# Notes for Summer Student Lecture on Precise Radio Astrometry
## 2 July 1976, by C. M. Wade

## I. What is Astrometry?

Astrometry is the precise measurement of the positions of astronomical objects on the celestial sphere. "Precise" in this context can be taken to mean an accuracy of 2" ($\sim 10^{-5}$ radian) or better. The best radio and optical astrometry achieve results accurate to a few hundredths of an arc second (say $\sim 10^{-7}$ radian).

The positions are expressed as coordinates in a spherical reference frame. Normally this is the equatorial system (right ascension and declination). All reference frames used in astrometry are defined kinematically, through the earth's rotation about its axis and its orbital motion about the barycenter of the solar system. Since the earth's rotational axis is not fixed in space, and since the parameters of the earth's orbit vary because of gravitational perturbations by the moon and the other planets, the reference frames change continually with time. Hence astrometry must determine the instantaneous orientation of the coordinate axes in space as well as the directions of celestial objects with respect to these axes. Moreover, the objects have intrinsic motions of their own, and their positions change with time regardless of how the coordinate systems are established.

For most purposes, astrometry treats celestial objects as if they were infinitely distant. It is concerned primarily with directions in space. In the case of relatively nearby objects, however, one must sometimes make allowance for parallax (the difference in direction as seen from opposite sides of the earth's orbit).

## II. Why is Astrometry Important?

Astrometry today is something of an orphan. Few graduate schools treat it seriously. It is not "exciting", in that it does little to one's adrenalin flow. With a little care, even a mediocre lecturer can turn it into a marvelously soporific subject. Nevertheless, astrometry is "basic" to an extent matched by few other branches of astronomy. Astrometry underlies celestial mechanics, and hence our quantitative knowledge of the structure and dynamics of the solar system. Without astrometry, we would know virtually nothing about the distances, masses and luminosities of the stars, or about the motion of the sun relative to its neighbors, or about the dynamics and mass of the Galaxy. We would lack the calibrations which are used to estimate the distances to other galaxies. Most contemporary astrophysics depends on the quantitative foundation which astrometry provides.

Optical astrometry goes back some 2000 years, to the work of Hipparchus in Greece and Claudius Ptolemy in Egypt. The measurements of Tycho Brahe in the late 16th century led to the discovery of the laws of planetary motion by Johannes Kepler, and this in turn led to Sir Isaac Newton's Principia and the systematic development of celestial mechanics. By the early 18th century, James Bradley at the Royal Greenwich Observatory was doing work of accuracy respectable by modern standards. Since Bradley's time, the subject has developed steadily to the point of diminishing returns; significant further improvements in accuracy are unlikely without some fundamentally new principles of measurement. In other words, optical astrometry has "matured", and unless something new is added, senility may be next.

Radio astrometry is providing this rejuvenation. It hasn't had much effect yet, since it is so new. But it will be very important, for the following reasons:

(i) Radio measurements will probably reach an accuracy in the near future which exceed that of the best optical work by a factor of 10 or more. This is because the radio measurements depend on the relative times of arrival of a signal at widely separated points (the essence of interferometry) rather than on the apparent direction of arrival. Thus the sources of systematic error are different for radio and optical astrometry, and these errors happen to be smaller in the radio case.

(ii) The accuracy with which an angle can be measured by radio methods is independent of the size of the angle. In optical work, the uncertainty is proportional to the size of the angle. For this reason optical astrometry has always had trouble in tying widely separated parts of the sky together--there have always been zonal errors which have been difficult to detect and eliminate.

(iii) At any given level of accuracy, the radio measurements are much easier to make and reduce. The radio work is easy to automate, the optical is not.

By determining very precise positions for a large number of sources with optical counterparts, well distributed over the sky, radio astrometry will provide a greatly improved net of standards for optical astrometry. It will not replace optical astrometry, of course, since most of the stars which the optical astrometrist measures are not radio sources. But I expect that in a few years most optical astrometry will depend on differential measurements referred to radio standards.

## III. What Accuracy is Now Attainable?

The best work done so far at NRAO, using the Green Bank interferometer with the 45-foot remote element (maximum baseline of 35 km), has internal errors of the order of 0".01 ($\sim 5\times10^{-8}$ radian). This work is not ready for publication, since the possible sources of systematic error have not yet been analyzed fully. Still, it is promising--the accuracy exceeds anything which has been achieved optically.

## IV. What does the Future Hold?

We expect that the VLA now under construction in New Mexico will permit systematic work accurate to 0".01. Carried over a span of years, this will make possible a redetermination of many of the fundamental constants of astronomy (e.g., precession, nutation, dynamical parameters of the rotating earth).

We have not considered VLBI measurements in this lecture. They are still in their infancy, and serious problems remain to be overcome. Nevertheless, the early work has been very promising, and it is not unlikely that measurements accurate to 0".001 will be possible within a few years.

## References

1. An excellent review of contemporary astrometry is:
   W. Fricke, Annual Review of Astronomy and Astrophysics, vol. 10, pp. 101-128, 1972.

2. The following two papers, both from "New Problems in Astrometry" (Proc. IAU Symposium No. 61, Reidel, 1974), provide a general review of the methods and results of radio astrometry:

   B. Elsmore, "Radio Astrometry Using Connected-Element Interferometers", pp. 111-117.

   C. M. Wade, "Radio and Optical Astrometry", pp. 133-139.

   (Xerox copies of both papers are attached.)

3. The methods of radio astrometry are developed in detail in the following papers:

   M. Ryle and B. Elsmore, Monthly Notices Royal Astron. Soc., 164, 223, 1973.

   C. M. Wade, Astrophys. J., 162, 381, 1970.

   P. Brosche, C. M. Wade and R. M. Hjellming, Astrophys. J., 183, 805, 1973.

4. Basic information on time, coordinate systems, and the general background underlying practical astrometry can be found in the following books. The first two are the "holy scriptures"--very complete, very accurate and definitely not light reading. The third is meant as an undergraduate text, but is meaty enough to be profitable reading for the Ph.D.

"Explanatory Supplement to the Ephemeris", Her Majesty's Stationery Office, London, 1961.

E. W. Woolard and G. M. Clemence, "Spherical Astronomy", Academic Press, New York and London, 1966.

D. McNally, "Positional Astronomy", John Wiley & Sons, New York, 1974.

# RADIO ASTROMETRY USING CONNECTED-ELEMENT INTERFEROMETERS

*(Invited Paper)*

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**Abstract.** The basic techniques of conventional radio interferometry are explained and a review given of the achievements and limitations of the methods as applied to radio astrometry.

The application of radio astronomy techniques to establish an astrometric system based on extragalactic radio sources has important consequences both in radio astronomy and in optical astrometry. In radio astronomy it will provide a reference frame of high accuracy for source catalogues and for the calibration of instruments for positional measurements. In optical astrometry, a comparison between the positions of compact, extragalactic objects determined by radio and optical methods may lead to a reduction in the errors in fundamental optical catalogues that arise from effects due to proper motion of individual stars and of galactic rotation, and hence, to the elimination of large scale inhomogeneities (Fricke, 1972).

Unlike the situation in optical astronomy, no radio astronomy instrument has yet been designed primarily for astrometric purposes but two distinct interferometer techniques exist that provide sufficient precision in measuring the positions of radio sources to merit the use of the term 'astrometric'.

One method, that uses interferometers of very large baselines, sometimes as much as a few thousand kilometres, has necessitated the development of a timing and recording technique to enable the signals to be recorded separately at each end of the baseline and compared at a later time. The accuracy of this VLBI system is potentially extremely high, but it has not yet been fully realized in practice; one serious limitation being caused by the difficulty in knowing precisely the delay due to the ionosphere and atmosphere above each of the two widely separated aerials. In addition to astrometric uses, VLBI measurements have important applications in geodesy.

Another interferometric technique, which is my main concern here, uses a baseline of a few kilometres, so that the aerials may be linked by cables to a receiving system that instantly measures and records the difference in phase between the signals arriving at the aerials. From measurements of this difference of phase at two aerials, fixed on the surface of the rotating earth, the positions of radio sources may be derived. This conventional type of interferometer has been widely used in recent years and an estimate of the accuracies that have been achieved may be obtained from an examination of Table I. It should be noticed that none of these measurements has been made in the southern hemisphere.

*Gliese, Murray, and Tucker, 'New Problems in Astrometry', 111–117.*

As the methods used in radio astronomy are very different from those of classical optical astrometry, an attempt will be made to give a broad outline of the principles involved.

The method of relating the measured phase difference to the position of a source on the sky clearly depends upon the orientation of the interferometer baseline and the geometry involved is perhaps the simplest when the interferometer baseline lies east-west. In the early days of radio interferometry, when the aerials were large fixed arrays, unable to track radio source, observations were made only at transit and the east-west interferometer was then analogous to the meridian circle, being subject to the usual level, azimuth and collimation errors. Now that much shorter radio wavelengths are used, and aerials track the sources as the earth rotates, the R.A. and Decl. may be determined independently.

TABLE I

Highest accuracies claimed for various instruments

| | | | "(arc) |
|---|---|---|---|
| NRAO Greenbank | Wade | (1970) | 0.6 |
| Cambridge One Mile | Smith | (1971) | 0.2 |
| RRE Malvern | Adgie *et al.* | (1972) | 0.4 |
| NRAO Greenbank | Brosche *et al.* | (1973) | 0.1 |
| Cambridge 5 km | Ryle and Elsmore | (1973) | 0.02 |

If a vector represents the baseline spacing between two aerials on the Earth's surface as shown in Figure 1a, it can be seen that the vector, due to the assumed uniform rotation of the earth, rotates in a plane at right angles to the polar axis; the sideways motion is of no consequence as it only produces an aberration. The path difference at any instant is the component of the vector in the line of sight, and hence, the measured

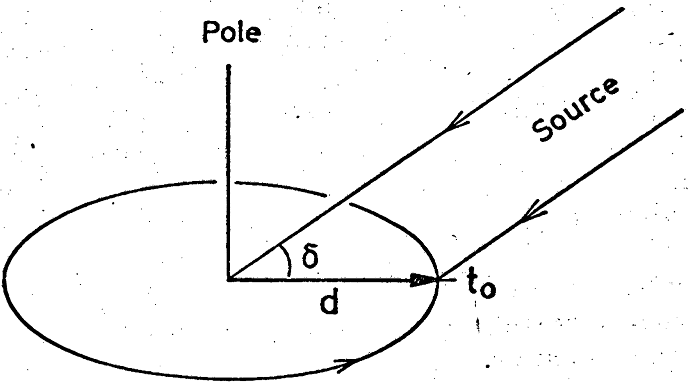

Fig. 1a. A vector representing the baseline spacing of an east-west interferometer is shown to sweep out a circle due to the rotation of the Earth. The measured phase difference at any instant is proportional to the component of this vector in the line of sight from the source.

phase difference is

$$\phi(t)=\frac{2\pi}{\lambda}\, d\cos(t-t_0)\cos\delta+C$$

where $C$ is the electrical collimation error; a delay within the receiving system caused, for example, by unequal cables connecting the aerials to the receiver. If we now plot the measured $\phi$ against time we obtain a curve of the form shown in Figure 1b.

Declination may be derived from the amplitude of the sinusoidally varying component, provided that the spacing d is known. A disadvantage can be seen of the east-west interferometer system in that the accuracy of measuring declination varies with $\sin\delta$. However, with the 5 km telescope at Cambridge (Ryle, 1972) this fact has been put to an advantage and used to determine the aerial spacing independently of the initial land survey of the instrument, by observing radio sources of only approximately known declinations lying near the equator. Thus the distance, for example, between two of the aerials has been measured to be $3\,430\,828.7\pm0.25$ mm (i.e. to 1 in $10^7$ which is about 10 times more accurate than that available for the measurement of 3.4 km by conventional means). Using this value for spacing, declinations of other sources may then be determined absolutely.

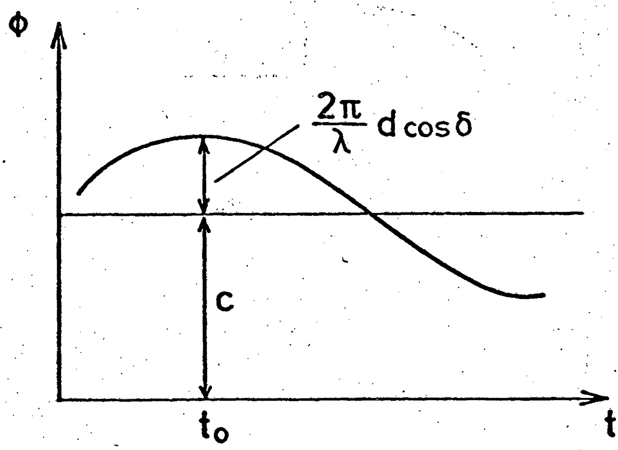

Fig. 1b. The measured phase difference, plotted against time, for an east-west interferometer system.

Hour angle may be derived from the phase of the $\phi(t)$ plot, but the relation between hour angle and right ascension presents more difficulty. The lack of suitable bright and compact radio objects in the solar system makes it difficult to establish the ecliptic and hence the equinox from radio observations, and so for the 5 km telescope the zero point of R.A. has been established from observations of $\beta$ Persei (Algol) which intermittently radiates sufficiently strongly at cm wavelengths, thereby enabling the RA scale to be related directly to FK4 (Ryle and Elsmore, 1973). A redetermination of the optical position of $\beta$ Persei at the Royal Greenwich Observatory and at the Institute of Astronomy, Cambridge has shown the FK4 right ascension to be in error by less than 3 ms (Tucker *et al.*, 1973).

With an interferometer not aligned east-west, the baseline vector sweeps out a cone as the Earth rotates as shown in Figure 2a. This fact has been utilised ingeniously by Wade (Wade, 1970; Brosche *et al.*, 1973) of NRAO to determine declinations without prior knowledge of the baseline or of any source declinations. As before, the phase difference is $2\pi/\lambda$ times the component of the vector in the line of sight. As $B$ describes a circle, the phase varies sinusoidally, hence

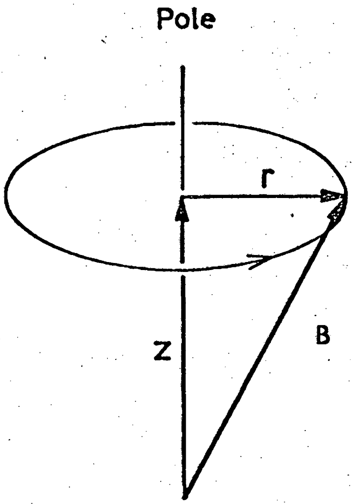

Fig. 2a. The baseline vector for a non east-west system describes a cone as the Earth rotates. The vector may be resolved into a component $z$, parallel to the polar axis, and a rotating component $r$.

$$\phi(t)=\frac{2\pi}{\lambda}\left[z\sin\delta+r\cos\delta\cos(t-t_0)\right]+C.$$

Three measured parameters may be derived from a plot of $\phi(t)$, as shown in Figure 2b.

(1) A constant term, $\phi_c=(2\pi/\lambda)\, z\sin\delta+C$.

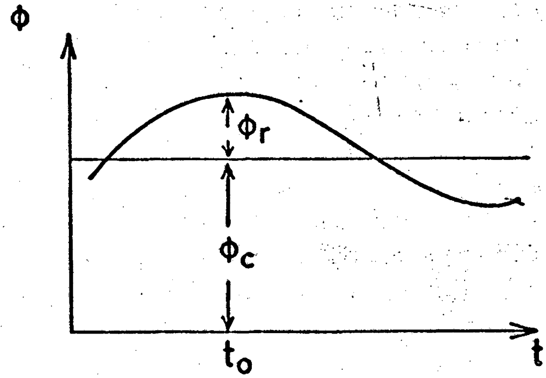

Fig. 2b. The measured phase difference, plotted against time, for a non east-west system.

(2) The amplitude of the oscillatory term, $\phi_r = (2\pi/\lambda)\, r \cos\delta$.

(3) The time $t_0$, which is related to hour angle.

For several sources, say $N$, with $\delta_i$, $z$, $r$ and $C$ unknown. Solving for declination:

$N+3$ unknowns: $\delta_i$, $z$, $r$ and $C$
$2N$ equations from (1) and (2) (i.e., from $\phi_c$ and $\phi_r$).

Therefore, a solution may be obtained for declinations using observations of three sources, and furthermore, the method remains accurate at low declinations. Relative right ascensions may be derived from the phase of the $\phi(t)$ plot and at NRAO, the zero of R.A. has been established for their instrument from observations of four extragalactic radio sources for which there are accurate optical positions, (Brosche *et al.*, 1973).

On careful examination of the properties of these two differently orientated interferometers, it can be seen that there is another difference, in that for the east-west system, the measurement of declination and right ascension is independent of the instantaneous position of the pole.

Consider an east-west interferometer situated on the Greenwich meridian. The $x$ component of polar motion changes the latitude, which has no effect on the phase difference between the two ends of the baseline. The $y$ component will rotate the baseline to give a small displacement parallel to the mean polar direction that contributes to $C$, the electrical collimation term, but will, to first order, make no change in the component parallel to the equatorial plane. Therefore, the measurement of R.A. and Decl. remains unchanged. The polar motion that occurs during the observations is sufficiently small as to not affect the measurements. For an interferometer not at Greenwich a similar argument holds, since the instantaneous polar coordinates can be resolved in, and at right angles to the local meridian.

This is not the case for a non east-west interferometer that has an appreciable component of the baseline vector parallel to the polar direction.

An advantage over all other methods of the technique that utilize moderately spaced interferometers stems from the fact that only *differences* between the paths to the two aerials are involved, hence atmospheric corrections are only of second order and are typically 1″. With a horizontally stratified atmosphere the path differences, and hence the corrections, would be zero, although the individual aerials would have to be steered to allow for refraction. However, in practice, path differences occur due to the curvature of the atmosphere and ionosphere, for which correction is possible. Although the aerials of an interferometer may only be a few kms apart, irregularities in the water vapour content of the troposphere produce phase variations for which no correction is possible. These effects, which are most severe during the daytime in summer, are the dominant factor that limit the accuracy that can be achieved by the 5 km telescope (Ryle and Elsmore, 1973). The magnitude of effects arising from various external causes for different interferometer spacings are shown in Table II (from Hinder and Ryle, 1971).

What of the future? The 5 km telescope at Cambridge has some features that make

TABLE II

Uncertainties of differential path in mm at 30° elevation for Cambridge

| Baseline (km) | Troposphere irregularities |  | Troposphere curvature | Ionosphere irregularities at 5 GHz |
|---|---|---|---|---|
|  | Summer | Winter |  |  |
| 1 | 2.6 | 0.66 | 0.12 | 0.03 |
| 10 | 2.8 | 0.71 | 1.2 | 0.35 |
| 100 | 2.8 | 0.71 | 12 | 2.8 |

*Note:* 1 mm at 1 km = 0″.2.

it especially suitable for astrometry, (for example, the declination and hour angle axes intersect at a point), but the instrument has been designed primarily as a fully automated instrument for providing high resolution maps of radio sources. For this reason, four of the eight aerials are mobile and are mounted on rails which renders them less stable, and so for astrometric observations, only the fixed aerials are used, giving spacings up to 3.4 km. From preliminary observations of much of the 3*C* catalogue north of Decl. 10°, only 12 sources have so far been found to be sufficiently compact for the highest precision astrometric purposes; all the other sources examined are either extended or complex. The positions of these 12 sources, with an accuracy of about ±0″.03 are about to be published (Ryle and Elsmore, 1973). It is estimated that there are another 100 suitable sources in the N hemisphere brighter than 0.2 f.u. In this connection, it is noted that a working party of Commission 40 has been set up to compile a list of sources suitable for positional calibration purposes.

It is particularly desirable that sources at low declinations should be observed so that the positions of sources in the N hemisphere may be linked to those which it is hoped will be measured in the S hemisphere. This cannot be done with east-west interferometer systems, such as those at Cambridge and Westerbork.

The contribution to astrometry from connected-element radio interferometry is only just beginning, now that large angles on the sky can be measured with great precision and that the positions of radio sources may be determined with comparable, if not greater accuracy than those of optical observations of stars. It is very important, in addition, to appreciate that the factors that limit the accuracies fortunately have different effects in radio and in optical astrometry, thus making the two types of observations complementary.

## References

Adgie, R. L., Crowther, J. H., and Gent, H.: 1972, *Monthly Notices Roy. Astron. Soc.* **159**, **233**.
Brosche, P., Wade, C. M., and Hjellming, R. M.: 1973, *Astrophys. J.* **183**, **805**.
Fricke, W.: 1972, *Ann. Rev. Astron. Astrophys.* **10**, **101**.
Hinder, R. A. and Ryle, M.: 1971, *Monthly Notices Roy. Astron. Soc.* **154**, **229**.
Ryle, M.: 1972, *Nature* **239**, 435.
Ryle, M. and Elsmore, B.: 1973, *Monthly Notices Roy. Astron. Soc.* **164**, **223**.

Smith, J. W.: 1971, *Nature Phys. Sci.* **232**, 150.
Tucker, R. H., Yallop, B. D., Argue, A. N., Kenworthy, C. M., Ryle, M., and Elsmore, B.: 1973, *Monthly Notices Roy. Astron. Soc.* **164**, 27P.
Wade, C. M.: 1970, *Astrophys. J.* **162**, 381.

# DISCUSSION

*Tucker:* Does the solution for three sources need three distinct $\delta$ values? Also there is a diurnal term in latitude variation that amounts to 0.″006.

*Elsmore:* The method does, in fact, rely upon the use of three different declinations. Concerning the second point, I am grateful to Mr Tucker for making me aware of this effect and one must consider what result it will have on the positional measurements.

*Eichhorn:* Will it sometimes be possible to use the radio emissions from Jupiter and Saturn to establish the position of the vernal equinox – should this be deemed desirable – or are their diameters too large for this purpose?

*Elsmore:* The difficulty here is that the sources are extended and furthermore the centre of radio emission may not coincide with that of the figure of these planets.

*Fricke:* Could our colleagues from Greenwich or Cambridge, England, tell us how they succeeded in checking the right ascension of $\beta$ Persei?

*Murray:* Algol was observed photographically at Herstmonceux and Cambridge relative to AGK3 stars, and also on the Herstmonceux Transit Circle relative to about fifteen FK4 stars on 20 different nights between September and December, 1972.

*Teleki:* We must compare, very carefully, the optical and radio observations. I suppose that there are many problems in this comparison including refractional problems. For this reason Prof. Fricke proposed to me, last year, the inclusion of a radio-astronomer in the Study Group on Astronomical Refraction. Dr W. J. Altenhoff (Max-Planck-Institut für Radioastronomie, Bonn), radioastronomer, has accordingly become a member of this Study Group. In his report prepared for the Study Group, I find this conclusion:

"In my feeling the astronomical refraction is not fully understood in the radio range. For single dish investigations one could think of some differential methods to measure the total refraction (with a dual beam system of fixed separation), the ionospheric refraction (with simultaneous observations at two frequencies) etc. Even though these measurements might help us to understand the refraction, it is not clear if they could help to improve positional (interferometric) work in radio astronomy."

Do you agree with this conclusion?

*Elsmore:* In interferometric work, the atmosphere does not present such a serious problem as you suggest, as only the difference between the paths to the two aerials is involved, and hence corrections are only of second order.

*Murray:* Could Mr Elsmore say something about the effect of the orbital motion of Algol? I believe you detected it at Cambridge.

*Elsmore:* Although the radio observations do not yet cover a complete period of the 1.ʸ9 orbit, it is clear that the emission is almost certainly related to the *AB* system rather than to component *C*, which is separated from *AB* by 0.″1.

# RADIO AND OPTICAL ASTROMETRY

*(Invited Paper)*

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**Abstract.** Radio positional measurements have achieved an accuracy as high as that of optical astrometry, with uncertainties no greater than a few hundredths of an arc second in each coordinate. Declinations and relative right ascensions are determined absolutely. Since the principal sources of systematic error are different for radio and optical astrometry, radio measurements can be useful in the preparation of future fundamental catalogues.

## 1. Introduction

A quarter of a century ago, the most accurate measurements of radio source positions were uncertain by several minutes of arc. The quality of radio work has since improved to the point that it now is possible to make systematic positional measurements of small-diameter sources which are as accurate as the fundamental optical catalogues. Interferometric methods developed at the Mullard Radio Astronomy Observatory in Great Britain and at the National Radio Astronomy Observatory in the United States yield precise absolute declinations and right ascension differences. It still is necessary to use optical information to fix the origin of the right ascensions, since no object in the solar system is suitable for radio astrometry. In all other respects, the radio measurements are fundamental in the astrometric sense.

Radio astrometry has several important advantages in comparison with optical astrometry. The effects of atmospheric refraction are relatively unimportant. Large angles can be measured with about the same accuracy as small ones, so regional systematic errors are not a serious problem. Observations of a large number of sources well distributed on the sky can be analyzed simultaneously in a way that fixes most of the instrumental parameters along with the source positions, so the measurements are largely self-calibrating. The observations determine the radii of the diurnal circles of the sources, thus referring the declinations directly to the Earth's instantaneous axis of rotation. Most sources of small angular size are extragalactic, and a grid of fundamental positions which are not affected by proper motions can be established readily. Finally, the observations can be made rapidly, and analyzed entirely by electronic computers.

We shall consider here only the most recent fundamental radio interferometric measurements. Much excellent radio work has been done by reference to optically determined calibration positions, but such measurements are outside the scope of the present paper since they are essentially differential. We shall also ignore lunar occulta-

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* Operated by Associated Universities, Inc., under contract with the National Science Foundation.

*Gliese, Murray, and Tucker, 'New Problems in Astrometry', 133–139. All Rights Reserved.*

tion measurements because their accuracy is limited by instrumental noise and the uncertainty of the lunar limb corrections.

## 2. Fundamental Radio Measurements

Radio interferometric techniques for absolute position measurements have been developed independently at the Mullard Radio Astronomy Observatory (Elsmore and Mackay, 1969; Smith, 1971; Ryle and Elsmore, 1973) and at the National Radio Astronomy Observatory (Wade, 1970; Brosche *et al.*, 1973). Although the underlying principles of the measurements at the two observatories are much the same, the actual methods differ considerably because the instruments are quite dissimilar (Ryle, 1972; Hogg *et al.*, 1969). The difference which has the greatest influence on method is in the baseline orientations. Some of the characteristics of the interferometers are summarized in Table I.

TABLE I

Characteristics of the Mullard and NRAO interferometers

| | Mullard | NRAO |
|---|---|---|
| Number of antennas | 8 | 3 |
| Antenna diameter | 13 m | 26 m |
| Maximum baseline | 4.6 km | 2.7 km |
| Baseline azimuth | 93°19′ | 62°02′ |
| Latitude | 52°.2 N | 38°.4 N |
| Operating wavelength | 6 cm | 4+11 cm |
| Min. fringe spacing | 2″.7 | 2″.9 |

The accuracy of both instruments is limited primarily by random fluctuations in the signal paths through the troposphere (and to a lesser extent, through the ionosphere). The effect is analogous to optical scintillation, although the characteristic time scale is longer by two to three orders of magnitude. Since the process is random, the errors can be reduced considerably by averaging the results of repeated observations. The uncertainties due to instrumental noise are negligible except for very weak sources.

Atmospheric refraction is not an important source of error in radio astrometry, since the positional information lies in the relative arrival times of the signals at the different antennas rather than in the apparent direction of arrival. Small corrections are necessary to compensate for the slightly differing elevations of the antennas above sea level and for the curvature of the atmosphere, but these are easily calculated and introduce little uncertainty.

The Mullard and NRAO methods both determine declinations absolutely, since in effect they measure the radii of the diurnal circles of the sources. Differences of right ascension between sources are also found absolutely, with an accuracy which is independent of the magnitude of the difference. Unfortunately, the angular sizes of the Sun and the planets are too large for their positions to be measured accurately with radio interferometers, and there is therefore no way to refer the right ascensions directly to the vernal equinox by purely radio methods. Instead, it is necessary that the observations include at least one object whose right ascension has been found by optical means. The Mullard right ascensions are referred to the FK4 position of $\beta$ Persei, which is an intermittent radio source (Wade and Hjellming, 1972). At NRAO, the right ascensions have been adjusted to agree in the mean with optical measurements of a large number of sources, mostly quasars (Kristian and Sandage, 1970; Murray *et al.*, 1971).

The major source of systematic error is imperfect instrumental calibration, particularly in the determination of the collimation error (which reflects departures from symmetry in the paths of the signals collected by the different antennas). Small changes in baseline length and orientation due to differing thermal effects at the various antennas can also cause diurnal errors which are difficult to evaluate. Careful calibration procedures can keep such errors small, but they cannot be eliminated entirely.

Accuracies of a few milliseconds in right ascension and a few hundredths of an arc second in declination have been achieved at Mullard (Ryle and Elsmore, 1973). This is fully as good as the FK4. Accuracies of about 0.″15 have been attained at NRAO (Brosche *et al.*, 1973). The higher precision of the Mullard measurements is the result of averaging a large number of observations, which much reduces the phase noise caused by atmospheric inhomogeneities. The instruments and methods used at the two observatories are inherently capable of similar accuracy.

## 3. Comparison of Radio and Optical Results

Several fairly extensive lists of accurate optical positions of radio sources have been published in the last 5 years (e.g., Bolton, 1968; Kristian and Sandage, 1970; Hunstead, 1971; Véron, 1972; Barbieri *et al.*, 1972). Perhaps the most accurate measurements have been made at Cambridge (Argue and Kenworthy, 1972; Argue *et al.*, 1973). These are essentially on the AGK3 system, although they are based partly on earlier measurements in the FK4 system by Murray *et al.* (1969, 1971).

We shall restrict the present discussion to comparing the Cambridge optical positions with the most recent fundamental radio measurements at Mullard and NRAO (Ryle and Elsmore, 1973; Brosche *et al.*, 1973). Table II gives the measured positions for 1950.0 with their mean errors; the errors of the optical positions are the external errors. The NRAO right ascensions have been adjusted slightly to agree in the weighted mean with the optical values.

It is clear from Table II that the agreement between the three sets of measurements is generally good. This is confirmed by Table III, which gives the weighted (by the inverse squares of the mean errors) average differences between the sets. The overall systematic agreement is at least as good as 0.″1, which is less than the claimed mean errors of the individual NRAO and optical measurements.

TABLE II

Radio and optical positions (1950.0) for 14 radio sources

| Source | | Radio | | Optical |
|---|---|---|---|---|
| | | Mullard | NRAO | |
| 0056-00 | $00^{h}56^{m}$ | | $31^{s}.761\pm0^{s}.020$ | $31^{s}.771\pm0^{s}.017$ |
| | $-00°09'$ | | $18''.72\pm0''.52$ | $18''.08\pm0''.30$ |
| 3C48 | $01^{h}34^{m}$ | $49^{s}.827\pm0^{s}.002$ | $49^{s}.817\pm0^{s}.012$ | $49^{s}.819\pm0^{s}.012$ |
| | $+32°54'$ | $20''.63\pm0''.04$ | $20''.74\pm0''.18$ | $20''.40\pm0''.12$ |
| 3C84 | $03^{h}16^{m}$ | $29^{s}.562\pm0^{s}.003$ | | $29^{s}.548\pm0^{s}.014$ |
| | $+41°19'$ | $51''.99\pm0''.04$ | | $52''.19\pm0''.12$ |
| 3C138 | $05^{h}18^{m}$ | $16^{s}.526\pm0^{s}.005$ | $16^{s}.520\pm0^{s}.011$ | $16^{s}.521\pm0^{s}.011$ |
| | $+16°35'$ | $27''.06\pm0''.09$ | $27''.29\pm0''.25$ | $27''.06\pm0''.15$ |
| 3C147 | $05^{h}38^{m}$ | $43^{s}.503\pm0^{s}.003$ | $43^{s}.498\pm0^{s}.016$ | $43^{s}.492\pm0^{s}.016$ |
| | $+49°49'$ | $42''.87\pm0''.02$ | $42''.94\pm0''.15$ | $43''.10\pm0''.12$ |
| 0736+01 | $07^{h}36^{m}$ | | $42^{s}.510\pm0^{s}.017$ | $42^{s}.496\pm0^{s}.010$ |
| | $+01°44'$ | | $00''.29\pm0''.44$ | $00''.29\pm0''.21$ |
| 4C39.25 | $09^{h}23^{m}$ | | $55^{s}.296\pm0^{s}.014$ | $55^{s}.310\pm0^{s}.015$ |
| | $+39°15'$ | | $24''.30\pm0''.17$ | $22''.94\pm0''.14$ |
| 3C279 | $12^{h}53^{m}$ | | $35^{s}.821\pm0^{s}.021$ | $35^{s}.842\pm0^{s}.018$ |
| | $-05°31'$ | | $07''.36\pm0''.30$ | $07''.28\pm0''.30$ |
| 3C286 | $13^{h}28^{m}$ | $49^{s}.653\pm0^{s}.004$ | $49^{s}.681\pm0^{s}.012$ | $49^{s}.657\pm0^{s}.012$ |
| | $+30°45'$ | $58''.79\pm0''.05$ | $58''.32\pm0.19$ | $58''.46\pm0''.12$ |
| 3C309.1 | $14^{h}58^{m}$ | $56^{s}.644\pm0^{s}.005$ | $56^{s}.616\pm0^{s}.038$ | $56^{s}.648\pm0^{s}.033$ |
| | $+71°52'$ | $11''.17\pm0''.02$ | $11''.26\pm0''.16$ | $11''.33\pm0''.16$ |
| 3C345 | $16^{h}41^{m}$ | $17^{s}.603\pm0^{s}.002$ | $17^{s}.609\pm0^{s}.014$ | $17^{s}.606\pm0^{s}.013$ |
| | $+39°54'$ | $10''.89\pm0''.03$ | $11''.26\pm0''.16$ | $10''.72\pm0''.12$ |
| BL Lac | $22^{h}00^{m}$ | $39^{s}.362\pm0^{s}.007$ | $39^{s}.370\pm0^{s}.014$ | $39^{s}.397\pm0^{s}.021$ |
| | $+42°02'$ | $08''.69\pm0''.04$ | $08''.77\pm0''.16$ | $08''.47\pm0''.14$ |
| CTA 102 | $22^{h}30^{m}$ | $07^{s}.793\pm0^{s}.005$ | | $07^{s}.823\pm0^{s}.029$ |
| | $+11°28'$ | $22''.89\pm0''.20$ | | $23''.07\pm0''.25$ |
| 3C454.3 | $22^{h}51^{m}$ | $29^{s}.510\pm0^{s}.006$ | | $29^{s}.533\pm0^{s}.011$ |
| | $+15°52$ | $54''.54\pm0''.09$ | | $54''.98\pm0''.15$ |

TABLE III

Weighted mean differences

| | Mullard-NRAO | Mullard-Optical | NRAO-Optical | Notes |
|---|---|---|---|---|
| $\langle\Delta\alpha\rangle$ | $-0^{s}.002\pm0^{s}.006$ | $-0^{s}.002\pm0^{s}.005$ | – | (1) |
| $\langle\Delta\alpha\cos\delta\rangle$ | $-0''.01\pm0''.07$ | $-0''.01\pm0''.05$ | $-0''.01\pm0''.05$ | (2) |
| $\langle\Delta\delta\rangle$ | $-0''.08\pm0''.09$ | $-0''.01\pm0''.08$ | $+0''.13\pm0''.09$ | |
| No. of sources | 7 | 10 | 10 | |

*Notes:*

(1) The NRAO right ascensions were adjusted to make the weighted mean difference from the optical equal to zero.

(2) 4C39.25 was omitted because the NRAO and optical declinations differ by $1''.36$, which is excessive in relation to the differences for the other sources.

The number of sources in Table II is far too small to permit a search for possible regional differences between the radio and optical positions. Even in this small sample, however, there appears to be a consistent trend in the difference between the Mullard and optical right ascensions, as a function of right ascension. This can be seen in Figure 1. The right ascensions of the sources common to the Mullard and optical lists fall into three loose groups. Within each group, the values of $\Delta\alpha$ are about the same, but there is a noticeable difference from group to group. The effect appears to be real, but one cannot say at this time whether the source of the discrepancy is in the optical or the radio measurements. The NRAO measurements are not accurate enough to resolve the matter with any certainty.

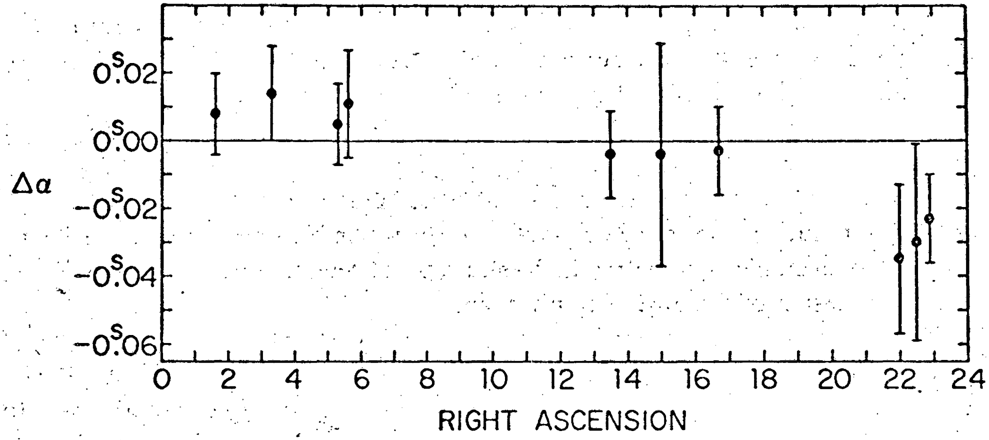

Fig. 1. Distribution of the differences between the Mullard and optical right ascension measurements, as a function of right ascension.

## 4. Conclusions

Radio astrometry, although new, has achieved a level of accuracy which rivals the best optical work. It has several valuable advantages over existing optical astrometric methods, of which the most important are:

(a) the relative unimportance of atmospheric refraction;

(b) the ability to measure absolute declinations which are automatically referred to the Earth's instantaneous axis of rotation; and

(c) the ability to measure large angles with essentially the same accuracy as small angles.

The principal disadvantage, from the standpoint of fundamental astrometry, is the inability of existing instruments to determine the location of the vernal equinox without recourse to optical data. This is a temporary deficiency, however, since the Very Large Array (VLA) now under construction in the United States will be sensitive enough to observe the brightest minor planets.

The major source of random error in radio astrometry is the phase instability caused by tropospheric inhomogeneities. Under average conditions, the accuracy attainable in a single observation with a single pair of antennas is limited to about 0″.2. The error can be reduced considerably by taking the mean of a large number of observations. This is shown by the much higher accuracy of the Mullard measurements in Table II as compared to the NRAO measurements. The Mullard positions are the result of many observations with 16 antenna pairs, while the NRAO positions are derived from single observations (each consisting of 5 min of integration at three widely separated hour angles) with 3 antenna pairs.

The systematic errors of radio astrometry are due to imperfect instrumental calibration. In a well-designed interferometer system, all of the relevant parameters except the collimation error are highly stable, and most are readily calibratable from radio observations alone. The collimation error is troublesome, however, since it is the result of unstable asymmetries in the electronic system. Its effects can be minimized by careful design of the observing program.

Most of the radio sources which are suitable for astrometric measurement with interferometers are extragalactic, mostly quasars and the nuclei of active galaxies. These objects are numerous and well distributed over the sky. Thus radio astrometry can readily establish a set of precise extragalactic reference points for the differential measurement of proper motions. The faintness of the optical counterparts of most of these objects (15th to 19th mag.) will cause some difficulty in their practical employment as proper motion standards, but the problem is not fundamental. Finally, repeated measurement of the positions of extragalactic sources over a sufficiently long span of time will permit a refinement of the precessional constants without the complications caused by proper motions and galactic rotation.

Radio observations can make valuable contributions to the field of astrometry as a whole, particularly in establishing a uniform positional reference frame over the entire sky. I believe that radio astrometry will soon play an integral part in the construction of fundamental catalogues, because of its ability to give results which are free of regional systematic errors as well as its inherently high precision. It is unfortunate that there is no radio interferometer in the southern hemisphere at present which is usable for astrometric work. Such an instrument would be invaluable in tying the astrometric systems of the two hemispheres together.

## References

Argue, A. N. and Kenworthy, C. M.: 1972, *Monthly Notices Roy. Astron. Soc.* **160**, **197**.
Argue, A. N., Kenworthy, C. M., and Stewart, P. M.: 1973, *Astrophys. Letters* **14**, **99**.
Barbieri, C., Capaccioli, M., Ganz, R., and Pinto, G.: 1972, *Astron. J.* **77**, **444**.
Bolton, J. G.: 1968, *Publ. Astron. Soc. Pacific* **80**, **5**.
Brosche, P., Wade, C. M., and Hjellming, R. M.: 1973, *Astrophys. J.* **183**, **805**.
Elsmore, B. and Mackay, D. C.: 1969, *Monthly Notices Roy. Astron. Soc.* **146**, **361**.
Hogg, D. E., Macdonald, G. H., Conway, R. G., and Wade, C. M.: 1969, *Astron. J.* **74**, **1206**.
Hunstead, R. W.: 1971, *Monthly Notices Roy. Astron. Soc.* **152**, **177**.
Kristian, J. and Sandage, A.: 1970, *Astrophys. J.* **162**, **391**.

Murray, C. A., Tucker, R. H., and Clements, E. D.: 1969, *Nature* **221**, 1229.
Murray, C. A., Tucker, R. H., and Clements, E. D.: 1971, *Roy. Obs. Bull.*, No. 162.
Ryle, M.: 1972, *Nature* **239**, 435.
Ryle, M. and Elsmore, B.: 1973, *Monthly Notices Roy. Astron. Soc.*, in press.
Smith, J. W.: 1971, *Nature Phys. Sci.* **232**, 150.
Véron, M. P.: 1972, *Astron. Astrophys.* **20**, 471.
Wade, C. M.: 1970, *Astrophys. J.* **162**, 381.
Wade, C. M. and Hjellming, R. M.: 1972, *Nature* **235**, 270.

# DISCUSSION

*Eichhorn:* I am not particularly worried about not all of the error boxes overlapping since the true values of a quantity will be within one sigma in only about two of three cases.

*Wade:* In fact the rate of agreement within the claimed mean errors is about two thirds, as one would expect.

*Eichhorn:* I am also happy about the apparent approaching demise of the vernal equinox as a fundamental point in astrometry. The vernal equinox is, after all, defined only by the kinematics – and only very incidentally the dynamics – of the Earth rotation and revolution. It is thus very intimately tied to the Earth and the solar system, and really not at all germane to matters galactic and cosmological.

*Van Herk:* How well should optical positions coincide with radio sources? It has been said that the position of the radio sources depends on the frequency. Have radio sources to be 'compact' to have the two positions (optical and radio) coinciding?

*Wade:* It is hard to give a general answer, since the radio sources are highly diverse in their properties. In cases where there is a spectral gradient across the source, the radio position will of course be a function of wavelength. When the spectrum is quite uniform over the source, such effects should be negligible. This is true also when the angular extent of the source is appreciably smaller than the error of the positional measurements.

*Baars:* Yesterday Prof. Bok urged radio astronomers to find all radio stars. I would like to inform you of a project in which Dutch and German radio astronomers use the 100-m telescope in Bonn to search for radio radiation at 3 cm wavelength. After a possible detection an accurate position is determined with the Synthesis Radio Telescope in Westerbork at 6 cm wavelength. Several stars have been detected in Bonn, while the Westerbork follow-up is now being made.

I have one new radio star to report, detected by Dr H. Wendker (Hamburg Observatory) and myself with the Westerbork telescope. It is P Cygni and it shows a flux density at 6 cm wavelength of $10^{-28}$ $\mathrm{Wm}^{-2}$ $\mathrm{Hz}^{-1}$. The derived radio position lies within $0\farcs6$ from the AGK3 position of the star. The estimated error in the radio position is $\pm 0\farcs5$.

*Brosche:* As long as radio astrometry is using the present constant of precession, 'artificial' proper motions have to be introduced also for extragalactic objects due to the necessary correction of this constant.