried). Surveys of shorter wavelengths have more quasars per galaxy; deeper surveys have much more empty fields (up to 70%), but also more quasars/galaxy. Table 13.5 gives some data about luminosity and frequency of occurrence. From the latter, one derives lifetimes between 10^2 years for the brightest and 10^9 years for the faint radio galaxies, assuming that each large elliptical (plus some other) galaxy goes once through an active phase; quasars then would give lifetimes between 0.1 and 10^3 years. These are only lower limits, since lifetimes are longer if the active phase occurs in some very special types of galaxy only. From estimated energies, divided by the luminosities, one derives lifetimes of 10^4 to 10^8 years. In the following we always assume lifetimes << 10^{10} years, which means class evolution only.

Table 13.5. Luminosities L and Space Density ρ of Radio Sources

	L_{rad}(178 MHz) W/Hz	L_{rad}/L_{opt}	$\bar{\alpha}$(rad/opt)	ρ $(Mpc)^{-3}$
opt. { all galaxies				4×10^{-2}
opt. { bright ellipticals				2×10^{-3}
normal galaxies	$10^{17} - 10^{23}$	0.01 - 10	0.0	2×10^{-2}
radio galaxies	$10^{23} - 10^{28}$	100 - 10,000	-0.4	2×10^{-4}
optically select. quasars	$\leq 10^{22}$	$\leq$ 0.001	$\geq$ +0.5	3×10^{-7}
radio selected quasars	$10^{27} - 10^{29}$	100 - 10,000	-0.4	1×10^{-9}

For the radio sources, the ratio L_{rad}/L_{opt} goes up to 10^4 which certainly is an advantage, but it shows no correlation with the radio index α nor any other observable; there are some extremely luminous sources, but L_{rad} varies over 12 powers of ten with (almost) no luminosity indicator or standard candle; VLB experiments give extremely high resolution, but the linear sizes go from 10^{-3} to 10^5 pc, over 8 powers of ten, with (almost) no diameter indicator or standard rod. Thus, radio astronomy reaches extremely far out into space but yields (almost) no information.

b. The Radio Luminosity Function, $\phi(L)$, is derived from the $n(S,z)$ counts by the luminosity-volume method or a similar one. It is needed for evaluating the $N(s)$ counts regarding world models and evolution. Fig. 13.8 is a compilation of available data, which shows a large uncertainty, especially at both ends. Data at the bright end can only be obtained using assumptions about world models and evolution, and the latter may give factors up to 300 (and just as much uncertainty).

Another problem, not seen clearly by some authors, is the existence and relevance of several critical slopes of $\phi(L)$. For simplicity, we discuss the Euclidean case. From (13.41), and with $\phi(L)$, we find for the number of sources with luminosity L, and observed at flux S,

$$n(S,L)\, dS\, dL \quad = \quad \text{const}\, \frac{dS}{S^{5/2}}\, L^{3/2}\phi(L)\, dL. \qquad (13.81)$$

Since S and L are separated, we see at any S the same distribution of sources, $L^{3/2}\phi(L)$. Furthermore, the sources most probably seen, at any S, are those where $L^{3/2}\phi(L) = \max$; which means those sources where the slope

$$\gamma \quad = \quad d \log \phi \;/\; d \log L \quad = \quad -\,3/2. \qquad (13.82)$$

In a well-behaved luminosity function, these would be the ones to be called standard candles. But Fig. 13.8 shows that almost the whole range of radio galaxies and quasars has a slope of -3/2; thus, the sources seen at any S may have any L (which means any distance or any z). Actually, the "half-probability" width of the distribution $L^{3/2}\phi(L)$ in Fig. 13.8 is five powers of ten in L, from 10^{24} to 10^{29} W/Hz. This exactly explains why the z(S) relation just looks like a scatter diagram; it will be a decent Hubble diagram only if the range of observed S is much larger than the width of $L^{3/2}\phi(L)$, or if $S_{max}/S_{min} >> 10^5$.

But for obtaining n(S)dS, the slope must be steeper than -5/2 in order to make (13.81) integrable. With (13.39), the slope must be < -6/2 for obtaining a Hubble relation $\overline{z}(S)$, and for giving it any accuracy we even need

$$\gamma < -7/2, \quad \text{for finite rms}(z-\overline{z}). \tag{13.83}$$

Since the bright end does not look steep enough, it seems that the observed redshifts have been kept finite only by the grace of expansion, model and evolution effects, entering approximation (13.81) as terms of second and higher order.

I would like to emphasize that a luminosity function as bad as Fig. 13.8 and extrapolated in several ways within the large range of our uncertainty, is what should be used for evaluating the N(S) counts when checking models and evolution. As to my knowledge this has not been done. I think it would invalidate all conclusions.

There is one more problem. Like any decent distribution function, the luminosity function should be used normalized, with $\int \phi(L)dL = 1$. But Fig. 13.8 shows that this is clearly impossible at the faint end. Other normalizations may be used, or none, but then the distinction between density evolution and luminosity evolution becomes problematic (and, indeed, is a mess: many authors criticizing each other for not having done it properly).

c. Intrinsic Correlations. Our lack of luminosity and distance indicators results from the absence of strong, narrow correlations of L and D with distance-free observables like spectrum index α or surface brightness B. Also, for the theories of sources we would appreciate some strong correlations between intrinsic source properties. Only two correlations were found and they have a large scatter. First, Heeschen (1966) plotted L over B, Fig. 13.8, which shows a clear correlation but a scatter of $\pm$ 0.6 in log L for 90%, plus 10% outsiders far away. Confirmed by Braccesi and Erculiani (1967), as a correlation of L and D, and by Longair and Macdonald (1969) with a total of 150 sources at 178 MHz, giving a larger scatter of $\pm$ 1.0 in log L. The smooth continuity in Heeschen's plot, from 12 normal galaxies over 28 radio galaxies to 14 quasars, was used as argument for the cosmological distance of quasars (Section 13.1.4.).

Second, a weak correlation between L and α was claimed by Heeschen (1960), Braccesi and Erculiani (1967), Bash (1968), and Kellermann and Pauliny-Toth (1969). It shows better for radio galaxies and looks more doubtful for quasars. I found that the latter can be improved if only very straight-lined spectra are selected.

13.5.2. Correlations Involving Distance

a. The z(S) Relation for Quasars is shown in Fig. 13.11. It just looks like a scatter diagram, without a Hubble law; Hoyle and Burbidge (1966) thus concluded that quasars are not at cosmological distance, but a large scatter of $L^{3/2}\phi(L)$ explains it just as well. Furthermore, in critical cases one should use the median and not the average; at the bright end of the luminosity function we need $\gamma < -3$ for obtaining $\bar{z}$, and $\gamma < -7/2$ for its accuracy, the rms$(z-\bar{z})$; whereas the median and its accuracy, the quartiles, require both only $\gamma < -5/2$. Indeed, the median z_m in Fig. 13.11 shows a fairly good correlation with S. Checking world models, however, would need a luminosity indicator for reducing the scatter.

b. Angular Size. Legg (1970) collects data for 32 radio galaxies and 25 quasars having double structure and known redshifts. He finds a good correlation with a well-defined upper envelope of

$$\theta(z) \sim 1/z, \quad \text{and} \quad D_{max} = 400 \text{ kpc}. \tag{13.84}$$

Miley (1971) uses the largest angular size, LAS, (diameter of singles, separation of doubles) of 39 radio galaxies and 47 quasars, see Fig. 13.10, with the same results. Most puzzling in Fig. 13.10 is the fact that

$$\theta(z) \sim 1/z \text{ continues beyond any possible world model.} \tag{13.85}$$

From Fig. 13.6 one can show that the steady-state or de Sitter model ($q_o = -1$, $\sigma_o = 0$) gives of all possible models the smallest θ for large z; whereas $\theta \sim 1/z$, called "Euclidean" in Fig. 13.10, is actually "static Euclidean" which is not possible. Both Legg and Miley conclude that they need a "diameter evolution" for explaining the small θ; Legg suggests $D_{max} \sim (1+z)^{-3/2} = (R/R_o)^{3/2}$, in agreement with a theoretical estimate of Christiansen (1969).

About the opposite type of deviation is found by Longair and Pooley (1969); comparing the diameter distributions $n(\theta,S)$ of the 3CR and 5C surveys, they find too many large diameters for small S.

Let me add two remarks. First, there is a selection effect, since at large z we see only large L while L and D are correlated. A correction for this effect must be worked out and applied before checking models with diameters. Second, even if Fig. 13.10 could be explained in this way, it still remains a puzzle why any correction should just yield the (impossible) static Euclidean continuation of $\theta \sim 1/z$.

c. <u>Other Correlations</u> have been tried in considerable numbers by Bash (1968), but without much success. Hogg (1969) finds a good correlation between index α and size (steep ones being larger).

13.5.3. <u>The n(z,S) Counts and Luminosity-Volume Test</u>

One would like to get a large sample n(z,S), complete down to some faint S_o, and to derive from it the luminosity function, the source evolution, and finally the world model. Unfortunately, the data depend (as usually) much more on the first two than on the last one. And for quasars, a sample is limited in two ways: radio (detection) and optically (redshifts); this would be better for optically selected QSOs, having one limit only.

The n(z,S) plot, Fig. 13.11,a, yields luminosity function, N(z) and N(S) counts by summations along different lines; it yields the z(S) relation by taking the median. Strictly speaking, there are two types of luminosity function: first, the directly obtained one, which prevails along our past light-cone and thus contains different (and unknown) amounts of evolution for different L; second, if possible, one would like to obtain the full time-dependent luminosity function, $\phi(L,t)$. A glance at Fig. 13.11 shows immediately that this cannot be properly done because the range of observed S is much too small for splitting up the data into several groups of z. We badly need larger samples, complete to fainter limits.

The luminosity-volume test was suggested by Schmidt (1968) and Rowan-Robinson (1968), see also Arakelian (1970). For critical discussion, summaries and new data, see Longair and Scheuer (1970), Schmidt (1970), Rees and Schmidt (1971), Davidson, Davies and Cos (1971), and Rowan-Robinson (1971). This is the basic procedure:

(a) Estimate completeness limits of sample (radio and optical), omit all sources beyond.

(b) Apply the model-independent part of a redshift correction (K-correction) to S:

$$S_{cor} = \frac{S_{obs}}{(1+z)^{1+\alpha}} . \tag{13.86}$$

(c) Adopt a world model or two. Calculate distance ℓ and luminosity L for each source, and the distance ℓ_m where a source of this L would just be, if at the nearer one of both completeness limits.

(d) Calculate volume V of sphere with radius ℓ, and V_m with ℓ_m. Take ratio $f = V/V_m$. Get distribution n(f) and average $\overline{f}$.

(e) $1/V_m$ is the contribution of this source to the luminosity function $\phi(L)$ which then is obtained as the sum of all $1/V_m$ in each group of L.

(f) For a uniform world without evolution, we should have n(f) = const, and $\overline{f} = 0.5$. A result like Fig. 13.11,g then means that there were more sources in the past.

(g) The slope β of the N(S) plot is related to $\overline{f}$ as shown by Longair and Scheuer. In the static Euclidean approximation,

$$\overline{f} = -\beta/(-\beta + 3/2). \tag{13.87}$$

All authors agree that evolution is definitely needed. They mostly say that the co-moving density of sources (or their luminosity) was higher in the past by a factor $(1+z)^n$, with $n = 3 \ldots 14$. This leads to the problem of very short evolutionary time-scales, of only 10^9 years (Rowan-Robinson 1971). And Schmidt's data (1968) show large f already at small z, see Fig. 13.11,a, which looks to me more like a local hole than evolution.

13.5.4. The N(S) Counts

a. Observations. Just to count sources down to various flux limits sounds very easy but actually isn't. The first surveys were all badly resolution-limited; one might have corrected the counts for this effect but that was not done. If errors from noise plus resolution are larger than the statistical error, one needs an additional "spillover" correction, since the more numerous faint sources will spill over, via errors, to the fewer bright sources, more frequently than vice versa. These corrections can best be done by a Monte Carlo method, adding some known artificial sources to the record. The radio-equivalent of a K-correction, equation (13.86), cannot be applied to the data since the redshifts are not known, but it is taken care of on the model side when models are compared with the data. The faulty error bars of N(S) plots, and the preference for differential counts, n(S)dS, was discussed in Section 13.2.4.f.

Table 13.6 shows the negative N(S) slope, $-\beta$, for the bright end. We see that Janucey's proper maximum-likelihood method yields smaller values and larger error limits.

Table 13.6. Slope of N(S) counts, $-\beta$. (ci = very certain identifications only)

Type of Source	3CR, 178 MHz			6 cm Survey
	Veron (1966)	Jauncey (1967)	Jauncey (1967) ci	Pauliny-Toth and Kellermann (1972)
total	1.85 ± .05	1.78 ± .12		1.76 ± .11 (n=271)
rad. galaxies	1.55 .05	1.58 .14	1.26 ± .13	1.54 .18 (103)
quasars	2.20 .10	2.00 .29	1.56 .26	1.54 .17 (96)
empty fields	($\approx$ quasars)	2.50 .45		2.07 .33 (67)

The full range of the counts is shown in Fig. 13.12 for n(S), and in Fig. 13.13 for N(S); correcting for various wavelengths gives good agreement. We see the famous steep slope at the bright end, and the well-pronounced flattening at the faint end, where we finally must have $\beta > -1$ for avoiding Olbers' paradox ($\beta = -0.8$ reached already), and $\beta = 0$ for a finite total number.

b. Results and Interpretations. I want to emphasize four points. First, a slope of $\beta = -1.50$ does not mean "no evolution" as sometimes stated; the numbers given in (13.40) show that the bright part of N(S), with an average redshift of 0.20, say, yields $\beta = -1.23$ without evolution, see also Fig. 13.14. Second, the flatness of the bright part is model-independent, and depends only on spectrum index, luminosity function, and evolution (Section 13.2.4.f.); but evolution at the bright part would mean evolution in the recent past. Third, in case of large-scale clustering (de Vaucouleurs 1970) there is nothing wrong with a local hole for explaining the flatness of the bright part. Actually, we should not sit at the average density $\overline{\rho}$, but at $\overline{\rho} \pm \mathrm{rms}(\rho-\overline{\rho})$. Fourth, a slope of $\beta = -1.50$ may be explained by a local hole or evolution, but it would still impose the same type of puzzle as (13.85) does; Kellermann

(1972) lists even four of such puzzles or paradoxa.

Maybe the puzzle can be solved by considering a luminosity function with a near-critical slope, where S shows only very little correlation with z, which may give $\beta = -1.50$ for the bright part. But it would not help for a steeper slope.

In Table 13.6, only the first line (total) is actually relevant. That the empty fields show a steeper slope seems trivial: since radio and optical fluxes are correlated, the radio-faint sources will be optically undetectable more frequently than the bright ones. For the same reason, the identified sources then must have a flatter slope. Their "true" slope could only be obtained with due regard to the optical detection limit, which then would spoil the basic idea of the N(S) counts as a simple radio-device. Once we need optical identification, we'd better go one step further and get redshifts, too, and then work with all tests indicated in Fig. 13.11. Either the N(S) count contains unidentified sources, then the slopes of "galaxies" and "quasars" (and their difference) are not meaningful; or all sources of a sample are identified, then the single slopes can be used but they do not contain all available information and are confined to a small sample only.

c. Evolution. Detailed calculations and checks have been done by many authors. All of them agree that evolution is needed for explaining the steep slope of the bright part. Longair (1966) finds that only quasars evolve, but Rowan-Robinson (1967) includes weak sources, too. Schmidt (1968) supports density evolution where the number per co-moving volume is $\rho_c \sim (1+z)^n$, with a constant luminosity distribution; but a luminosity evolution is claimed by Rowan-Robinson (1970) instead of, and by Davidson, Davies and Cox (1971) in addition; while Longair and Scheuer (1970) say it does not matter whether in the past the density of sources was higher or their luminosity.

Fig. 13.8 shows that the luminosity function cannot be normalized unambiguously, that most of it follows a straight line of critical slope, and that our uncertainty is large. This means that a clear distinction between evolution types is hopeless. The easiest then is density evolution with about

$$\rho_c \sim (1+z)^5. \tag{13.88}$$

Since this diverges for large z, while actually the counts get flatter at the faint end ($\beta = -0.8$), one needs a strong reduction for large z, and the easiest is a cut-off at some z* beyond which there are no sources, of about

$$z \leq z^* = 5. \tag{13.89}$$

This approach has only two free parameters, n and z*. If that does not fit the data well enough, one needs more parameters; 3 are used by Longair, and 5 by Davidson et al. Both achieve rather good fits, see Fig. 13.14.

A different approach (to be preferred if we had more and better data) is the one of Ringenberg and McVittie (1969, 1970). Instead of assuming a steep increase as (13.88) and a sharp cut-off as (13.89), with maybe some more free parameters, they introduce a free evolution function to be determined numerically from the data. The result is a steep increase again, much steeper for bright than for faint L, but a more gradual decrease for large z.

What I miss is the use of several near-to-critical luminosity functions, and a good discussion of (13.88) versus local hole. A cut-off, however, is needed anyway: if it takes $t_g = 6 \times 10^8$ years (3 rotations, say) to make a galaxy and let it get an explosive core, or to make strong sources and quasars in any other way, then there are no sources beyond

$$z^* = (\frac{2}{3} H_o^{-1}/t_g)^{2/3} - 1 = 4.0 , \qquad (13.90)$$

for Einstein - de Sitter model and $H_o = 100$.

13.5.5. The 3 °K Background Radiation

A thermal background radiation was predicted for all models with a hot and dense beginning, see Section 13.2.5.c. It was found independently by Penzias and Wilson (1965); they observed a total of 6.7 °K at $\lambda = 7.3$ cm, of which 2.3 °K was attributed to the atmosphere, about one degree to spillover, and the remaining $T = 3.5 \pm 1.0$ °K to a cosmic origin. Measurements at other wavelengths gave about the same temperature.

This can be regarded as an argument for big-bang and against steady-state theory. Therefore some people tried whether a background of many faint discrete sources could explain the observations, too (see Wolfe and Burbidge 1969). The observed spectrum can be explained if a new type of source is postulated with the right kind of spectrum. But the observed isotropy (lack of bumpiness) would demand a space density of these sources much larger than that of galaxies, and the idea now is mostly dropped; see Penzias, Schraml and Wilson (1969), and Hazard and Salpeter (1969).

Is it a thermal black-body spectrum? The observable range is limited by galactic and atmospheric radiation, see Fig. 13.15. The older measurements covered only the Rayleigh-Jeans part, which even seemed to continue too far up. But the most recent observations yield a good confirmation of the thermal spectrum, with a value of

$$T = (2.7 \pm 0.1) \text{ °K}. \qquad (13.91)$$

The observed isotropy is remarkable; see Partridge (1969), Wolfe (1969), Pariiskii and Pyatunina (1971). No deviation ΔI of the intensity I was found:

$$\begin{array}{l|c|c|c||c} \text{down to} & 5\text{ arcmin} & 2\text{ arcmin} & 10\text{ arcsec} & \text{polarization} \\ \hline \Delta I/I \leq & 0.03\% & 1\% & 15\% & 1\% \end{array} \qquad (13.92)$$

This radiation defines a very distant frame of rest (surface of last scattering). If we have a velocity v, we observe a small daily anisotropy of $T(\theta) = T_o(1 + (v/c) \cos \theta)$, where v = 100 km/sec yields $\Delta T = T_o v/c = 1$ mK (millidegree). This was actually observed by Conklin (1969) and Henry (1971). The resulting motion agrees so well with estimates from the redshifts of surrounding galaxies, that our local supercluster can only have a small or no peculiar velocity ($\leq$ 200 km/sec, say); see Table 13.7.

Table 13.7. Solar Motion from 3 °K Background Radiation, and from Redshifts of Galaxies in Local Supercluster.

Measured	Anisotropy of 3 °K	Redshifts of Galaxies	Unit
frame of reference	surface of last scatter	local supercluster	-
solar velocity	320 ± 80	400 ± 200	km/sec
right ascension	10.5 ± 4	14 ± 2	hours
declination	- 30 ± 25	- 20 ± 20	degrees

13.6.0 Summary

a. Oddities of GR. First, empty space has an amazing degree of physical reality, see Table 13.1; actually GR is less relative than Newtonian physics. The model $\Lambda = \sigma = 0$ has space curvature ($k = -1$) and thus a well-defined frame of rest, it also starts with a singularity (infinite curvature), and both in spite of being completely empty and force-free.

Second, at the singularities (big-bang and collapse) the velocities $v = u\dot{R}$ become infinite; $v > c$ then leads to a particle horizon and thus to the rather serious mixmaster problem (13.74). A good theory should avoid $v \geq c$, if necessary by postulate. Third, GR seems to be too similar to Newtonian cosmology. Equation (13.58) is identical with (13.66) for $p = \Lambda = 0$; in this case ρ is the matter density only, and what I miss is the gravitational contribution of the energies, $(E_{kin} + E_{pot})/c^2$ as demanded by (13.63). Fourth, our physics cannot continue before a big-bang and after a collapse (no bouncing), see (13.62).

Fifth, in equation (13.66) we have $p = 0$ for a matter universe, and $\rho = 3\ p/c^2$ for a radiation universe. Thus, for equal total energy ρ, radiation is twice as "attractive" as matter.

b. Disentangling. Unfortunately, most observations have turned out to be "model-free evolution tests" or nearly so. Truly model-free are the surface brightness b(z), see (13.30), and the flatness of the bright end of the N(S) counts, see (13.42).

Evolution-free model tests are hard to find, because of the rarity of evolution-free observables. Since the redshift is one, Sandage suggested $\dot{z}(z)$ but found it needs over a million years, Section 13.2.4.a. Another test is the parallax distance, $\ell_{par}(z)$, see (13.32), but it would need an accuracy better than 10^{-10} arcsec.

If no evolution-free model test can be found, one needs a theory of source evolution, meaning class evolution for radio sources, which does not exist at present.

c. More Unsolved Problems. First, are quasars at cosmological distance? Second, do we have clusters of antimatter? Third, the problem of the hidden mass. Fourth, we badly need indicators of luminosity and linear size, for standards. Fifth, we need some test for evolution versus local hole. Sixth, develop a hierarchical cosmology.

d. Most Urgent Needs. For various models, Table 13.8 shows:

(1) the redshift z where angular size ϕ has its minimum;

(2) redshift z where source is at equator for closed models, $u = \pi/2$; also for open models at $u = \pi/2$;

(3) u of z = 2, divided by u of particle horizon;

(4) u of z = 2, divided by u_{ap} (π = u of antipole, if closed).

Table 13.8. Several Models and Observational Ranges.

q_o	σ_o	k	λ_o	(1) $z(\phi_{min})$	(2) $z(u=\pi/2)$	(3) $u(z=2)/u_{ph}$	(4) $u(z=2)/u_{ap}$
-1	.3	+1	+1.3	1.10	3.50	.445	.397
-1	1	+1	+2	.75	1.61	.522	.546
0	0	-1	0	∞	3.80	0	.350
0	.3	0	+.3	1.40	(∞)	.397	0
+1	0	-1	-1	∞	3.2	0	.396
+1	.3	-1	-.7	1.65	12	.373	.273
+1	1	+1	0	1.00	∞	.465	.232
+.5	.5	0	0	1.30	(∞)	.423	0

The columns of Table 13.8 have the following meaning:

(1) This should be reached for model tests working with angular size or separation.

(2) To be reached for model tests working with m, S, N, n. All these quantities depend mainly on space curvature; we must see a good part of the total curvature before model differences get really large.

Compare (1) and (2): angles are the most sensitive observable.

(3) At present, observational limit is about $z = 2$ (Wills, 1971, finds 19 of 208 QSRs have $z \geq 2$).

Column (3) says: we see now already 1/2 of the distance which can possible be seen.

(4) We see now about 1/3 of the total curvature.

From (3) and (4) we find:

Most important for observational cosmology is not to extend our range to very large z, but to obtain:

higher accuracy,
complete samples to fainter limits,
theory of sources, intrinsic correlations,
theory of class evolution,
distance of quasars.

e. Our Meager Results. First, the 3 °K background seems now well established, see Fig. 13.15. It follows naturally from a hot, dense early phase, and the big-bang seems the only way to get one. This excludes the lower left-hand part of Fig. 13.3, below and with the asymptotic models A_2. (In steady-state, it would mean that not only matter is created all the time the right amount, but radiation, too.)

Second, the authors evaluating N(S) counts and luminosity-volume tests agree that evolution is definitely needed but do not agree on its details. I miss the use of several near-to-critical luminosity functions, and a test for evolution versus local hole. The simplest evolution is about as $(1+z)^5$, for density and/or luminosity of sources, with a cut-off at about $z^* = 5$. The latter is nicely explained if it takes a few galactic (rotation) time-scales for making sources after the big-bang. It seems to me that the whole curve of Fig. 13.14 (not just its bright end) speaks fairly strong against steady-state.

Third, we see no antipole-images up to $z = 2$, say. This excludes a somewhat larger left-hand part of Fig. 13.3, starting somewhat above A_2. Fourth, the density is somewhere between $\sigma_o = 0.01$ for visible galaxies, and $\sigma_o = 0.30$ for hidden mass according to virial equilibrium of clusters, both limits being rather uncertain. Fifth, the optical z(m) relation of cluster galaxies, Fig. 13.6, yields about $-2 \leq q_o \leq +1$ and $0.5 \leq \sigma_o \leq 4$, with very uncertain limits as indicated by Fig. 13.5.

In summary, the universe started most probably with a big-bang about 10^{10} years ago, and radio sources appeared about 5×10^8 years thereafter. Either these sources were then much more numerous (and/or brighter) than they are today, or we happen to sit in a local hole of a spatially fluctuating density. Both q_o and σ_o (and thus λ_o, too) are of the order of unity, but the sign of q_o and λ_o is not known. The following big questions are still unsolved:

Is 3-space closed, flat, or hyperbolical?
Is the expansion monotonic or oscillating? (13.93)
Is the cosmological constant zero or not?

But our inability to answer these questions implies a positive and significant statement, too, for which I do not know any a-priori reason:

The universe is not very different from the simplest non-empty model, the Einstein - de Sitter model with $q_o = \sigma_o = 1/2 (k = \Lambda = 0)$. (13.94)

LITERATURE (CHAPTER 13)

1) The following general literature on cosmology is recommended:

Some Textbooks

R. Tolman, "Relativity, Thermodynamics and Cosmology" 1934(1950),
Classical introduction to Relativity, detailed formulae.

O. Heckman, "Theorien der Kosmologie" 1942(1968) Springer,
Good comparison of various theories, history, formulae.

E. A. Milne, "Kinematic Relativity" 1948, Oxford Univ. Press,

G. C. McVittie, "Cosmological Theory" 1937(1952),

G. C. McVittie, "Fact and Theory in Cosmology" 1961.

H. Bondi, "Cosmology" 1950(1960),
Discussion of various basic philosophies.

G. C. McVittie, "General Relativity and Cosmology" 1956(1965) Univ. Illinois.
Frequently quoted for formulae, short derivations.

W. Rindler, "Essential Relativity", 1969, Van Nostrand Reinhold.

Summaries, Lectures

Brandeis Summer Inst. "General Relativity" 1965, Prentice-Hall.
(Contr.: Trautman, Pirani, Bondi) Differential geometry.

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(Contr.: Field, Greenstein, Greisen, Lin, Layzer, Lynden-Bell, Misner, Moffet, Sachs)

H. Y. Chiu, Lecture Notes "Relativity Theory & Astroph." Goddard, NASA.

H. Y. Chiu "Cosmology of our Universe" Science Journal.
Good short summary, history, tables, figures.

W. Kundt "Survey of Cosmology" 1971, Springer Tracts Mod. Ph. 58.
Especially early phases.

Enrico Fermi School, "General Relat. & Cosmology", 1971, Academ. Press
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FIGURE CAPTIONS

Fig. 13.1. Distance and time; Einstein - de Sitter model.

z = redshift; v = expansion speed at emission

τ = travel time of light; t = world age at emission;

ℓ = present proper distance (radar) of object;

ℓ_H = present distance of horizon; t_o = present world age.

Fig. 13.2. Newtonian Cosmology.

a) Attraction of arbitrary particle P to arbitrary origin O by mass M.

b) Three resulting types of expansion: elliptic ($E < 0$), parabolic ($E = 0$), hyperbolic ($E > 0$). Identical with general relativity for $p = \Lambda = 0$.

Fig. 13.3. The q_o, σ_o plane of all pressure-free uniform models, with their different types of expansion, $R(t)$.

Fig. 13.4. The early phases (Kundt 1971). The outer boundary is our past light-cone.

Fig. 13.5. The $z(m)$ relation for brightest cluster galaxies. The best fit (assuming $\Lambda = 0$) gives $q_o = 1.54$; the straight line, $q_o = 1$, is drawn for comparison. Copied from Peach (1970).

Fig. 13.6. The σ_o, q_o plane of world models, with results from $z(m)$ tests. A, B, C, D: best-fitting values; a, c, d: 64% confidence $\approx$ probable error.

A, a: 20 brightest cluster galaxies (Solheim 1966);

B: shift of point A for evolution correction, with P = fraction of light from unevolving stars (Solheim);

C, c: 15 quasars (Solheim);

D, d: 46 brightest cluster galaxies (Peach 1970).

Fig. 13.7. In non-empty expanding models, the angular diameter $\theta(z)$ has a minimum, θ_m, at a certain redshift z_m. A linear diameter of 10 kpc is used. Copied from Refsdal, Stabell and de Lange (1967).

Fig. 13.8. Radio luminosity function $\phi(L)$; all data. The straight lines are critical slopes.

Fig. 13.9. Luminosity and surface brightness, for radio galaxies and quasars, at 1400 MHz. Copied from Heeschen (1966).

Fig. 13.10 Angular size versus redshift. Copied from Miley (1971).

Fig. 13.11 The n(z,S) count and its derivates.

a) Radio-complete sample of 40 quasars (Schmidt 1968).
Open circles: beyond optical completeness limit.
Right and left of "$\frac{1}{2} V_m$" should be about equal numbers.

b) Volume, and cumulative numbers N(z).

c) Light travel time $\tau(z)$, used for evolution.

d) Luminosity function, number per volume.

e) The z(S) relation, using the median and not the average.

f) The cumulative N(S) plot.

g) Luminosity-volume test; without evolution, n(f) = const, and $\overline{f} = 0.5$.

Fig. 13.12 The differential counts, n(S), normalized with static Euclidean $n_o \sim S^{-5/2}$. Copied from Kellermann (1972).

Fig. 13.13 The cumulative counts, N(S). Ryle (1968).

Fig. 13.14 The cumulative counts, N(S), normalized with static Euclidean $N \sim S^{-3/2}$; a) without evolution, b) with evolution. Ryle (1968).

Fig. 13.15 The "3 °K background" radiation.

o Radio measurements, quoted by Wolfe and Burbidge (1969).

● Rotational CN absorption; Heygi, Traub and Carleton (1972)

x Several older rocket measurements.

⊗ New rocket (balloon) measurements: Blair et al. (1971), Beckman et al. (1972).