1972 Summer Student Lecture - N.R.A.O.

Experimental Tests of General Relativity

R. Sramek

1) General Relativity is Einstein's theory of gravitation. Unlike Newtonian gravitational theory, it is invariant under the Lorentz transformation, and, in a very beautiful way, accounts for the universality of the accelation of freely falling bodies. Rather than describe gravity as a force, in General Relativity theory (GRT), freely falling particles are in natural motion in a curved space; gravity becomes a property of space. It is the curvature of space rather than a force which brings bodies together.

2) The curvature of space-time is obtained from the metric of space-time $g_{\mu\nu}$, and its derivatives.

3) For flat space $g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} = \eta_{\mu\nu}$ the Minkowski metric tensor.

$$
\begin{aligned}
ds^2 &= g_{\mu\nu}\, dx^\mu dx^\nu \\
&= dx^0 dx^0 - dx^1 dx^1 - dx^2 dx^2 - dx^3 dx^3 \\
&= dt^2 - dx^2 - dy^2 - dz^2
\end{aligned}
$$

4) Curvature is expressed by $R_{\mu\nu}$, the Ricci tensor and R, the curvature scalar, which are functions of $g_{\mu\nu}$ and its derivatives.

5) The Einstein field equations relate the curvature of space-time to the distribution of matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} = \begin{pmatrix} \left(\begin{smallmatrix}\text{Energy}\\ \text{density}\end{smallmatrix}\right) & \left(\begin{smallmatrix}\text{Momentum}\\ \text{density}\end{smallmatrix}\right) \\ & \left(\begin{matrix}\text{stress}\\ \text{tensor}\end{matrix}\right) \end{pmatrix}$$

$T_{\mu\nu}$ = stress-energy tensor.

) GRT is only one of over 20 relativistic metric theories of gravity.

) A metric theory:

A) endows space with a metric

B) tests bodies move along geodesics in the metric

c) in a freely falling frame, special relativity holds

) The many metric theories differ only in the way in which matter or other scalar, vector, or tensor fields ~~grad~~ determine $g_{\mu\nu}$.

9) Example: in GRT

$$\begin{pmatrix} \text{Matter + energy dist} \\ \to T_{\mu\nu} \end{pmatrix} \Rightarrow \begin{pmatrix} \text{elements of} \\ g_{\mu\nu} \end{pmatrix}$$

in scalar-tensor theory of Brans + Dicke

$$T_{\mu\nu} \Longrightarrow (\text{scalar field}, \phi)$$

$$(T_{\mu\nu}, \phi) \Longrightarrow (g_{\mu\nu})$$

10) to determine particle motion under various theories, find weak field limits and write very general metric with parameters determining influence of grav. energy, pressure rest mass, momentum, etc. on metric. Each theory of gravity will determine a value for these parameters.

11) Express motion of particles ~~in terms~~ in terms of these parameters and experimentally find their value, thereby supporting some theories for a true theory of gravity

Example: simple parametrized post-Newtonian metric

(Eddington - Robertson)

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & & & \\ & g_{11} & & 0 \\ & 0 & g_{22} & \\ & & & g_{33} \end{pmatrix}$$

$$g_{00} = 1 - 2U + 2\beta U^2$$

$$g_{11} = g_{22} = g_{33} = -(1 + 2U\gamma)$$

parameters β, γ

β measures non-linearity of superposition of fields

γ " curvature of space

for Newtonian gravitation $\gamma = \beta = 0$

GRT $\gamma = \beta = 1$

scalar-tensor (B.D.) theory — $\gamma = \dfrac{1+\omega}{2+\omega}$ $\beta = 1$

ω = coupling constant ~ 6

$\therefore \gamma \sim 0.875$

13) Experiments.

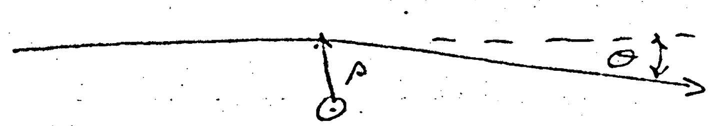
light bending $\Delta\theta = \frac{1}{2}(1+\gamma)\,\dfrac{1.75}{\rho}$ $\rho = \dfrac{R}{R_\odot}$

predictions:

$\lambda = \frac{1}{2}(1+\gamma) = 1.00$ GRT

$= 0.94$ B.D.

$(\omega \sim 6)$

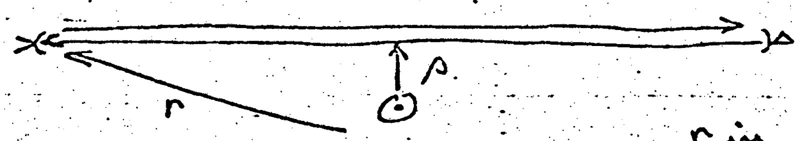


observations: radio measurements.

1969 $\lambda = 1.04^{+.15}_{-.10}$ J.P.L. 1970, 1971 } $\lambda = 0.94 \pm 0.06$ N.R.A.O.

$\lambda = 1.01 \pm .12$ OVRO.

14) <u>time delay:</u>



r in A.u.

$$\Delta T = \lambda \left[250 - 20 \ln \frac{\rho^2}{r} \right] \quad \mu sec$$

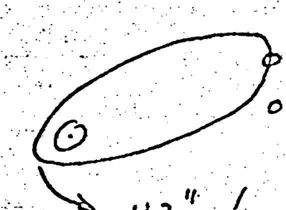
passive radar to Mercury $\lambda = 1.02 \pm 0.05$

active radar to Mariner VI VII $\lambda = 1.00 \pm 0.04$

15) <u>Perihelion advance:</u>

$$\Delta\omega = 43 \left[\tfrac{1}{3}(2\gamma + 2 - \beta) \right] + 4 \left[\frac{J_2}{3 \cdot 10^{-5}} \right]$$

where J_2 = quadriopole moment of Sun.



$\Delta\omega$ measured $= 43 \pm 0.4$ arc sec / century

assume $\beta = 1$

if $J_2 = 0$ $\quad \frac{1}{3}(2\gamma + 2 - \beta) = 1$

$\therefore \gamma = 1.0$ GRT is right !

if $J_2 = 3 \cdot 10^{-5}$ $\quad \frac{1}{3}(2\gamma + 2 - \beta) = \frac{39}{43}$

$\therefore \gamma = 0.86$

B.D. is right !

measured $J_2 \sim 3 \cdot 10^{-5}$ but interpretation is still questioned.

TABLE 5.1. Metric Theories of Gravity and Their PPN Parametric Values

Theory and its adjustable parameters[†]	γ	β	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
1. General Relativity[L]	1	1	0	0	0	0	0	0	0
2. Scalar-Tensor Theories									
a. Bergmann-Wagoner-Nordtvedt (ω, Λ)[L]	$\frac{1+\omega}{2+\omega}$	$1+\Lambda$	0	0	0	0	0	0	0
b. Dicke-Brans-Jordan (ω)[L]	$\frac{1+\omega}{2+\omega}$	1	0	0	0	0	0	0	0
3. Vector-Metric Theory (K)[L]	1	1	0	$\frac{K^2}{1+\frac{1}{2}K^2}$	0	0	0	0	0
4. Conformally Flat Theories									
a. Nordstrøm-Einstein-Fokker[L]	-1	$\frac{1}{2}$	0	0	0	0	0	0	0
b. Ni's Lagrangian Theory (q)[L]	-1	1-q	0	0	0	0	0	0	0
c. Ni's General Theory (p,q)	-1	1-q	0	0	0	0	4+p-2q	0	0
d. Nordstrøm	-1	1	0	0	0	0	0	0	0
e. Littlewood-Bergmann	-1	$\frac{1}{2}$	0	0	0	0	-1	0	0
f. Whitrow-Morduch (q)	-1	1-q	0	0	0	0	-2q	0	0

TABLE 5.I (cont'd)

Theory and its adjustable parameters†	γ	β	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
5. Stratified Theories (with Time-Orthogonal Conformally Flat Space Slices)									
a. Rosen (λ)[L]	λ	$\frac{1}{4}(3+\lambda)$	$-4(1+\lambda)$	0	0	0	0	0	0
b. Papapetrou[L]	1	1	-8	-4	0	0	0	0	0
c. Ni's Lagrangian Stratified Theory[L]	1	1	-8	0	0	0	0	0	0
d. Ni's General Stratified Theory (p, q)	1	1-q	-8	0	-4	0	p-2q-2	0	-2
e. Yilmaz	1	1	-8	0	-4	0	-2	0	-2
f. Page-Tupper (a, c)	a	1+c	-4(1+a)	0	-2(1+a)	0	1+a+2c	0	-(1+a)
g. Coleman (p)	1	1	-8	0	-4	0	p-2	0	-2
h. Einstein's 1912 Theory	0	0	-4	0	-2	0	-1	0	-1
i. Whitrow-Morduch Stratified Theory	0	-1	-4	0	-2	0	-3	0	-1
6. Whitehead's Theory	1	1	0	0	0	-6	0	-1	-1
						(plus additional metric terms)			

† The superscript [L] refers to theories which are Lagrangian-based.