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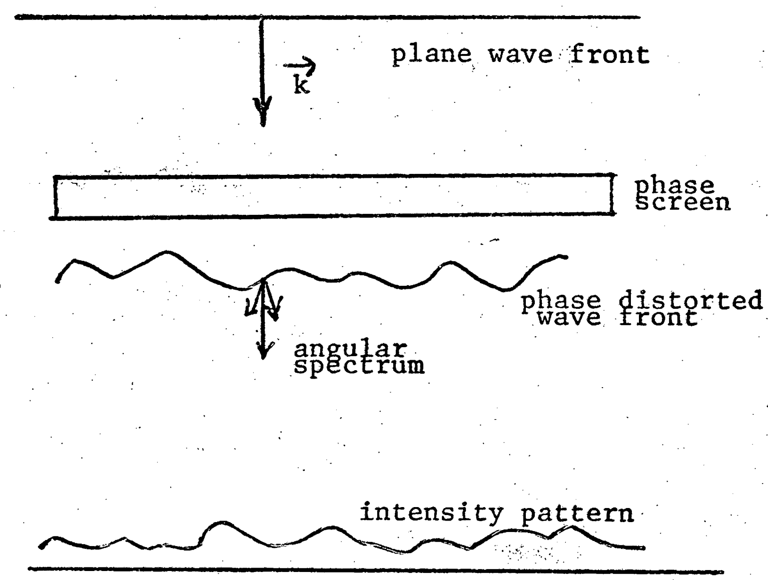
Student Lecture Notes

RADIO SCINTILLATIONS

Radio radiation from celestial sources must pass through several media (e.g., solar wind, ionosphere) before being received at the earth's surface. Propagation through these media can change the observed properties of the radiation. These notes deal with one aspect of these propagation effects: scintillation of the sources caused by small angle scattering of the radiation off irregularities in the various media.

General Ideas: Monochromatic Point Source

A useful model of the physical situation is the "thin phase screen". In this approximation, the true medium is replaced by a geometrically thin layer of turbulence located a diastance z from the observer. Suppose that the source is a monochromatic point source and at a diatance $\gg$ z on the other side of the layer. The index of refraction is assumed to vary in a random way within the layer and is describeable only statistically, say by its correlation function. Radiation passing through the layer suffers position-dependent phase shifts as it encounters differing values of the refractive index. At the "bottom" of the layer there is a corrugated phase front. The spatial gradients in the phase at the bottom of the layer launch waves having propagation vectors slightly inclined with respect to thenormal of the layer. This bundle of plane waves is called an angular spectrum and in the cases of interest here is very sharply forward directed (rms width $\lesssim 10^{-6}$ rads). After propagating through free space to the receiver, the waves may interfere constructively and destructively to give spatial intensity variations at the observer's plane.



For the point source situation considered here, intnesity scintillations (IS) will be either weak or strong depending on the distance, z, from the observer to the screen compared with the diatance, l_ϕ, over which the phase fluctuations are correlated at the bottom of the layer. From diffraction theory, we know that the region of the screen that contributes radiation to the observer that is approximately (geometrically) in phase is roughly

$\sqrt{\lambda z}$ in radius (Fresnel zone). Thus, if the observer is close to the screen ($\sqrt{\lambda z} \ll \ell_\phi$), the phase is nearly uniform over the region of the screen important for diffraction and only small IS are observed. On the other hand, if the observer is in the "far field" ($\sqrt{\lambda z} \gg \ell_\phi$) a Fresnel zone contains many uncorrelated phase patches giving strong IS.

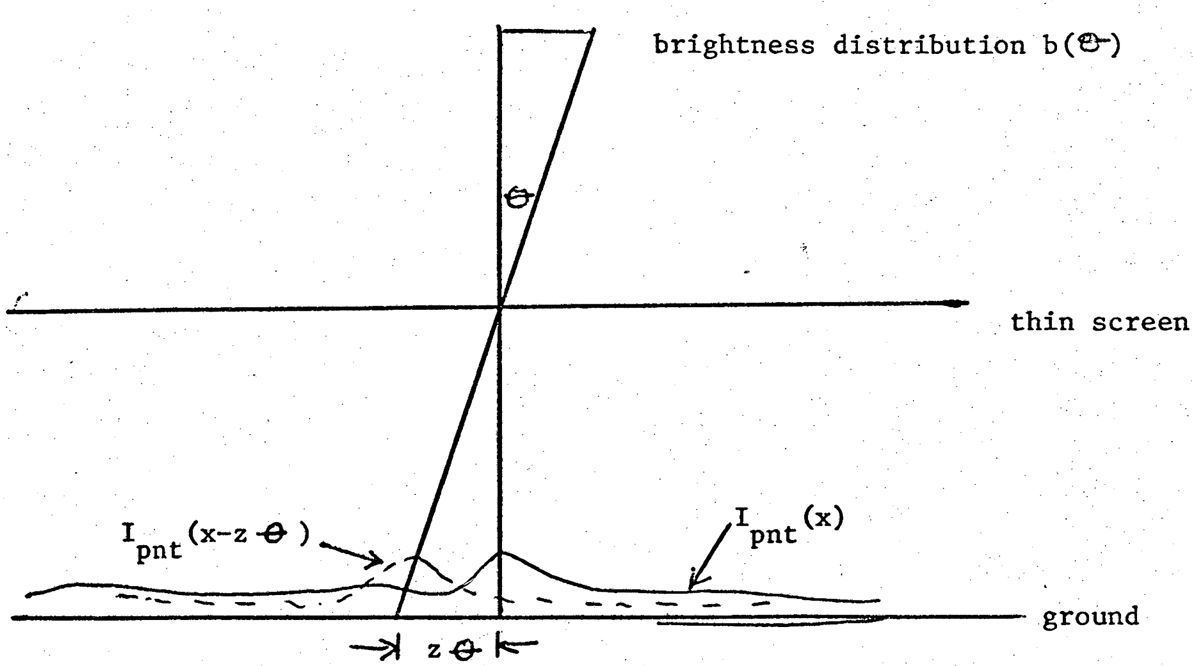
It is useful to define the scintillation index, m, as the rms intensity/mean source intensity. For weak scintillation, m <u>for a point source</u> is much less than 1; strong scintillation is when m <u>for a point source</u> is of the order of unity.

The observer usually has control of λ rather than z. Because the refractive index of a plasma is frequency dependent, changing λ not only changes the Fresnel zone radius, but it can also in some cases change l_ϕ. In weak scattering, l_ϕ is the same as the correlation length for refractive index fluctuations. If the scattering is strong however one can think of a radio wave as being scattered several times and hence having its propagation vector deflected more from the normal than it would for weak scattering. This is the same as saying that l_ϕ is smaller than the refractive index correlation length in the case of strong scattering.

Time variation is introduced by supposing that the layer moves parallel to the observer's plane and without internal rearrangement. In this "frozen flow" case, spatial intensity variations map simply to temporal variations through the pattern velocity, $\vec{u}$.

<u>Source Size, Bandwidth, Intregation Time</u>

Small source size is clearly important for IS: stars twinkle; planets don't. Within the framework of the thin screen, source size effects can be treated elegantly. Imagine an incoherent source with (one dimensional) brightness distribution $b(\theta)$ (see figure). Since $b(\theta)$ is non-zero only over a region much smaller than a radian, each



element of the brightness distribution produces the same intensity pattern on the observer's plane, but offest in space by a distance θz. The net intensity pattern is the sum of these offset "point source" patterns, e.g., $I_{ext}(x) = I_{pnt}(x) * b(\theta z)$ where I_{ext} and I_{pnt} are the extended and point source patterns, x is a transverse coorinate, and * indicates convolution. This is the Cohen-Salpeter equation. Clearly if $b(\theta)$ is too extended, the pattern will be severely smoothed and IS will be reduced below detectability.

An equivalent statement of the Cohen-Salpeter equation can be made by recalling from Fourier transform theory that the transform of the convolution of two functions is the product of the transforms of the individual functions. The transform of the brightness distribution is the visibility function, V. Hence

$$\tilde{I}_{ext}(q) = \tilde{I}_{pnt}(q) \quad V(qz)$$

thus $$\left\langle \left|\tilde{I}_{ext}(q)\right|^2 \right\rangle = \left\langle \left|\tilde{I}_{pnt}(q)\right|^2 \right\rangle \cdot \left|V(qz)\right|^2$$

power spectrum observed — point source power spectrum — squared magnitude of source visibility

where a tilde indicated Fourier transformation and angle brackets denote ensemble averages. The observed power spectrum is a "filtered" version of the square of the source visibility function. IS analyzed in this way give information similar to that obtained from lunar occultations or intensity interferometry.

Instrumental considerations such as bandwidth and integration time also influence the observibility of IS. Bandwidth decorrelation is important if the angular spectrum is broad or if the distance to the screen is large. Scattered power in angular spectrum components greater than $\theta^* \simeq (2\ c/Bz)^{1/2}$ (c is the speed of light, B is the bandwidth of the receiver) cannot interfere to cause IS. Another way of thinking about this bandwidth limit is that the receiver performs a time average of duration B^{-1}. If the time required to go the extra path length $z\ \theta^{*2}/2$ is greater than B^{-1} no interference (hence IS) can occur.

Another way to destroy IS is to integrate too long. The pattern has a typical intensity correlation length l_I and speed u. If the time constant of the receving system is long compared with l_I/u then many uncorrelated samples of the diffraction pattern will be in the receiver at once and the fluctuations will again be smoothed.

Applications of Scintillations to the Study of Random Media

The most common current use of IS is to study the medium causing the scattering. For example, the velocity of the medium can be measured by measuring the pattern velocity of the IS. This is usually done by spacing three or more receivers in the observer's plane and crosscorrelating the temporal intensity series recorded at each site. From the offsets of the cross correlation functions from zero time lag, the pattern speed and angle can be determined. In this way it has been demonstrated that the solar wind flows radially (to within a few degrees) between 0.3 and 1.3 au, independent of solar latitude, but that the flow speed increases with increasing (absolute) solar latitude by about 2.1 km/sec/degree of latitude (Coles and Maagoe 1972, JGR, 77, 5622). Three station observations have also proven useful in studying the temporal evolution and three-dimensional structure of large solar disturbances like the giant flares of August 1972.

The power spectrum of the density fluctuations in the medium can also be determined from IS. One looks at point sources in weak scattering for which it is well known that the observed intensity spectrum is a filtered version of the refractive index fluctuation spectrum. The filter function is well understood and the observed spectrum can be inverted to get the refractive index spectrum (Harmon 1975, Thesis, UC San Diego). Scintillations of pulsar signals have been used to study the microturbulence (scale size of 10^{10} cm) in the interstellar medium and to deduce pattern velocities (Rickett 1970, MNRAS, 150, 67).

Applications of Scintillations to the Study of Sources

Cohen (1969, An. Rev. Astr. Astrophys., 7, 619) compares scintillations, lunar occultation, and interferometric methods of source structure determination.

Immediate rough diameter estimates can be obtained by noting if a source scintillates in a particular medium or not. For example, a source must be smaller than or about equal to 10 arc min, 0.5 arc sec, and 20 microarcsec to scintillate appreciably in the ionosphere, solar wind, and interstellar medium, respectively. Several interplanetary scintillation surveys (e.g., Readhead and Hewish 1974, MRAS, 78,1) have shown that about one-half of all meter wavelength sources contain greater than 10% of their flux on angular scales less than about one arcsec. Most of these surveys folded in models of the solar wind to estimate both diameter and flux density of the compact component. Readhead and Hewish (1972, Nature, 236,440) have shown that, at 81.5 MHz, apparent source diameters increase markedly for low galactic latitude sources. They interpret this as due to angular broadening caused by scattering in the interstellar medium setting a lower limit on the apparent angular sizes of low-frequency sources.

Actual inversion of the IS spectrum to get $|V(qz)|^2$ (see Cohen-Salpeter equation, above) is widely regarded as impractical or impossible. However, in certain situations where the sources are known *a priori* to be circularly symmetric this procedure has been successful (Armstrong, Coles, Kaufman, and Rickett 1973, Ap. J., 186, L141).

Miscellaneous

No mention has been made of phase scintillations, but these exist and may be the ultimate limit to the accuracy of interferometers.

An interferometer measures the true source diameter of the scattering angle at the screen, depending on the situation. Cohen and Cronyn (1974, Ap. J., 192, 193) show that the true diameter is measured unless all of the following are true: (a) intrinsic diameter less than scattering angle, (b) no IS, and (c) point source shows strong IS.

A moving turbulent medium causes IS temporal variations. Thus, even an initially monochromatic source will at the observer have a finite bandwidth. Hence it is in principle impossible to transmit a monochromatic signal through the galaxy. In fact, however, the bandwidth broadening is not large if one goes to high enough radio frequencies. For the widely advertised "water hole" (18-21 cm), the broadening is expected to be only 0.01 - 0.1 Hz, depending on where the transmitter is located and which set of interstellar scintillation data is extrapolated.