Introduction to Radio Astronomy Jargon

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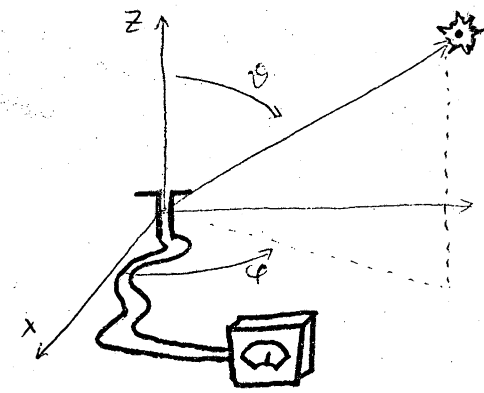


Consider a simple radio telescope, two bent wires attached to a meter, exposed to a point source at large distance.

The meter responds to the presence of the source (for reasons to be discussed below). We define $D(\theta,\phi)$ = meter deflection, where θ and ϕ are the sky coordinates of the point source.

There will be a sky position for the point source such that the meter deflection is a maximum. Define $D_{max} = D(\theta_{max},\phi_{max})$ and

$$\frac{D(\theta,\phi)}{D_{max}} \equiv \mathcal{P}(\theta,\phi) \leq 1$$

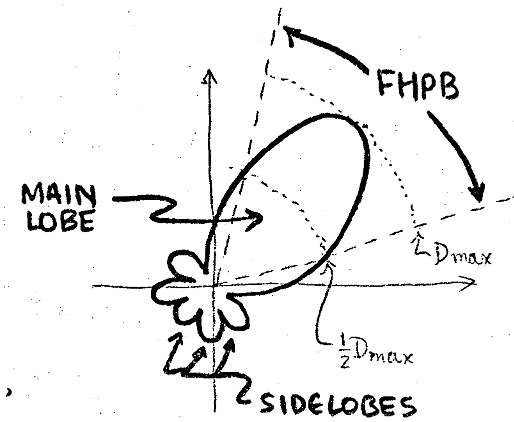
$\mathcal{P}$ is the BEAM PATTERN (or power pattern), a kind of weighting function for the sky. It has a "zeroth" moment:

$$\Omega_A \equiv \iint_{4\pi} \mathcal{P}(\theta,\phi)\, d\Omega \leq 4\pi$$

Ω_A is the (total) BEAM SOLID ANGLE, indicating roughly the total weighting of the sky. One can also define

$D \equiv 4\pi/\Omega_A$, the DIRECTIVITY (or GAIN)

Obviously, if $\mathcal{P}$ is nearly 1 over a lot of the sky, then Ω_A is larger (and D is smaller) than if $\mathcal{P}$ were nearly 1 only over a small part of the sky. We generally try to maximize D, usually by placing cunningly fashioned pieces of metal around our basic antenna.



Consider that part of the beam pattern seen in the plane defined by a great-circle cut through $(\theta_{max}, \phi_{max})$:

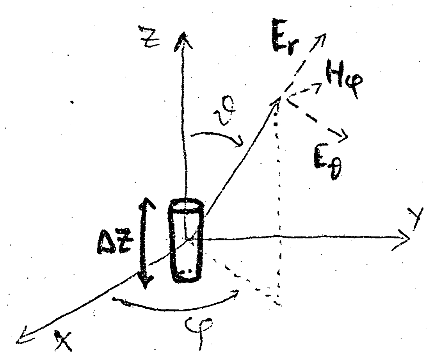
One can define

$$\Omega_M \equiv \iint_{\substack{\text{main}\\ \text{lobe}}} P(\theta,\phi)\, d\Omega \text{ , the MAIN-LOBE SOLID ANGLE,}$$

FHPB = FULL HALF-POWER BEAMWIDTH,

and $\eta_B \equiv \Omega_M/\Omega_A \leq 1$, the BEAM EFFICIENCY.

Note that $\eta_B = D\ \Omega_M/4\pi$, one of many amusing and often useless relations.



To understand the general features of the power pattern, we consider a small chunk of the antenna. For the moment, we will investigate the fields produced by the currents in this chunk, i.e., a transmitting antenna.

If the current is $\mathcal{I} = \mathcal{I}_o\, e^{i\omega t}$,
with $\omega = 2\pi$ (and assuming that we will only consider the real parts of the final quantities), then solution of Maxwell's equations gives

$$E_\theta = \frac{\mathcal{I}_o \Delta Z \sin\theta}{4\pi\varepsilon_o}\ \frac{i\omega}{c^2 r}\ e^{i\omega(t - r/c)} + \mathcal{O}\left(\frac{1}{r^2}\right) + \mathcal{O}\left(\frac{1}{r^3}\right)$$

$$H_\phi = \frac{\mathcal{I}_o \Delta Z \sin\theta}{4\pi}\ \frac{i\omega}{cr}\ e^{i\omega(t - r/c)} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$E_r \sim \mathcal{O}\left(\frac{1}{r^2}\right) + \mathcal{O}\left(\frac{1}{r^3}\right)$$

[$\mathcal{O}(\frac{1}{r^2})$ means "on the order of $\frac{1}{r^2}$"]

$$E_\phi = H_r = H_\theta = 0$$

(For a more detailed solution, follow chapter 9 of Jackson's <u>Classical Electrodynamics</u> - but watch out for the different units.)

In the "FAR-FIELD LIMIT", i.e., at large r, the fields are dominated by the terms of order 1/r. In particular,

$$\left.\frac{E_\theta}{H_\phi}\right|_{\text{far-field}} \sim \frac{1}{\varepsilon_o c} = 377 \text{ ohms (impedance of free space)}$$

So E_θ is the only significant quantity at large r.

The emitted power is obtained from the Poynting vector

$$\vec{S} = \frac{1}{2} (\vec{E} \times \vec{H}^*) = \frac{(\mathcal{I}_o \Delta Z \sin\theta)^2 \omega^2}{32\pi^2 \varepsilon_o c^3 r^2}$$

$$\text{the total power emitted} = \int_{\substack{\text{some surface}\\ \text{at } r \to \infty}} \vec{S} \cdot \hat{n} \, dA$$

$$dP = |\vec{S}| dA = \frac{(\mathcal{I}_o \Delta Z \sin\theta)^2 \omega^2}{32\pi \, \varepsilon_o c^3} \, d\Omega$$

$$\frac{dP}{d\Omega} = \frac{15\pi}{\lambda^2} (\mathcal{I}_o \Delta Z \sin\theta)^2$$

We recognize $\mathcal{P} = \dfrac{(dP/d\Omega)}{(dP/d\Omega)_{max}} = \sin^2\theta$

$$\Omega_A = \iint_{4\pi} \mathcal{P} d\Omega = \frac{8\pi}{3} \quad ; \quad D = {}^{4\pi}/_{\Omega_A} = {}^{3}/_{2}$$

$$\mathcal{P} = \tfrac{1}{2} \text{ at } \theta = 45^o, \text{ so FHPB} = 90^o$$

The total power emitted is

$$P_{em} = \iint_{4\pi} \frac{dP}{d\Omega} d\Omega = \frac{15\pi}{\lambda^2} (\mathcal{I}_o \Delta Z)^2 \Omega_A = \frac{40\pi}{\lambda^2} (\mathcal{I}_o \Delta Z)^2$$

At this point, one may legitimately ask about the relevance to the astronomical problem of formulas derived for transmitting antennas. The answer is that, by the principle of RECIPROCITY, the power pattern of a transmitting antenna is identical to that of a receiving antenna.

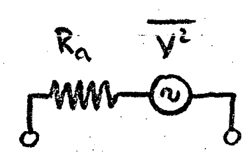
Note first that, in the derivation of the fields above, we dropped terms arising from the advanced potentials, i.e., terms in $\exp[i\omega(t + \frac{r}{c})]$. Assuming that the medium of propagation is linear and passive, this was done entirely on grounds of causality - we were thinking about the problem in terms of generating the fields by making the currents. Had we retained the advanced terms and dropped the retarded ones instead, we would have been solving the same boundary-value problem, but with a different causal interpretation: what incoming fields are required to stimulate the hypothesized currents?

More to the point here: suppose we have two identical antennas, one set up for transmitting and the other for receiving. We generate a current flow $\mathcal{I}_A$ in antenna A, and find that we measure a current flow $\mathcal{I}_B$ at antenna B. The principle of reciprocity asserts that, if we now generate a current flow $\mathcal{I}_B$ in antenna B, we will measure a current flow $\mathcal{I}_A$ in antenna A, and that this will be true for all relative orientations of the two antennas.

Kraus' Antennas, chapter 10, discusses the proof of this principle.

Note, by the way, that reciprocity is not a guaranteed thing. For example, the ionosphere is not a linear, passive medium. (We get around this by considering the ionosphere as part of the observed source; i.e., we cheat.)

Having satisfied ourselves of the significance of the transmitting antenna, we now consider it as an electrical system. The antenna is equivalent to a source of fluctuating voltage V (which we will characterize by the mean-square voltage $\bar{v}^2$), and some characteristic resistance.



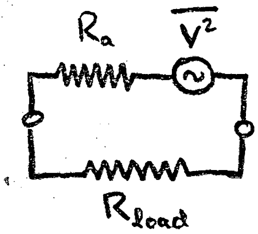
Nyquist (1928 Phys. Rev., 32, 110) showed that a resistance R_a, at a temperature T_a, will produce a constant noise power per unit frequency interval. Thus we will find it convenient to define the fluctuating voltage in our antenna as arising from some characteristic temperature of its characteristic resistance:

$$\overline{v^2} = 4kT_a \, \Delta\nu \, R_a .$$

Thus, the power emitted into frequency interval $\Delta\nu$ is

$$P_{em} = kT_a \, \Delta\nu$$

In the case of a receiving antenna, we connect a receiver (of characteristic resistance R_{load}) to the antenna and draw power in the amount of P_{load} $= \overline{v^2} R_a/(R_a + R_{load})^2$.



The maximum power drawn is obtained by matching the receiver to the antenna, which requires setting $R_{load} = R_a$ (cf. Kraus' Antennas, chapter 3)

$$P_{abs} = P_{load}(\max) = \frac{\overline{v^2} R_a}{(2R_a)^2} = \frac{\overline{v^2}}{4R_a}$$

From reciprocity, we can identify $\overline{v^2}$ arising in the receiving antenna with the $\overline{v^2}$ in the transmitting antenna.

By Nyquist's law, $P_{abs} = kT_a \Delta\nu$.

Thus, we have identified the power stimulated in the antenna by the presence of a source with that obtained by heating the characteristic antenna resistance to some temperature, T_a.

To clarify this, imagine the antenna and the source put into a closed black box. After some time, the system of antenna + source radiation will approach thermal equilibrium. Then, the power absorbed by the antenna (due to the radiation of the source) must be exactly equal to the power emitted by the antenna (due to its characteristic temperature). The equilibrium temperature of the antenna resistance, which we will now call T_A, is a good thermodynamical measure of the source intensity. T_A is called, naturally, the ANTENNA TEMPERATURE.

Radio astronomers often prefer to use temperature units, rather than fluxes or intensities, because they are the closest things to actual measured voltages (thanks to Nyquist's Law). As we shall see below, while the specific intensity is the most physical quantity with which to characterize a source, we can only infer it from measurement via the difficult deconvolution of power pattern and intensity distribution.

In real observing, T_A refers to intensity contributions of all things in the beam -- source, sky, birds, etc. Furthermore, the noise power associated with T_A is combined with noise power arising from the receiver, $P_{load} = kT_{load}\Delta\nu$, so that we actually measure

$$T_{sys} = T_{load} + T_A \quad , \quad \text{the SYSTEM TEMPERATURE}.$$

T_{load} is usually fairly constant, but because $T_{load} >> T_A$ in most cases, the actual measurement of T_A is usually difficult.

We may characterize the sensitivity of the system as follows: $\overline{v^2}$ is a mean sum of the potentials that arise from collisions among the thermally agitated electrons in the system. So we let the rate of collisions be $n_c \propto T_{sys}$. We measure n_c by just counting collisions for awhile. If we make a total of N counts, then the uncertainty in the derived value of n_c is

$$\frac{\Delta n_c}{n_c} \propto \frac{1}{\sqrt{N}} \qquad \text{(from Poisson statistics)}$$

The individual collisions are instantaneous, or nearly so. But because the detector has a finite bandwidth, $\Delta\nu$, its response to an individual collision has a time constant, $\frac{1}{\Delta\nu}$. So in an integration time of τ seconds, the maximum number of independent counts that one can get is $N = \Delta\nu\ \tau$.

$$\frac{\Delta n_c}{n_c} \propto \frac{1}{\sqrt{\Delta\nu\ \tau}}$$

or $$\Delta T = \frac{KT_{sys}}{\sqrt{\Delta\nu\ \tau}}$$, the RADIOMETER EQUATION.

K is a constant of proportionality, of order unity, which depends on the specific characteristics of the receiver and the mode of observation.

Unfortunately, the thermal noise is not the only factor limiting sensitivity. Continuum observations are affected by gain and bandpass instabilities, so that one should more properly write

$$\Delta T \propto T_{sys} \sqrt{\left(\frac{1}{\Delta\nu\ \tau}\right) + \left(\frac{\Delta G}{G}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

where G = system gain and B = $\Delta\nu$ (for neatness). They are also affected by CONFUSION, which is quite literally the inability to distinguish between various sources in the beam:

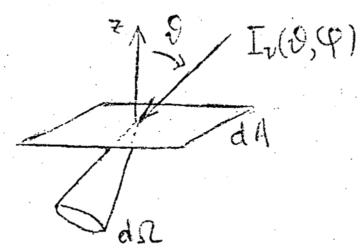
> There are $\frac{4\pi}{\Omega_M}$ independent pieces of sky for a telescope of given Ω_M. Suppose there are also $\mathcal{N}$ sources in the sky for which $T_A \geq (3 \text{ to } 5) \times \Delta T$. (The factor 3 to 5 depends on your choice of what represents statistical significance in the presence of thermal noise.)
>
> Is $\mathcal{N} > \frac{4\pi}{\Omega_M}$? If so, then there is likely to be more than one detectable source in a given beam.

Spectral line observations are affected by standing waves, reflections between the feed and various parts of the telescope that produce a frequency dependence in the gain. In usual spectral line observations, one subtracts off-source spectra from on-source spectra, and thus cancels out much of the standing wave. But the wave amplitude is generally $\propto T_{sys}$, so the cancellation is not perfect. Also, the standing wave is not constant with changing telescope orientation and thermal conditions. The various imperfections in cancelling out standing waves can add up to produce spurious signals that obscure or (worse) mimic the true signals.

Return to the problem of the telescope response to a source in the sky.

The incident radiation can be represented by the specific intensity, $I_\nu(\theta,\phi)$

The power passing through area dA, directed into solid angle dΩ, is



$$dP_{inc} = I_\nu(\theta,\phi)\ \cos\theta\ dA\ d\Omega\ d\nu$$

Thus the total power incident on the antenna is

$$P_{inc} = \int_\nu^{\nu+\Delta\nu} d\nu \int_{4\pi} d\Omega \int_{antenna} dA\ \cos\theta\ I_\nu(\theta,\phi)$$

We have allowed for the fact that the source may be extended in θ and ϕ, and have assumed that the intensities from each direction can be added linearly, i.e., that the source emits incoherently.

Assuming that I_ν is constant over $\Delta\nu$, and that $\cos\theta \simeq 1$ (i.e., that we point the antenna at the source)

$$P_{inc} = \Delta\nu \int_{4\pi} A\ I_\nu(\theta,\phi)\ d\Omega \qquad , \quad A = \text{physical area of telescope}$$

Of course, $P_{abs} \neq P_{inc}$ in general. We usually write

$$P_{abs} = \tfrac{1}{2}\ \Delta\nu \iint_{4\pi} d\Omega\ \ A_e(\theta,\phi)\ I_\nu(\theta,\phi)$$

Here, the factor ½ comes from assuming that the radiation has equal intensity in orthogonal polarizations, and that the telescope is sensitive to only one polarization. Also

$$A_e(\theta,\phi) = A_e \mathbf{P}(\theta,\phi)$$

A_e = EFFECTIVE AREA of the telescope

$\eta_A \equiv A_e/A$ = APERTURE EFFICIENCY, which depends on such details as how the feed illuminates the reflector, how much the feed legs block the reflector, etc.

We also define $S_\nu = \iint_{source} d\Omega\ I_\nu(\theta,\phi)$, the FLUX DENSITY of the source.

If the source is small compared to the size of the main lobe, then

$$P_{abs} \simeq \tfrac{1}{2}\,\Delta\nu\, A_e S_\nu .$$

One can derive the relation $\lambda^2 = A_e\Omega_A$ from a detailed analysis of the field and current distribution over the aperture (cf. Kraus' Radio Astronomy, chapter 6, esp. §§6-2, 6-7). This can be combined with the above relations to get such formulas as

$$D = \frac{4\pi}{\lambda^2} A_e \qquad \text{(our version of the Rayleigh formula)}$$

$$\frac{\eta_A}{\eta_B} = \frac{A_e}{A}\,\frac{\Omega_A}{\Omega_M} = \frac{\lambda^2}{A\Omega_M}$$

(etc.)

Combining $\lambda^2 = A_e\Omega_A$ with Nyquist's Law, we find the final expression for what we measure:

$$T_A = \frac{\lambda^2}{2k}\,\frac{1}{\Omega_A}\iint_{4\pi} d\Omega\, \mathcal{P}(\theta,\phi)\; I_\nu(\theta,\phi)$$

Now, recall that for a black-body, $I_\nu = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT_B} - 1\right)^{-1}$.

In the case $h\nu << kT_B$, $I_\nu \simeq \frac{2kT_B}{\lambda^2}$. This suggests that it will be convenient to define a RADIATION TEMPERATURE

$$J(T_B) \equiv \frac{\lambda^2}{2k} I_\nu .$$

Note that $J(T_B) \to T_B$ in the limit $h\nu << kT_B$. In fact, because $h\nu << kT_B$ was generally true in early radio astronomy, it was conventional to skip this step and just define the BRIGHTNESS TEMPERATURE $T_B = \lambda^2 I_\nu/2k$ (e.g., Wild 1952 Ap. J., 115, 206)

For the sake of convenience, we will also substitute T_B for $J(T_B)$ in the rest of these notes. But it is important to remember the distinction, because much radio astronomy is now done at millimeter wavelengths, where $h\nu << kT_B$ is not true. So before using the expressions below, check $h\nu/kT_B$ and decide whether or not to substitute

$$T_B \to J(T_B) = \frac{h\nu}{k}\left(e^{h\nu/kT_B} - 1\right)^{-1}$$

We now have the traditional expression

$$T_A = \frac{1}{\Omega_A} \iint_{4\pi} d\Omega \, \mathbb{P}(\theta,\phi) \, T_B(\theta,\phi)$$

We can consider two important limiting cases:

i)

$$\text{If } T_B(\theta,\phi) = \begin{cases} T_B, \text{ constant (over } \Omega_S \ll \Omega_M) \\ 0 \quad \text{(elsewhere)} \end{cases}$$

$$\text{then } T_A \simeq \frac{\Omega_S}{\Omega_A} T_B = \eta_B \frac{\Omega_S}{\Omega_M} T_B$$

Ω_S/Ω_M is called the DILUTION FACTOR.

ii)

$$\text{If } T_B(\theta,\phi) = \begin{cases} T_B \text{ (over } \Omega_S \gg \Omega_M, \text{ but } \Omega_S \ll \Omega_A) \\ 0 \quad \text{(elsewhere)} \end{cases}$$

$$\text{then } T_A \simeq \frac{\Omega_M}{\Omega_A} T_B = \eta_B T_B.$$

While these two cases can be used frequently, the true situation almost always involves the more complex convolution of $\mathbb{P}(\theta,\phi)$ with $T_B(\theta,\phi)$.

The actual physical conditions of the source are coded into I_ν via the EQUATION OF TRANSFER. We can show by inspiration (argument from first principles is much harder) that along a line of sight

$$dI_\nu = j_\nu ds - k_\nu I_\nu ds, \quad \text{where}$$

j_ν and k_ν are the volume emission and volume absorption coefficients, which depend on the physical conditions along the line of sight.

$$\frac{dI_\nu}{ds} = j_\nu - k_\nu I_\nu$$

We define the OPTICAL DEPTH, $d\tau_\nu = k_\nu ds$

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{k_\nu} - I_\nu$$

We can thus derive a formal solution for integration along a path from τ_1 to τ_2:

$$I_\nu(\tau_2) = I_\nu(\tau_1) \, e^{-(\tau_2-\tau_1)} + \int_{\tau_1}^{\tau_2} d\tau_\nu \frac{j_\nu}{k_\nu} e^{-(\tau_2 - \tau_\nu)}$$

Generally, we define $\tau_1 = 0$, radiation temperature $J(T_B) = \frac{\lambda^2 I_\nu}{2k}$,

and SOURCE RADIATION TEMPERATURE $J(T_S) = \frac{\lambda^2}{2k}\frac{j_\nu}{k_\nu}$. So

$$T_B(\tau_2) = T_B(0)e^{-\tau_2} + \int_0^{\tau_2} T_S(\tau)e^{-(\tau_2-\tau)}\,d\tau$$

where we have incautiously adopted $J(T_B) \to T_B$, $J(T_S) \to T_S$.

If the source function j_ν/k_ν is uniform along the path, i.e. $T_S(\tau)$ = const., then

$$T_B(\tau_2) = T_B(0)e^{-\tau_2} + T_S\,(1-e^{-\tau_2})$$

Note that, if $\tau_2 << 1$, $T_B(\tau_2) \approx T_B(0) + T_S\tau_2$

and if $\tau_2 << 1$, $T_B(\tau_2) \approx T_S$

Finally, we often observe a source by comparing a measurement on-source with one made nearby, but off the source. Defining $T_B(0) = T_{bg}$, the background brightness temperature, and dropping the subscript Z's that have been cluttering the equations, we have

$$T_B(\text{on-source}) = T_{bg}\,e^{-\tau} + T_S(1-e^{-\tau})$$

$$T_B(\text{off-source}) = T_{bg}$$

and $\Delta T_B = T_B(\text{on}) - T_B(\text{off}) = (T_S - T_{bg})(1-e^{-\tau})$.

ΔT_B is often called the EXCESS BRIGHTNESS TEMPERATURE.

Note that if $T_S > T_{bg}$, the source appears in emission, while if $T_S < T_{bg}$, the source appears in absorption.